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Letter

Simplified permeable surface correction for frequency-domain Ffowcs Williams and Hawkings integrals

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ABSTRACT

A simplified surface correction formulation is proposed to diminish the far-field spurious sound generated by the quadrupole source term in Ffowcs Williams and Hawkings (FW-H) integrals. The proposed formulation utilizes the far-field asymptotics of the Green's function to simplify the computation of its high-order derivatives, which circumvents the difficulties reported in the original frequency-domain surface correction formulation. The proposed formulation has been validated by investigating the benchmark case of sound generated by a convecting vortex. The results show that the proposed formulation successfully eliminates the spurious sound. The applications of the proposed formulation to flows with some special parameters are also discussed.

The Ffowcs Williams and Hawkings (FW-H) equation extends Lighthill's acoustic analogy method to flows with moving solid/permeable boundaries, resulting in an inhomogeneous wave equation that includes monopole, dipole and quadrupole sources on the right hand side. The solution to the FW-H equation can be expressed as integrals of the sources (hereinafter referred to as the FW-H integrals) by using the Green's function. The integrals of monopole and dipole sources are surface integrals and that of quadrupole sources is a volume integral [1].

The computation of the volume integral is much more challenging than that of surface integrals [2,3]. In computing the far-field sound generated by low Mach number flows, the volume integral is usually ignored with the assumption that the sound is dominated by the monopole and dipole sources. However, recent researches find that the ignoration of the quadrupole sources may generate spurious sound even though the flow is at low Mach numbers [4–8].

Different methods have been proposed to eliminate the spurious sound associated with the quadrupole sources [5,9,10]. In particular, Wang et al. [11] found that the spurious sound is caused by the eddy crossing the surface of the integral domain. Then a new surface integral (hereinafter referred to as surface correction) is proposed to correct the contribution of quadrupole sources to the far-field sound. The surface correction formulation is constructed with the assumption that the eddy is frozen when it is convected across the integral surface. The convection velocity is usually computed by scaling the freestream velocity [11]. The spurious sound can be correctly eliminated as long as the eddy leaves the integral surfaces with relatively uniform velocity [11]. The surface correction formulation is then improved by Nitzkorski et al. [12], Rahier et al. [8] and Ikeda et al. [4] by taking into account the effects of non-uniform convection velocity. The above surface correction formulation is derived in a convective frame of reference and employed in the time-domain method for the FW-H integrals. The application of the above surface correction to the frequency-domain method is usually not straight forward, because most of the frequency-domain methods are derived in a laboratory frame of Ref. [11].

Ikeda et al. [13] examined the far-field approximation for the surface correction between the laboratory frame of reference and convective frame of reference and derived a frequency-domain formulation via a Fourier transform of the time-domain formulation. Lockard and Casper [12] proposed an alternative method to derive the surface correction formulation in the frequency domain. This alternative method converts the volume integral into a series of surface integrals by repeatedly using the formulation of integration by parts. The proposed surface correction is validated by computing the sound generated by a 2D convecting vortex. The advantage of this method is that the surface correction formulation is directly derived in the frequency domain without referring to the

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surface correction formulation in the time domain. The disadvantage is that the surface correction formulation involves high-order derivatives of the Green's function. The computation of the high-order derivatives of the Green's function are quite complicated and nontrivial [14].

The objective of this letter is to propose a simplified surface correction formulation based on the work of Lockard and Casper [14]. We simplify the computation of the high-order derivatives of the Green's function based on its far-field approximations and validate the proposed formulation by using the 2D convecting vortex.

The solution to the FW-H equation in the frequency domain can be given by the FW-H integrals as follows

$$p'(\mathbf{x},\omega) = -\underbrace{\int_{f=0} \mathrm{i}\omega Q(\mathbf{y},\omega)G(\mathbf{x};\mathbf{y})\mathrm{d}S}_{I_T} - \underbrace{\int_{f=0} F_i(\mathbf{y},\omega)\frac{\partial G(\mathbf{x};\mathbf{y})}{\partial x_i}\mathrm{d}S}_{I_L} - \underbrace{\int_{f>0} \mathrm{i}\omega T_{ij}(\mathbf{y},\omega)G(\mathbf{x};\mathbf{y})\mathrm{d}V}_{I_Q}, \quad (1)$$

where the I_T and I_L terms are the surface integrals of monopole sources and dipole sources, respectively, I_Q term is the volume integral of quadrupole sources. $Q = (\rho(u_i+U_i) - \rho_o U_i)n_i$, $F_i = (P_{ij} + \rho(u_i - U_i)(u_j + U_j) + \rho_o U_i U_j)n_j$, $T_{ij} = \rho u_i u_j + P_{ij} - c_0^2\rho'\delta_{ij}$, u_i is the ith component of the velocity of the fluid, U_i is the ith component of the freestream velocity, $P_{ij} = (p - p_0)\delta_{ij} - \tau_{ij}$ is the compressive stress tensor and τ_{ij} is viscous stress tensor. Here, we ignore the viscous part according to the work of Lockard and Casper [14]. f is a level set function where $f=0$ indicates the FW-H surface and $f>0$ is the region outside the FW-H surface.n_j is the unit normal of the FW-H surface. ρ_o, c_o and p_o represent the density, speed of sound and pressure in the background flow, respectively. $\rho' = \rho - \rho_o$ and $p' = p - p_o$ are perturbations of density and pressure, respectively. In accordance with the work of Lockard and Casper [14], we set the $o - y_1$ direction as the freestream flow direction. Hence, the Green's functions G in Eq. (1) for 2D and 3D flows are

$$G_{2D}(\mathbf{x};\mathbf{y}) = \frac{\mathrm{i}}{4\beta}\exp\left(\frac{\mathrm{i}Mk(x_1-y_1)}{\beta^2}\right)\mathrm{H}_0^{(2)}\left(\frac{k}{\beta^2}R\right), \quad (2)$$

$$G_{3D}(\mathbf{x};\mathbf{y}) = \frac{-1}{4\pi d}\exp(\varphi_{3D}(\mathbf{x};\mathbf{y})), \quad (3)$$

respectively. Here $\beta = \sqrt{1-M^2}$, $R = \sqrt{(x_1-y_1)^2 + \beta^2(x_2-y_2)^2}$, $d = \sqrt{(x_1-y_1)^2 + \beta^2(x_2-y_2)^2 + \beta^2(x_3-y_3)^2}$, $k = \frac{\omega}{c_o}$ and $\varphi_{3D}(\mathbf{x};\mathbf{y}) = -\mathrm{i}k\frac{d - M(x_1-y_1)}{\beta^2}$. M is the Mach number of the freestream flow. $\mathrm{H}_0^{(2)}$ is the zero-order Hankel function of the second kind.

The terms I_T and I_L in Eq. (1) are surface integrals. I_Q is the volume integral over the region $f>0$. The computation of I_Q is much more challenging than I_T and I_L, because the Lighthill stress tensor at all grid points within the region $f>0$ is needed to be experimentally measured or numerically simulated. Usually, the domain $f>0$ extends to far downstream of the wake for high Reynolds number flows. It is very difficult to measure or compute the large domain of $f>0$ in flows of practical interest. To compute the contribution of the quadrupole term efficiently, many efforts have been devoted to transforming the quadrupole term from a volume integral to surface integrals [10,11,13]. The frequency-domain surface correction proposed by Lockard and Casper [14] is as follows,

$$I_Q(\mathbf{x};\omega) \approx -\sum_{l=1}^{n}\int_{f=0}\left(\frac{U_1}{\mathrm{i}\omega}\right)^l T_{ij}(\mathbf{y},\omega)\frac{\partial^{l-1}}{\partial y_1^{l-1}}\left(\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right)\mathrm{d}S. \quad (4)$$

To ensure the accuracy of the series, $n \geq 2$ is necessary [14]. However, as pointed out by Lockard and Casper [14], the computation of high-order derivatives of the Green's function is quite complicated. They refer to the work of Gloerfelt et al. [15] to compute the second derivatives of the Green's function in two dimension and use a symbolic algebra package to obtain the higher-order derivatives.

We propose a simplified surface correction formulation as follows to diminish the spurious far-field sound generated by the eddy crossing the FW-H integral surface,

$$I_Q(\mathbf{x};\omega) \approx -\sum_{l=1}^{n}\int_{f=0}\left(\frac{U_1}{\mathrm{i}\omega}\right)^l T_{ij}(\mathbf{y},\omega)\left(\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{l-1}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}S, \quad (5)$$

where $G = G_{3D}$, $\varphi = \varphi_{3D}$ for 3D flows and $G = G'_{2D}$, $\varphi = \varphi'_{2D}$ for 2D flows. G'_{2D} is the asymptotic Green's function in 2D space

$$G'_{2D}(\mathbf{x};\mathbf{y}) \approx \frac{\mathrm{i}}{4\beta}\left(\frac{2\beta^2}{\pi kR}\right)^{\frac{1}{2}}\exp\left(\varphi'_{2D}(\mathbf{x};\mathbf{y})\right),$$
$$\varphi'_{2D}(\mathbf{x};\mathbf{y}) = \mathrm{i}\left[\frac{Mk(x_1-y_1)}{\beta^2} + \frac{\pi}{4} - \frac{k}{\beta^2}R\right]. \quad (6)$$

Equation (5) employs the approximations as follows to simplify the computation of the high-order derivatives of the Green's function at the far-field

$$\frac{\partial^k}{\partial y_1^k}\left(\frac{\partial^2 G'_{2D}(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right) \approx \left(\frac{\partial\varphi'_{2D}(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^k\frac{\partial^2 G'_{2D}(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}, \quad (7)$$

$$\frac{\partial^k}{\partial y_1^k}\left(\frac{\partial^2 G_{3D}(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right) \approx \left(\frac{\partial\varphi_{3D}(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^k\frac{\partial^2 G_{3D}(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}. \quad (8)$$

The proof of Eq. (7) is briefly reported as follows. The kth-order derivative of G'_{2D} with respect to y_1 is

$$\frac{\partial^l G'_{2D}(\mathbf{x};\mathbf{y})}{\partial y_1^l} = \frac{\mathrm{i}}{4\beta}\left(\frac{2\beta^2}{\pi k}\right)^{\frac{1}{2}}\sum_{k_1=0}^{l} C_l^{k_1}\frac{\partial^{k_1}\exp\left(\varphi'_{2D}(\mathbf{x};\mathbf{y})\right)}{\partial y_1^{k_1}} \times \frac{\partial^{l-k_1}\left(R^{-\frac{1}{2}}\right)}{\partial y_1^{l-k_1}}, \quad (9)$$

where $C_l^{k_1}$ is the binomial coefficient. We use the far-field condition $|\mathbf{x}| >> |\mathbf{y}|$ to approximately compute the hth-order partial derivatives of R and $R^{-\frac{1}{2}}$ with respect to y_1

$$\frac{\partial^h R}{\partial y_1^h} \approx \alpha_1\frac{1}{R^{h-1}}\frac{\partial R}{\partial y_1},$$
$$\frac{\partial^h\left(R^{-\frac{1}{2}}\right)}{\partial y_1^h} \approx \alpha_2\frac{1}{R^{h-1}}\frac{\partial R^{-\frac{1}{2}}}{\partial y_1}, \quad (10)$$

where $h \geq 1$. In the far field, α_1 and α_2 are $O(1)$ and independent of R [16]. By using the first line of Eq. (10), the s_1-order derivative of $\exp(\varphi(\mathbf{x};\mathbf{y}))$ is approximated as follows

$$\frac{\partial^{s_1}\exp(\varphi(\mathbf{x};\mathbf{y}))}{\partial y_1^{s_1}} \approx \left(\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{s_1}\exp(\varphi(\mathbf{x};\mathbf{y}))(s_1 \geq 1). \quad (11)$$

Equation (7) is obtained by ignoring the small terms in Eq. (9) according to Eqs. (10) and (11). More details of the proof can be found in the supplementary material [14]. Thus, Eq. (7) is proved. Eq. (8) can be proved by using the similar method. Compared with Eq. (4), the computation of the high-order derivatives $\frac{\partial^{l-1}}{\partial y_1^{l-1}}\left(\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right)$ are replaced by $\left(\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{l-1}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}$. The computation of high-order derivatives of the Green's function is circumvented.

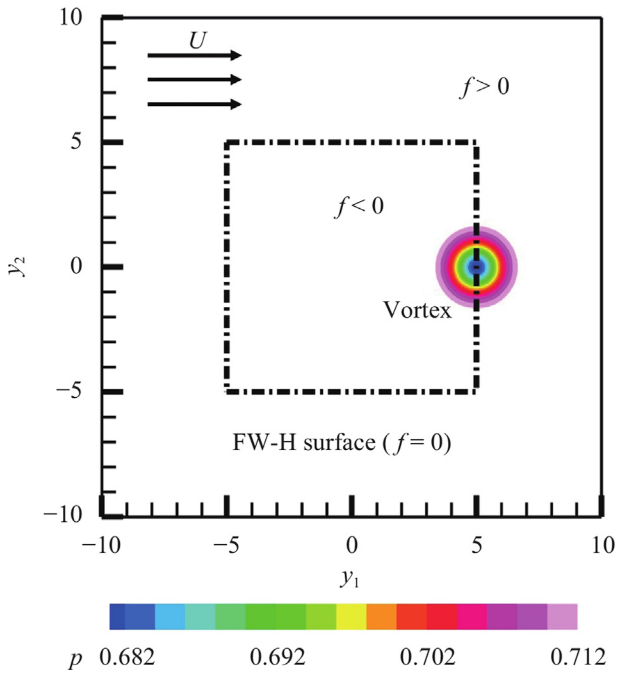

Fig. 1. Schematics of the FW-H surface position and pressure contours of the vortex.

In accordance with the work of Lockard and Casper [14], we validate the simplified surface correction formulation (Eq. (5)) by using the benchmark case of spurious sound generated by a 2D convecting vortex crossing the FW-H integral surface. The normalized pressure and velocity of the 2D convecting vortex is given by [12]

$$\begin{aligned} &p' = \frac{1}{\gamma}\left[1 - a_2\exp\left(1 - r^2\right)\right]^{\frac{\gamma}{\gamma-1}},\\ &\rho = \left(\frac{p}{Po}\right)^{\frac{1}{\gamma}},\\ &u_1 = U_1 - a_0 a_1 y_2 \exp\left(\frac{1-r^2}{2}\right),\\ &u_2 = a_0 a_1 (y_1 - Mt)\exp\left(\frac{1-r^2}{2}\right), \end{aligned} \tag{12}$$

where $a_0 = 1$, $a_1 = 1/(2\pi)$, $a_2 = (\gamma - 1)a_0^2 a_1^2/2$ and $r^2 = (y_1 - Mt)^2 + y_2^2$. $\gamma = 1.4$ is the specific heat ratio of air. Po is the pressure in the background flow.

A permeable square with each side of 10 unit lengths centered at origin is used as the FW-H integral surface, as shown in Fig. 1. The convecting vortex moves at the Mach number of $Ma = 0.2$. The observer location is at $(100, 0)$ in the downstream of the convecting vortex. A spurious sound is generated when the vortex crosses through the FW-H surface. The spurious sound can be computed by using $I_S = I_T + I_L$ for this special flow, because sound pressure approaches to zero exponentially in the far-field. The negative of the spurious sound pressure generated by the vortex crossing the FW-H interface is plotted in Fig. 2. The surface corrections computed by using Eq. (5) with $n = 1$, 2 and 3 are also plotted in Fig. 2. The spurious sound pressure is correctly diminished by the proposed simplified surface correction with $n \geq 2$. The result is consistent with that of Lockard and Casper [14].

We also compared the details of the simplified surface correction (Eq. (5)) with the original surface correction (Eq. (4)) proposed by Lockard and Casper [14]. The surface corrections computed by using these two equations are plotted in Fig. 3. The results show that the surface corrections computed by using the simplified surface correction (Eq. (5)) are in good agreement with the corresponding results of the original surface correction (Eq. (4)) with $n = 1$, 2 and 3 at the downstream observer $(100, 0)$. Figure 4 plots the surface correction computed by Eq. (5) with $n = 2$ and 3 at

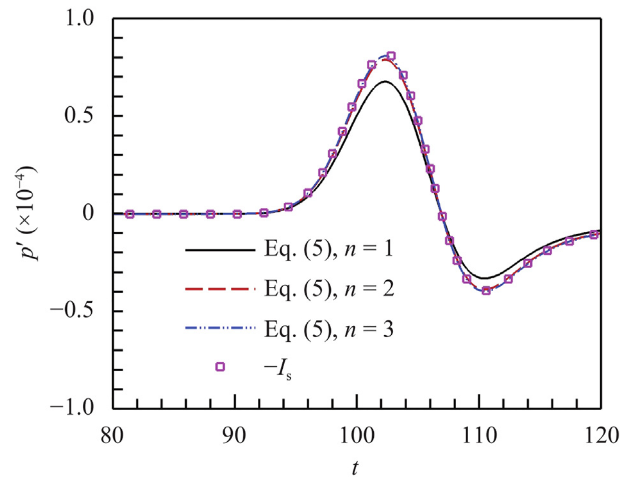

Fig. 2. Negative of the spurious sound pressure and the correction computed by using the proposed simplified surface correction for the sound generated by a convecting vortex crossing the FW-H integral surface.

different downstream observers when the non-dimensional time is $t = 120$. The distance between the downstream observer and the initial vortex center is denoted by R. It is observed that the surface correction computed by using Eq. (5) with $n = 2$ and 3 can correctly diminish the spurious sound when $R > 60$. The results show that the simplified surface correction proposed by this work is valid for computing the far-field sound.

The simplified surface correction formulation (Eq. (5)) not only provides a simplified surface correction but also helps to identify the limitations of the previous surface correction. As pointed out by Lockard and Casper [14], the series of surface integrals of Eq. (4) is divergent when the convecting vortex moves at $Ma = 0.6$. The divergence of the series can be clearly inferred from Eq. (5), since Eq. (5) is the approximation of Eq. (4) at the far field. We note that Eq. (5) is a geometric series with a ratio of $\frac{U_1}{i\omega}\frac{\partial\varphi}{\partial y_1}$. The series is divergent when $|\frac{U_1}{i\omega}\frac{\partial\varphi}{\partial y_1}| > 1$. Therefore, the divergence of Eq. (4) may occur when the convection velocity is larger than the phase velocity ($|\frac{U_1}{i\omega}\frac{\partial\varphi}{\partial y_1}| > 1$).

It is worth noting that $\frac{U_1}{i\omega}\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1} \neq 1$ for subsonic flows. This inequality can be interpreted by analysing the 2D subsonic flows as follows. According to Eq. (6), $\frac{U_1}{i\omega}\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1}$ can be simplified as follows,

$$\frac{U_1}{i\omega}\frac{\partial\varphi_{2D}(\mathbf{x};\mathbf{y})}{\partial y_1} = \frac{U_1}{i\omega}\left[-\frac{ik(M+A)}{\beta^2}\right] = 1 - \frac{1+AM}{1-M^2}, \tag{13}$$

where A is $(y_1 - x_1)/R$. We have $-1 \leq A \leq 1$ since $R = \sqrt{(x_1 - y_1)^2 + \beta^2(x_2 - y_2)^2}$. Thus $-1 < AM < 1$ and $0 < (1 + AM)/(1 - M^2) < 2/(1 - M^2)$ for subsonic flows due to $0 < M < 1$. Using the inequality $0 < (1 + AM)/(1 - M^2)$, we obtain

$$\frac{U_1}{i\omega}\frac{\partial\varphi_{2D}(\mathbf{x};\mathbf{y})}{\partial y_1} < 1. \tag{14}$$

Equation (14) shows that $\frac{U_1}{i\omega}\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1} \neq 1$ for 2D subsonic flows. We can also prove $\frac{U_1}{i\omega}\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1} \neq 1$ for 3D subsonic flows by using the similar method.

For the special case of $\frac{U_1}{i\omega}\frac{\partial\varphi(\mathbf{x};\mathbf{y})}{\partial y_1} = -1$, the I_Q term in Eq. (1) can be expressed as follows by using the integral by parts repeatedly [12]

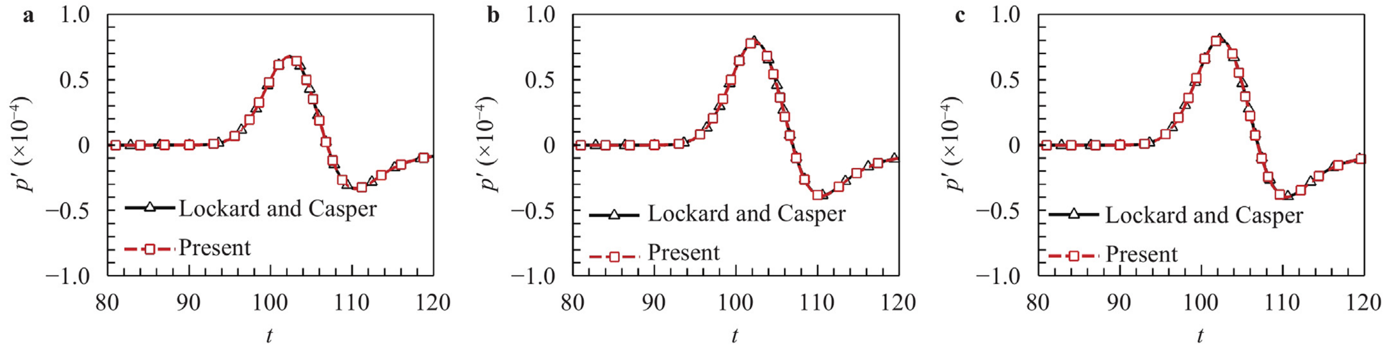

Fig. 3. Comparison between the simplified surface correction proposed in the present work (Eq. (5)) and the surface correction proposed by Lockard and Casper [12] (Eq. (4)) with **a** $n = 1$, **b** $n = 2$, **c** $n = 3$.

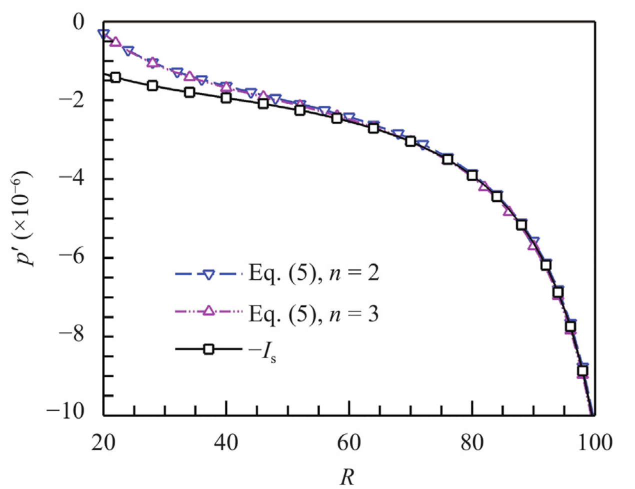

Fig. 4. The instantaneous surface correction at the downstream observer at $t = 120$.

$$I_Q(\mathbf{x};\omega) = -\int_{f>0} (-1)^n T_{ij}(\mathbf{y},\omega)\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}V - \sum_{l=1}^{n}\int_{f=0} (-1)^l T_{ij}(\mathbf{y},\omega)\left(\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{-1}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}S. \tag{15}$$

When n is even, Eq. (15) reduces to the identity $I_Q = I_Q$. When n is odd, Eq. (15) reduces to

$$I_Q(\mathbf{x};\omega) = \frac{1}{2}\int_{f=0} T_{ij}(\mathbf{y},\omega)\left(\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{-1}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}S. \tag{16}$$

Equation (16) is much simpler than Eq. (5) for the surface correction, because Eq. (16) consists of only one surface integral instead of the series of surface integrals in Eq. (5). Equation (16) can be utilized for surface correction for the special case of $\frac{U_1}{\mathrm{i}\omega}\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1} = -1$.

It is also worth noting that another much simpler surface correction formula consisted of only one surface integral for surface correction can be derived for the special case of nearly uniform Lighthill stress distribution ($\frac{\partial T_{ij}}{\partial y_1} \approx 0$) near the FW-H integral surface. The simplified surface correction for this special case is

$$I_Q(x;\omega) \approx \int_{f=0} T_{ij}(\mathbf{y},\omega)\left(\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{-1}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}S. \tag{17}$$

Equation (17) is derived from the definition of I_Q and the product rule for derivatives with the assumption $\frac{\partial T_{ij}}{\partial y_1} \approx 0$ as follows,

$$\begin{aligned} I_Q(\mathbf{x};\omega) &= -\int_{f>0} T_{ij}(\mathbf{y},\omega)\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}V \\ &= -\int_{f>0} \frac{\partial}{\partial y_1}\left(T_{ij}(\mathbf{y},\omega)\frac{\partial^2 G^1(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right) - \frac{\partial T_{ij}(\mathbf{y},\omega)}{\partial y_1}\frac{\partial^2 G^1(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}V \\ &\approx -\int_{f>0} \frac{\partial}{\partial y_1}\left(T_{ij}(\mathbf{y},\omega)\frac{\partial^2 G^1(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\right)\mathrm{d}V \\ &= \int_{f=0} T_{ij}(\mathbf{y},\omega)\frac{\partial^2 G^1(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}\mathrm{d}S, \end{aligned} \tag{18}$$

where $\frac{\partial^2 G^1(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j} = \int_{\infty}^{y_1} \frac{\partial^2 G(\mathbf{x};\xi_1,y_2)}{\partial y_i \partial y_j}\mathrm{d}\xi_1$. When the observer is located in the far field, the derivatives of G^1 can be approximated by the equation as follows [16],

$$\frac{\partial^2 G^l(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j} \approx \left(\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1}\right)^{-l}\frac{\partial^2 G(\mathbf{x};\mathbf{y})}{\partial y_i \partial y_j}, \tag{19}$$

with $l = 1$. More detailed derivation of Eq. (19) can be found in the supplementary material [14]. Combination of Eq. (19) with Eq. (18) results in the simplified surface correction Eq. (17).

We have proposed a simplified surface correction formulation for the quadrupole source term of the Ffowcs Williams and Hawkings integrals in frequency domain. The proposed surface correction consists of a series of surface integrals and is applicable to eliminating the spurious sound at far field. The simplified formulation improves the original surface correction by circumventing the difficulties in computing the high-order derivatives of the Green's function. An easy-to-use expression is derived to compute the high-order derivatives by referring tothe far-field asymptotic of the Green's function. The spurious sound generated by a convecting vortex crossing the FW-H integral surface is investigated to validate the proposed surface correction formulation. The results show that the proposed formulation can successfully eliminate the spurious sound generated by the quadrupole term. The terms of the simplified surface correction formulation are consistent with these of the original surface correction formulation at the far field. The proposed formulation also helps to analyse the failure of the original surface correction formulation and further simplify the surface correction to a formulation with only one surface integral when the magnitude of the convection velocity equals to the phase velocity ($\frac{U_1}{\mathrm{i}\omega}\frac{\partial \varphi(\mathbf{x};\mathbf{y})}{\partial y_1} = -1$) or the flow with nearly uniform Lighthill stress distribution ($\frac{\partial T_{ij}}{\partial y_1} \approx 0$) near the FW-H integral surface.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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