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RESEARCH

Proof-of-principle Experimental Demonstration of Time-delay-interferometry for Chinese Space-borne Gravitational Wave Detection Missions

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Abstract

Due to the laser frequency fluctuation, the unequal arm-length of the laser interferometer will give rise to a significant phase noise. This laser frequency fluctuation noise dominates the current space-borne gravitational wave detection missions. To tackle this problem, a post data processing method called time delay interferometer was invented. This method was experimentally verified in the previous work with the scale of a few meters. By kilo-meter long optical fibers, an experimental verification of the time delay interferometer for Chinese space-borne gravitational wave detection mission was carried out in this work. The results showed that the laser frequency fluctuation noise could be suppressed by more than 6 orders in the frequency band from 1 mHz to 0.1 Hz.

Keywords Space-borne gravitational wave detection · Laser frequency fluctuation noise · Time delay interferometry · Experimental verification

Introduction

The space-borne gravitational wave (GW) detectors, such as the LISA (Amaro-Seoane et al. 1702), Taiji (Hu and Wu 2017; Luo et al. 2020, 2021) and Tianqin (Luo et al. 2016), were proposed to use gigameter arm length laser interferometer to probe milli-hertz gravitational waves (The LISA Consortium 2013; Gong et al. 2015). However, due to the orbital motions of the three different space-crafts, the arm lengths of these space-based interferometer can differ by a few kilometers (Luo et al. 2020, 2021). When we comparing the two split beams propagate forth and back along the different arms, the laser frequency fluctuations cause the two beams under different delay can not cancel out, which gives rise to a non-negligible laser frequency noise (Bender et al. 1998).

To solve this problem, a post data processing technics called time delay interferometry (TDI) was invented (Tinto and Dhurandhar 2014). Its mathematical approach, algorisms and implementation in space-borne GW detection missions were detailed discussed in many aspects (Dhurandhar et al. 2002, 2010; Cornish and Hellings 2003; Dhurandhar 2009). However, an experimental verification of TDI to suppress the laser frequency fluctuation noise by more than seven orders is challenging. Despite of the difficulty in realizing gigameter delay line, several attempts has been done by LISA collaboration related to the verification of TDI in a laboratory environment (de Vine et al. 2010). It was shown that, on an optical benchtop, with meter-scale arm length, the laser frequency noise was suppressed by 9 orders (de Vine et al. 2010).

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The Chinese space-borne GW detection missions, include Taiji and Tianqin, both adopted the LISA measurement scheme (Tinto and Dhurandhar 2014), so that the laser frequency noise was also the primary noise. To tackle this problem, multifaceted investigations on TDI theory, algorisms and simulations were carried out by Chinese scientists (Wang et al. 2013, 2021; Zhou et al. 2021). The following urgent task was to set up an experimental system to verify the specific implementation of TDI to Chinese space-borne GW detection missions. In this work, we proposed to use optical fiber delay lines to simulate the arms of space-based interferometer, in which the arm length could been extended to several kilometers. A proof-of-principle experimental demonstration of TDI for Chinese space-borne GW detection missions was firstly presented.

Principle of the Experiment

The space-based interferometers of Chinese GW detection missions were consisted of three satellites (see Fig. 1a) (Luo et al. 2020, 2021). We take the Michelson type interferometer as an example. The laser beams were emitted from the satellite A and reach the satellites B and C. The satellites B and C were served as the two laser transponders that the local lasers in the satellites B or C were phase locked to the received beam and transmitted back to the satellite A. In principle, the satellites B and C could be simplified as the reflecting mirrors (see Fig. 1b) (Bender et al. 1998). The two beams back from the satellites B and C were then interfered with the local oscillators in the satellite A. Taking the subtraction of φ_1 and φ_2 at the satellite A, we can obtain the Michelson type interferometer data.

Then $\varphi 1$ and $\varphi 2$ could be written as.

$$\varphi_1(t) = C\left(t - 2 \bullet \frac{L_1}{c}\right) - C(t) + h_1(t) \tag{1}$$

$$\varphi_2(t) = C\left(t - 2 \bullet \frac{L_2}{c}\right) - C(t) + h_2(t) \tag{2}$$

where c is the speed of light and $h_i(t)$ is the measured signals due to the gravitational wave. An ordinary Michelson type interferometer data could be obtained by,

$$\varphi_1(t) - \varphi_2(t) = C\left(t - 2 \bullet \frac{L_1}{c}\right) - C\left(t - 2 \bullet \frac{L_2}{c}\right) + h_1(t) - h_2(t) \tag{3}$$

The laser frequency fluctuation noise $C(t)$ would not cancel out due to the unequal arm length between L_1 and L_2. The difference of L_1 and L_2 will be a few kilometers. One can also notice, taking a linear combination of the delayed data of $\varphi 1$ and $\varphi 2$, such as $\varphi 1(t\text{-}2{\cdot}L_2/c)$ and $\varphi 2(t\text{-}2{\cdot}L_1/c)$, one obtains

$$\begin{aligned} &\varphi_1\left(t - 2 \bullet \frac{L_2}{c}\right) - \varphi_2\left(t - 2 \bullet \frac{L_1}{c}\right) \\ &= C\left(t - 2 \bullet \frac{L_1}{c}\right) - C\left(t - 2 \bullet \frac{L_2}{c}\right) \\ &+ h_1\left(t - 2 \bullet \frac{L_2}{c}\right) - h_2(t - 2 \bullet \frac{L_1}{c}) \end{aligned} \tag{4}$$

in which the frequency fluctuation noise is as same as in Eq. (3). Combining Eqs. (3) and (4), a so called TDI variable could be constructed

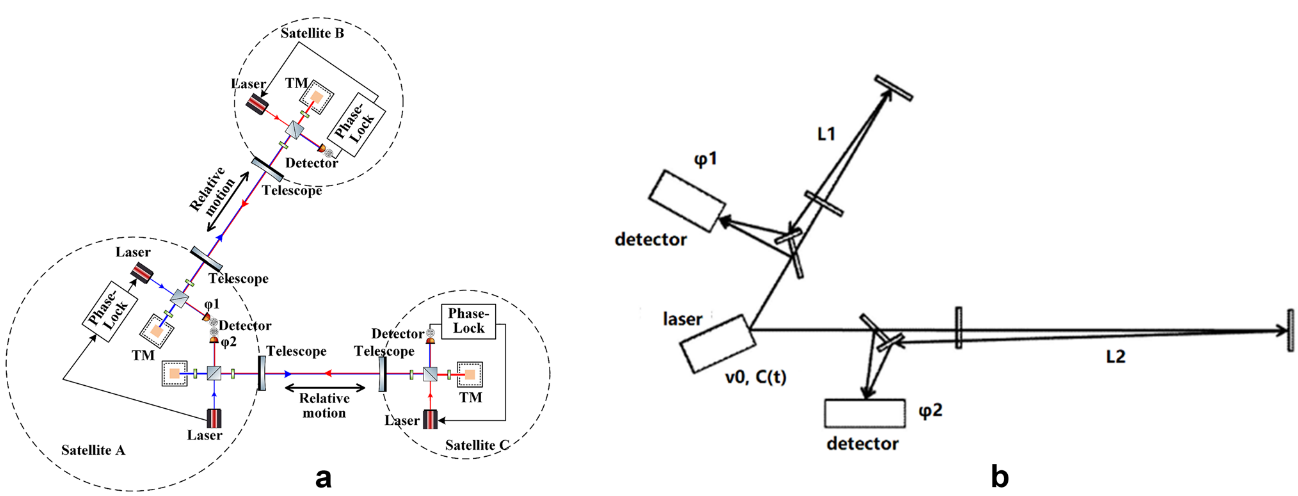

Fig. 1 **a** The schematic diagram of the Taiji laser metrology system, where φ_1 is the phase readout of arm 1 interferometer, φ_2 is the phase readout of arm 2 interferometer. **b** The simplified interferometer for principle demonstration, where v_0 is the laser frequency, $C(t)$ is the phase noise due to the laser frequency fluctuation, L_1 and L_2 represent the arm length which will be approximately 3 million kilometers for Taiji

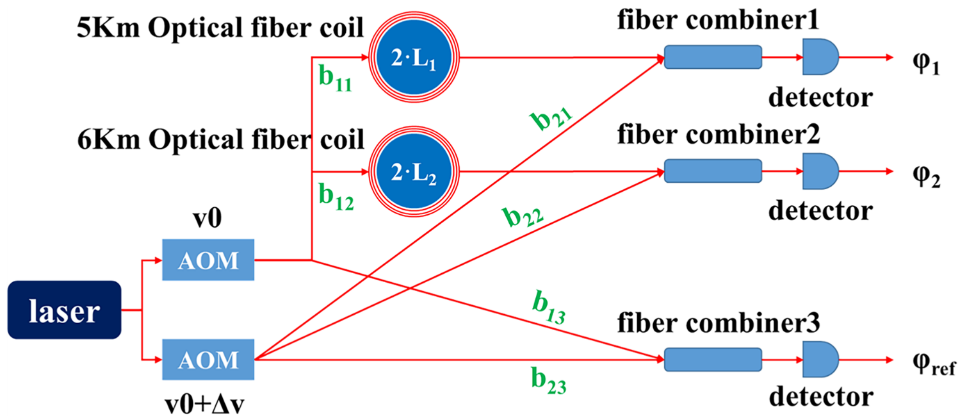

Fig. 2 The principle demonstration diagram of the experimental setup, where b_{ij} ($i=1,2$ and $j=1,2,3$) were the laser beams. Two optical fiber coils, one is 5 km and the other is 6 km, were used to represent the arm length $2{\cdot}L_1$ and $2{\cdot}L_2$ respectively

$$\varphi_{TDI} = \left[\varphi_1(t) - \varphi_2(t)\right] - \left[\varphi_1\left(t - 2\bullet\frac{L_2}{c}\right) - \varphi_2\left(t - 2\bullet\frac{L_1}{c}\right)\right]$$
$$= \left[h_1(t) - h_2(t)\right] - \left[h_1\left(t - 2\bullet\frac{L_2}{c}\right) - h_2(t - 2\bullet\frac{L_1}{c})\right] \quad (5)$$

It can be seen that the laser frequency fluctuation noise could be eliminated.

Experimental Setups

A principle demonstration diagram of the experimental setup is shown Fig. 2. After coming out of the laser source, the laser beam was split into two. Each of the laser beam was modulated by an acoustic optical modulator (AOM) to create an offset frequency Δv between the two beams.

After the AOMs each beam was split into three (see Fig. 2). The beams b_{1i} and b_{2i} combine at the ith fiber combiner to form three interferometers. φ_1 and φ_2 were to simulate the phase readout of Taiji's two arm interferometers. φ_{ref} was the phase readout of a reference interferometer which was used to monitor the noise from and before AOMs.

The experimental setup was laid on an optical table in a vacuum tank (see Fig. 3). The optical table was equipped with an air spring vibration-isolation system, and the vacuum can provide a thermal shield for the experimental system. To prevent the possible temperature fluctuation, the laser source was put outside the vacuum tank and the laser beams were injected in the tank via fiber feed-through. The interferometric beatnote from three detectors were sent to a phase-meter (Liu et al. 2021) from with the phase measurement data, φ_1, φ_2 and φ_{ref} were read out.

Defining

$$\Phi_1(t) = \varphi_1(t) - \varphi_{ref}(t) \quad (6)$$

$$\Phi_1(t) = \varphi_1(t) - \varphi_{ref}(t) \quad (7)$$

We can eliminate the noise of AOM modulation and the noise picked up from the fibers before entering the vacuum tank.

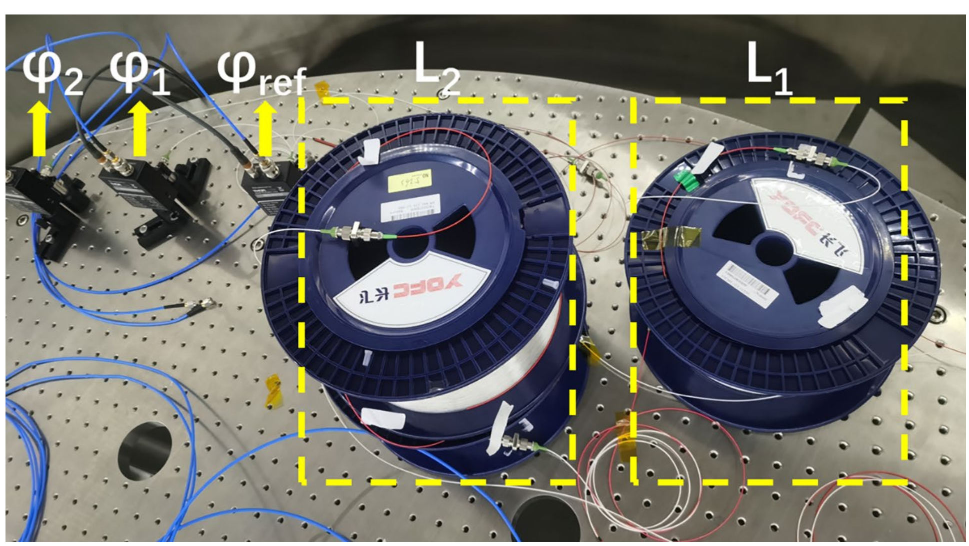

Fig. 3 The layout of the experimental system, which consisted of two optical fiber coils and three optical fiber interferometers. The experimental setup was put in a vacuum tank which provide thermal shield and vibration isolation

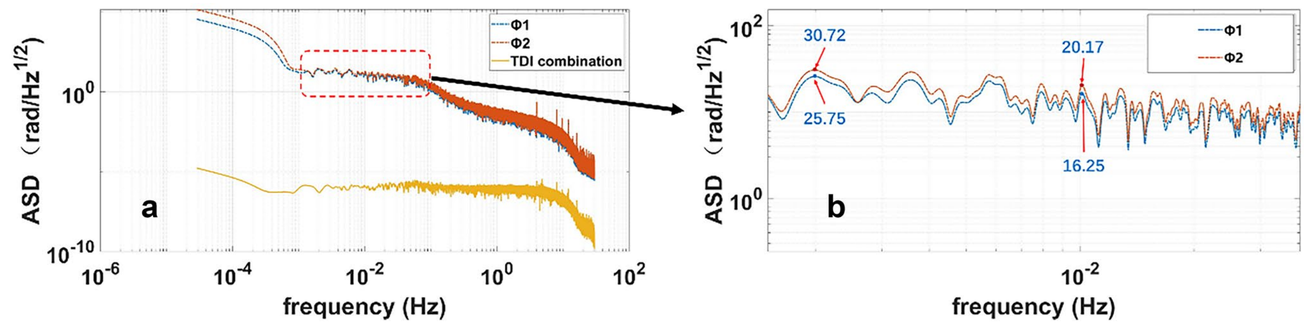

Fig. 4 **a** The amplitude spectrum density of $\Phi_1(t)$, $\Phi_2(t)$ and $\Phi_{TDI}(t)$. **b** A zoom-in comparison of $\Phi_1(t)$ and $\Phi_2(t)$, the numbers are the corresponding value of $\Phi_1(t)$ and $\Phi_2(t)$ at 2 mHz and 10 mHz

Results and Discussion

A customized laser was used in this experiment, with center wavelength $\lambda = 1064.5\ nm$. The frequency fluctuation $\delta\nu$ of the laser is about a few tens of kilohertz at 0.1 Hz. Considering the phase noise C due to laser fluctuation (Bender et al. 1998)

$$C = \frac{\delta\nu}{\nu_0} \bullet L \bullet \frac{2\pi}{\lambda} \tag{8}$$

the phase noise will be approximately a few radians for L_1 and L_2.

By collecting the data $\varphi_1(t)$, $\varphi_2(t)$ and $\varphi_{ref}(t)$, one could obtain $\Phi_1(t)$ and $\Phi_2(t)$ by using Eqs. (6) and (7). $\Phi_1(t)$ and $\Phi_2(t)$ contained the phase noise due to the laser frequency fluctuation. With the aid of an interpolation algorithm invented in Shaddock et al. (2004), the delayed data $\Phi_1(t\text{-}2\cdot L_2/c)$ and $\Phi_2(t\text{-}2\cdot L_1/c)$ could be constructed. Then by applying Eq. (5), the TDI variable $\Phi_{TDI}(t)$ could be obtained, in which the phase noise of laser frequency fluctuation should be suppressed.

In this article, the Amplitude Spectra Density (ASD) (Tröbs and Heinzel 2006) was used to evaluate the results of the TDI experiment. The ASDs of $\Phi_1(t)$, $\Phi_2(t)$ and its TDI combination $\Phi_{TDI}(t)$ were plotted in Fig. 4a. It was noted that, the ASD of $\Phi_2(t)$ was slightly greater than the ASD of $\Phi_1(t)$. The proportional ratio between $\Phi_2(t)$ and $\Phi_1(t)$ was about 1.2 ± 0.03 within the frequency band from 1 mHz to 0.1 Hz (Fig. 4b). This proportional factor was in accord with the Eq. (8).

It was also obvious that the ASD values of $\Phi_1(t)$ and $\Phi_2(t)$ at 0.1 Hz were around 2 rad/Hz$^{1/2}$ which was also in accord with Eq. (8). After applying the TDI algorism, it could be seen from the ASD of TDI variable $\Phi_{TDI}(t)$ that the phase noise due to the laser frequency fluctuation could be suppressed to 1μrad/Hz$^{1/2}$ in the frequency band between 1 mHz to 1 Hz. The noise floor is mainly limited by the phase readout noise of the phasemeter (Liu et al. 2021). It implied that the laser frequency fluctuation noise was suppressed by 5×10^4 times at 1 Hz, by 6 orders at 0.1 Hz and by 7 orders at 10 mHz and 1 mHz.

The TDI method which was employed in this experiment is so-called 1st generation TDI algorism. It only applied to the satellite constellation which had no relative motion. When there was relative velocity between each two satellite, the 2nd generation TDI algorism should be used (Tinto and Dhurandhar 2014). In this article, we only discussed the 1st generation TDI algorism, the experimental verification of 2nd generation TDI algorism will be investigated in our later research.

Conclusion

The 1st generation TDI algorism was experimentally verified in this work. Compared to the previous work, we firstly proposed to use optical fibers to simulate the arms of Taiji constellation. The simulated optical path was extended from a few meters to 5 and 6 km that the delayed time was significantly improved.

It also could be seen from the above that by employing the TDI algorism, the laser frequency fluctuation noise can be effectively suppressed for more than 6 orders within frequencies from 1 mHz to 0.1 Hz. Since there would always be relative motion between satellites of Taiji constellation, the 2nd generation TDI algorism might be necessary, and its experimental verification should be investigated in further works.

Author Contribution Xiaokang Li wrote the main manuscript. Heshan Liu performed the experiment and prepared the figures. Pengzhan Wu and Haosi Li performed the data analysis and provide constructive discussions. Peng Xu contributed significantly to analysis. Ziren Luo contributed to the conception of the study and manuscript preparation. All authors reviewed the manuscript.

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Availability of Data and Material The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Ethics Approval and Consent to Participate Not applicable, because this article does not contain any studies with human or animal subjects.

Consent for Publication Not applicable.

Competing Interests The authors declare no competing interests.

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