



A single-camera synthetic Schlieren method for the measurement of free liquid surfaces

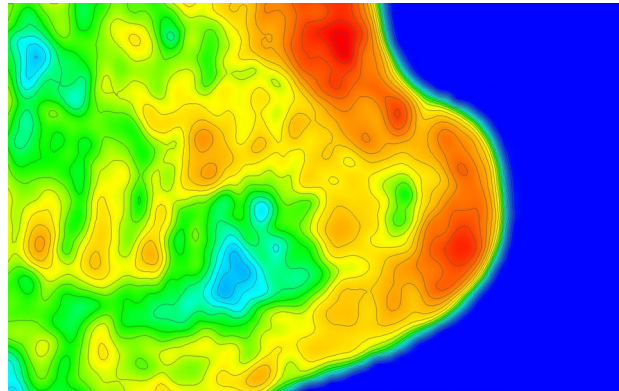
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Abstract

A single-camera synthetic Schlieren method is introduced to measure the height of a dynamic free liquid surface in a container with a flat bottom. Markers placed on the bottom, seen through the free surface, are virtually displaced due to light refraction at the surface. According to Snell's law, the marker displacements depend on the refractive indices of the transparent liquid and the air, and on the surface height and its spatial gradients. We solve the resulting governing nonlinear equation with the Newton–Raphson method to obtain the surface height. Our method does not require a reference surface height, which allows the measurement of surfaces topography in situations where the liquid depth is unknown. We demonstrate the accuracy of the method by performing experiments of surface ripples and dam-break flows and discuss the measurement uncertainty, errors and limitations.

Graphic abstract



1 Introduction

The quantitative measurement of a liquid-free surface is fundamentally important in many applications. For example, in marine and coastal engineering, wind-generated surface waves have substantial impacts on marine vessels and offshore platforms Holthuijsen (2007). In chemical engineering, falling liquid films with dynamic free surface are employed for the efficient heat exchange in steam condensers Karimi and Kawaji (1998). The liquid sloshing with dynamic free surface motions is essential in propellant tank in aerospace devices and in liquid natural gas cargos in ship industry Ibrahim (2005). With better-controlled

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flow conditions, laboratory experiments are carried out for these flows through, i.e., measuring the topography structures of the wind-generated surface waves in a tank with a flat bottom Paquier et al. (2015), and measuring the falling film thickness of a flow over an inclined flat plate Charogiannis et al. (2015), to offer insights into the engineering flows. In the laboratory experiments, quantitative optical imaging measurements of the surface height are advantageous because of their non-intrusive nature.

One imaging methodology associates the height of the free surface with light absorption in the liquid. Given that image magnification is independent of surface height, Jähne et al. (2005) used a telecentric optical system to measure the surface height, which is a function of the light absorption rates from light-emitting diodes (LEDs) with red and near-infrared lights. Here, a challenging task is to precisely align the collimated beams in different wavelengths. Aureli et al. (2014) used a camera with two charged-coupled devices (CCDs) including a monochrome near-infrared sensor in order to implement the surface measurements with the working principle of light absorption. They took the advantage of high absorption capacity of water in the near-infrared range. Their method can reach an accuracy in the order of 1 mm; however, the measurement sensitivity decreases with the water depth.

Planar laser-induced fluorescence (LIF) with a laser sheet has also been used to capture the profiles of the free surface Duncan et al. (1999); André and Bardet (2014). A fast-scanning of a laser sheet can be used to obtain and reconstruct the surface topography in two dimensions van Meerkerk et al. (2020). The scanning needs to be fast enough so that the surface structure can be approximately assumed to be frozen. In the LIF method, edge-detection algorithms are required to extract the surface profiles from the LIF images, and the physical length corresponding to an image pixel determines the measurement resolution. The measurement dynamic range (the ratio of the maximum measured value to the smallest measured value) is determined approximately by the image resolution.

Light reflection can also be used for surface measurements. For example, the slope of two-dimensional surfaces can be measured with a polarimeter by correlating the surface orientation with changes of light polarization Zappa et al. (2008). This technique can be used not only in laboratory but also in rivers with unpolarized skylight or moon-glade Zappa et al. (2008); Vinnichenko et al. (2020). Alternatively, symbol patterns and fringe patterns are projected onto the surface, and the deformation of the patterns due to the surface topography is exploited for the surface measurements Tsubaki and Fujita (2005); Cobelli et al. (2009); Hu et al. (2015). The intensity of the light reflection is crucial to the measurement accuracy, and additives

mixed with the liquid (water) are employed to improve the light reflection at the surface.

A group of imaging techniques relies on light refraction. Kurata et al. (1990) used a camera to capture a reference and a refraction image of a striped grating, which was placed below a shallow water channel. When the water surface had ripples, the imaged grating was virtually displaced with respect to the still, horizontal water surface, due to the changed light refraction. In their method, the topography of the free surface is obtained from the virtual displacements. The surface height can also be obtained provided that the height of a reference surface point is known. In their flow, there is a point that the surface height is nearly unchanged in time, so that they use the height at this point as the reference height to reconstruct the surface depth from the topography data. Fouras et al. (2008); Moisy et al. (2009); Ng et al. (2011) further developed this method by replacing the grating with random-dot patterns, so that optimized cross-correlation particle image velocimetry (PIV) algorithms can be used to compute the virtual displacements precisely. Alternatively to the dot patterns, Zhang and Cox (1994) introduced a color-coded pattern, where colors were designed to correlate with surface slopes one-to-one Dabiri et al. (1997).

The advantages of refraction methods are the use of low-cost illumination, simple optical configuration and a single camera. Despite their simplicity and precision, a drawback of these methods is that a reference height is needed to reconstruct the height of the surface from the surface topography. In some situations, the liquid depth is not known, and the method cannot be applied. Morris (2004) and Gomit et al. (2013) used two or more cameras to obtain stereoscopic views of the same pattern. They could directly obtain the topography and the height of the free surface by matching the surface orientation at each surface point from the measurements of multiple cameras Morris (2004) (see Table 1). However, using multiple cameras raises the hardware costs, results in more time-consuming calibration of the camera positions, and requires more computation power in the post-processing. Recently, Engelen et al. (2018) proposed a method with a single camera for the surface measurements, which relies on the condition that the surface topography can be depicted by a parametric model.

We present an extension of the free-surface synthetic Schlieren (FS-SS) method of Moisy et al. (2009), which allows to directly measure the topography and the height of the free liquid surface in laboratory experiments using a single camera, without neither reference height nor parametric models. In Sect. 2, the working principle is introduced. Two sets of experiments were carried out to test our method. Experiments of surface ripples, where the spatially averaged height is approximately constant, are shown in Sect. 3. Experiments of dam-break flows, where the averaged height

Table 1 A brief list of refraction methods for free surface measurements.

Reference	Quantity of camera	Method	Object and pattern	Illumination	Measurability of liquid height
Zhang and Cox (1994)	1	Refraction	Random colored blocks	Halogen light	Yes, with reference height
Dabiri et al. (1997)	1	Refraction / Reflection	Random colored blocks	–	Yes, with reference height
Morris (2004)	2	Refraction and pattern	Checkerboard	LED	Yes
Jähne et al. (2005)	1	Refraction and light absorption	Gridded dots	LED	Yes, with complex telecentric illumination
Fouras et al. (2008)	1	Refraction and Image correlation	Random dots	LED	Yes, with reference height
Moisy et al. (2009)	1	Refraction and Image correlation	Random dots	LED	Yes, with reference height
Gomit et al. (2013)	3	Refraction	Random PIV tracers	Laser sheet	Yes
Aureli et al. (2014)	1	Refraction and light absorption	Random colored blocks	LED	Yes, with coupled RGB Bayer array and Monochrome sensor
Engelen et al. (2018)	1	Refraction and pattern match	Checkerboard	LED	Yes, with parametric surface model
Kolaas et al. (2018)	2	Refraction and image correlation	Random dots	bichromatic LED	Yes, with reference height and telecentric lens

changes in time, are presented in Sect. 4. A detailed discussion and conclusions are given in Sects. 5 and 6, respectively.

2 Methodology

The presented methodology exploits the light refraction at the air–liquid interface. A schematic of the working principle, based on the synthetic Schlieren method Moisy et al. (2009) and the background-oriented Schlieren method Raffel (2015), is shown in Fig. 1. When there is no liquid, the incident light ray at a point P follows a straight trajectory (shown in black) toward the camera. When there is a liquid–air interface, the incident light ray at point P' follows the dashed gray trajectory toward the surface and then follows the same black trajectory after leaving the surface. The light refraction is governed by the Snell’s law for refractive index of the liquid n_l and that of the gas n_a (in this study $n_l = 1.334$ and $n_a = 1$, corresponding to water and air). The displacement $\delta \mathbf{r}$ from the point P to the point P' depends on the liquid height $h(x, y)$, and its spatial gradients $\nabla h(x, y)$, according to the equation

$$\delta \mathbf{r} = h [\tan(\tan^{-1}(\nabla h) + \sin^{-1}[(n_a/n_l) \cdot \sin(\beta - \tan^{-1}(\nabla h))]) - \tan(\beta)], \tag{1}$$

where $\beta = \beta_x(x, y)\hat{x} + \beta_y(x, y)\hat{y}$ denotes the angle of the camera viewing the point P . The detailed derivation of this equation is given in Appendix. When the camera is set to satisfy the paraxial approximation, $\beta(x, y)$ is small so that

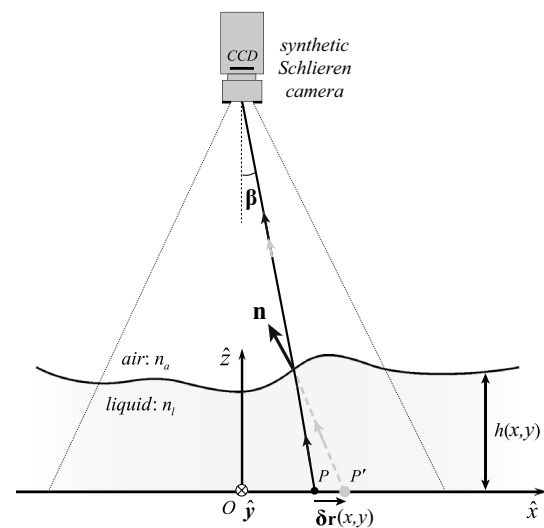


Fig. 1 Schematic of working principle of measuring the topography and the height of free air-liquid surface $h(x, y)$ with a surface normal direction \mathbf{n} . $(\hat{x}, \hat{y}, \hat{z})$ and O denote the Cartesian coordinate and the origin, respectively. The displacement from a point P to the point P' (due to the surface refraction) is $\delta \mathbf{r}$. β is the angle between the light (black line with an arrow) and the camera axis

$\tan(\beta) \approx \beta$. Similarly, $\tan(\nabla h) \approx \nabla h$ when ∇h is small, e.g., $\nabla h < 0.5$, see Sect. 5.5. Then, Eq. (1) can be simplified to

$$\delta \mathbf{r} = h \cdot (1 - n_a/n_l)(\nabla h - \beta), \tag{2}$$

where $h(x, y)$ is the only unknown to be determined provided that $\delta\mathbf{r}$ and $\boldsymbol{\beta}$ have been pre-computed, as explained in what follows.

The displacement $\delta\mathbf{r}(x, y)$ can be obtained by taking the cross-correlation operation over the reference and the refraction images of dot objects (see Fig. 2b, c). Image cross-correlation algorithms, as commonly used in two-dimensional PIV, are applied in this study. In this study, the multi-step algorithm is used with interrogation window $96 \times 96 \text{ pixel}^2$ at the initial step reducing to the window $24 \times 24 \text{ pixel}^2$ with 75% window overlap at the final step, an optimized setting in Lavisision Davis©. The interrogation window at the final step includes approximately 8 dots for accurate displacement tracking Raffel et al. (2018), given that the number of dots in each interrogation window between the paired images is approximately unchanged. In this work, the dot objects are printed (randomly distributed) on a transparent sheet. The random distribution of the dots helps the PIV correlation algorithm reduce fortuitous pattern coincidence and renders better accuracy. We carefully designed the pattern, where the diameters of individual dots are approximately 5 to 6 pixels in images.

The camera viewing angles, $\boldsymbol{\beta}(x, y) = \beta_x(x, y)\hat{x} + \beta_y(x, y)\hat{y}$, are determined by the spatial position of the camera to the object pattern (e.g., see Fig. 2a) and can easily be obtained in a calibration procedure. For a leveled flat bottom and a still (flat) water surface (i.e., $\nabla h = 0$, known n_l), the surface height h_c can be measured with a caliper precisely.

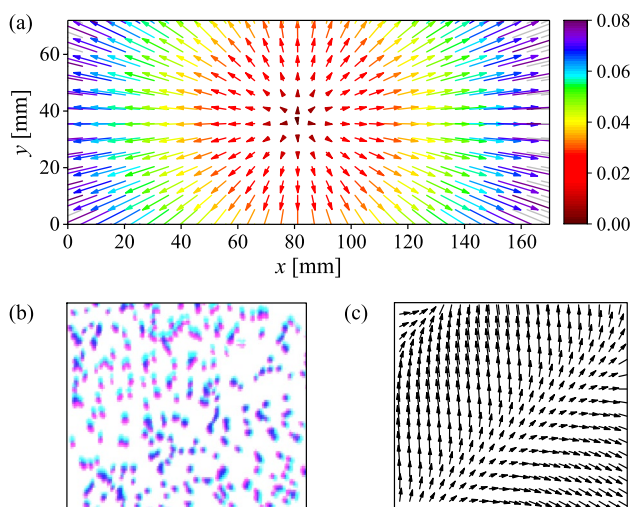


Fig. 2 (Color online) **a** The vector map of $\boldsymbol{\beta}(x, y)$ obtained in the calibration procedure, where the direction of a vector is given by β_x and β_y , and the color shows the magnitude $|\boldsymbol{\beta}|$. Every other vector along each direction is shown for clarity. **b** A portion of dot pattern is shown: pink dots (tuned from black) are from a calibration image, while cyan dots (tuned from black) are from a measurement image of surface ripples, and the overlap is in blue. **c** The displacements $\delta\mathbf{r}$ (amplified 4 times in length for clarity) between the pink and the cyan dots in (b)

According to Eq. (2), $\boldsymbol{\beta}(x, y) = -\delta\mathbf{r}_c(x, y)/[h_c(1 - n_a/n_l)]$ with $\delta\mathbf{r}_c(x, y)$ obtained from the cross-correlation operation. Note that the height h_c can be freely chosen and does not depend on the details of the experiments (e.g., water fill level) to be performed in the tank.

We solve for $h(x, y)$ in Eqs. (1) and (2) using the MATLAB intrinsic function *fsolve* (Levenberg–Marquardt algorithm), which requires long computing time and large memory. To reduce the need of computation power and time, an *in-house* MATLAB code implementing the Newton–Raphson method was scripted to solve Eq. (2) (with reduction of the computation time by two orders of the magnitude). The gradient ∇h is discretized with central, second-order finite differences at interior grid points and with forward/backward second-order differences at the boundaries. The displacements $\delta\mathbf{r}(x, y)$ span $M \times N$ grid points, and at each grid point, there are two components of $\delta\mathbf{r}$ and one h . This leads to a linear system including $2MN$ equations and MN unknowns. This over-determined linear system can be solved without imposed boundary conditions by Newton iterations from an initial guess (a positive constant) to the converged solution $h(x, y)$. Convergence is assured if the maximum of the least-squared residual errors in the linear system drops below 10^{-5} .

We note that $\delta\mathbf{r}$ is measured with respect to the point P in Fig. 1, whereas the coordinates of the intersection (surface) point for the solved h differ from P . The coordinates of the solved $h(x, y)$ thus need to be remapped according to $(x, y)^* = (x, y) - h(x, y) \tan[\boldsymbol{\beta}(x, y)]$.

3 Liquid ripples

In this section, we report on experiments of surface ripples, where the spatially averaged surface height is approximately constant.

3.1 Experiments

The tank that was manufactured with transparent acrylic plates has dimensions of $1000 \times 150 \times 350 \text{ mm}^3$ in length, width and height. The random dot pattern in dimension of $200 \text{ mm} \times 120 \text{ mm}$ was printed on an overhead transparency (in thickness of 0.16 mm) in a printer (Ricoh MP C3504). The dot pattern was attached upon the inner flat bottom of the tank. A white LED with a light diffuser was applied to illuminate the dot pattern from the bottom of the tank, while a camera (Phantom VEO 640L equipped with a Zeiss lens of focal length 100 mm) in resolution $2560 \times 1600 \text{ pixel}^2$ was placed approximately 1 meter above the pattern. A plate was employed to flap back and forth to generate surface ripples (see Fig. 3).

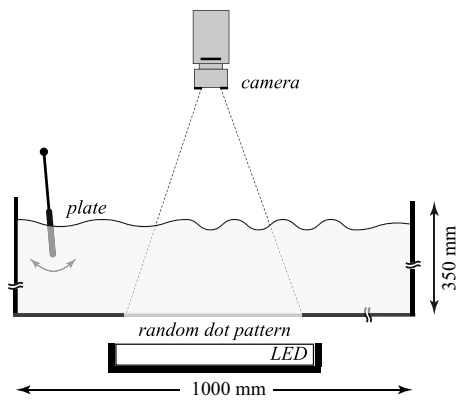


Fig. 3 Schematic of measuring topography and height of liquid ripples in a tank. The dimensions are not in scale

The experiments were carried out in the following order. The tank bottom was leveled; an image was captured as the reference image I_0 when there was no water in the tank; the water was filled into the tank to a height of 20 ± 0.1 mm which was measured by a caliper, and an image I_{h_0} was captured when the water surface was still. When the water ripples were generated, a series of images $I(t)$ was recorded.

When the images $I(t)$ are correlated with the image I_0 to get $\delta r(x, y, t)$, Eq. (1) is solved to give $h(x, y, t)$ from the present method. For comparison, when the images $I(t)$ are correlated with the image I_{h_0} to get the displacements, $h(x, y, t)$ can be obtained with the FS-SS method Moisy et al. (2009). The script of FS-SS method of Moisy et al. (2009) was downloaded from their website and compared with our script of the FS-SS method to confirm that we understand their method correctly. In their script, the solution of the topographic height at the bottom left point of the computation domain is set to be zero, and then, the summation of the solved topographic height and the reference height gives the surface height.

3.2 Results

A snapshot of the surface ripples measured with our method is shown in Fig. 4b, and it is visually in good agreement with the result of the FS-SS method shown in (a). This comparison is performed on the surface topography by removing the spatially averaged height $\bar{h}(t)$ from $h(t)$. The particular consideration behind this comparison is that although the reference height h_0 is constant, the averaged height within the field-of-view measurements may vary temporally, which may break the fundamental assumption of the FS-SS method Moisy et al. (2009). About 8, 000, 000 individual spatial points of $h(x, y) - \bar{h}(x, y)$ from 200 temporal snapshots are shown as the gray squares in Fig. 4c, where a black dot shows the bin-averaged value of the individual points. Good quantitative agreement between the two methods is obtained.

3.3 Effect of initial guesses

The effect of initial guesses for the Newton–Raphson method on the computed $h(x, y)$ is shown in Fig. 5a. For the same snapshot shown in Fig. 4b, the spatially averaged heights \bar{h} computed from initial guesses $h_0(x, y) = 15, 20, 25, 30$ converge to the same value $\bar{h} \approx 19.93$ (see the inset). The contours of the topographic difference on $h(x, y)$ between the result with $h_0 = 20$ (shown in Fig. 4b) and those with other initial guesses are shown in Fig. 5b–d, where the difference is only about 10^{-6} . This shows that the present method does not depend on different initial guesses. When the initial guesses are meaningful, i.e., $h_0 > 0$, the present method does not have multi-solution issues as reported in Gomit et al. (2013). In practice, we suggest to take $h_0(x, y)$ slightly larger than the maximum height of the surface from *a-priori* knowledge of the flow.

Fig. 4 (Color online) The contours of topography from the FS-SS method (a) and from our method (b). c Comparison of $h - \bar{h}$ between present method and FS-SS method in experiments of dynamic ripples, where the gray squares denote individual data in space. A black dot shows the average of the individual points in a bin with the size of 0.3 mm, and an error bar shows the standard deviation of the data in the bin

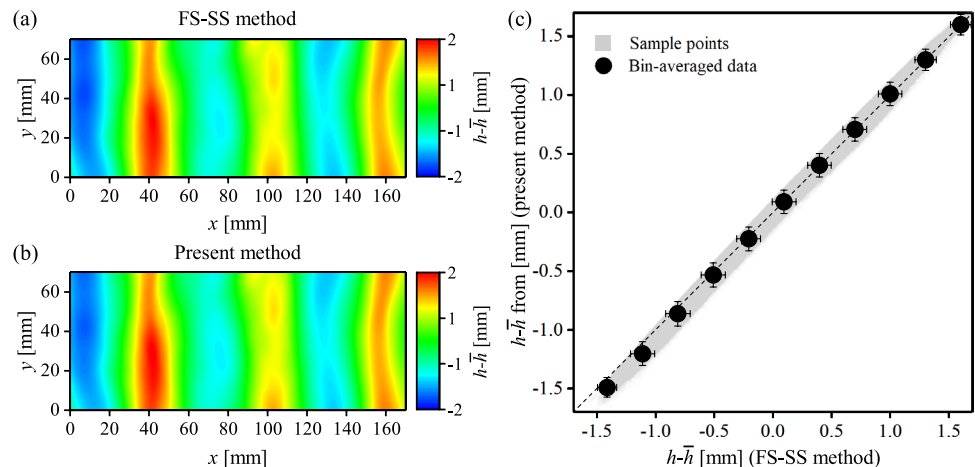
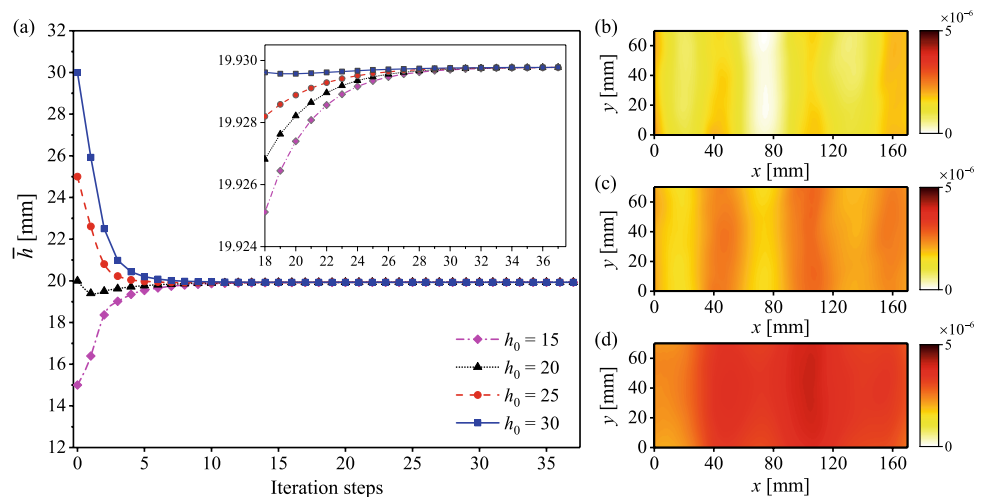


Fig. 5 (Color online) The effects of initial guesses in Newton method on $h(x, y)$ in solution. **a** The mean height \bar{h} resulted from initial guesses $h_0 = 15, 20, 25, 30$ against iteration steps, where the inset shows a zoom for iteration steps larger than 18. **b–d** The contours of topography differences, $h(x, y, h_0 = 30) - h(x, y, h_0 = 20)$, $h(x, y, h_0 = 25) - h(x, y, h_0 = 20)$ and $h(x, y, h_0 = 15) - h(x, y, h_0 = 20)$ respectively



4 Dam-break problem: propagation of a current

A dam-break problem involves a liquid current propagating downstream with the averaged height temporally changing. The experiments of the dam-break problem introduced in this section demonstrate that with our method flows with temporally evolving averaged height can be measured.

4.1 Experiments

The experiments of the dam-break problem were carried out in a lock-exchange setup as sketched in Fig. 6a. The tank is the same as used in the experiments of the liquid ripples. The experimental procedure is as follows. Water was filled into the locked portion of the tank only (the left side of the gate) to the height marked by a dashed line (45 mm in height); The gate was rapidly, manually moved away vertically, and the water flowed into the right side of the gate driven by gravity formed a dynamical current.

The dot pattern was attached to the inner bottom of the tank, while a camera was placed approximately 1.5 m above the bottom (see Fig. 6a). The details of the camera-illumination system are given in Sect. 3. An example of the current body, the surface height upstream of the current front, is shown in Fig. 7a, where large spatial variations of the current heights can be observed. In time, the averaged current height \bar{h} changes by 3 mm in a short time interval (300 ms). As consequence, the FS-SS method is inapplicable in this flow condition.

4.2 Measurement validation

For validation purposes, a visualization system was additionally set up to capture the height profile of the current at the same time. The visualization camera (also a Phantom VEO 640L) was placed approximately 0.7 m away from a sidewall of the tank. A white paper was attached outside another sidewall of the tank to produce an approximately uniform background. Every liter of water was premixed with 18 ml ink to increase the contrast of the imaged current to

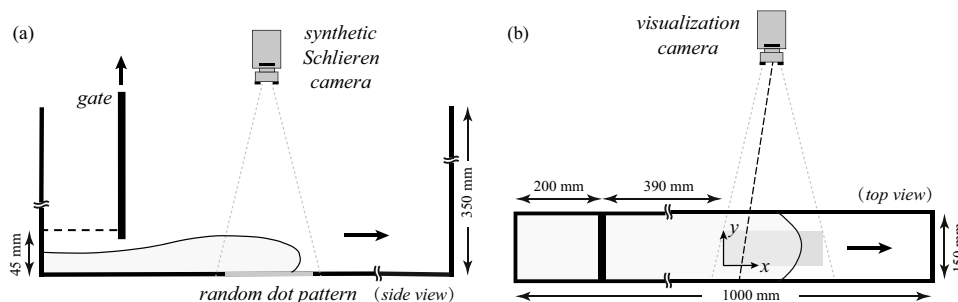
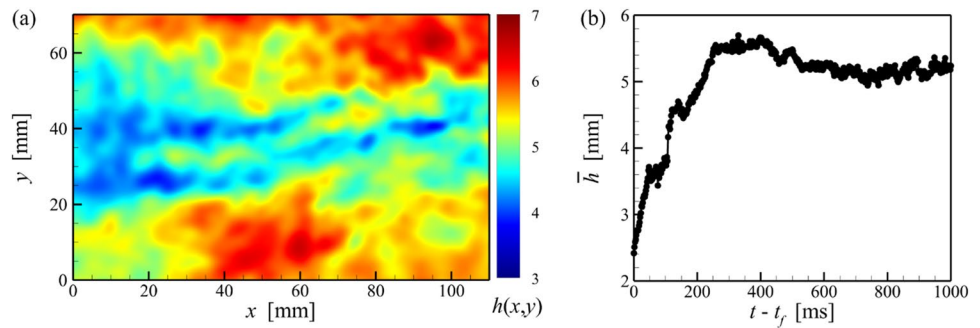


Fig. 6 **a** Side view schematic of a lock-exchange experimental setup for propagation of current fronts in a dam-break flow. The camera and illumination systems are the same as in Fig. 3. **b** Top-view schematic of visualization measurements. The thick black arrow indicates the

flow direction, and the gray rectangular marks the area where $h(x, y)$ is measured. The dot lines enclose the field-of-view of the visualization camera, and the dash line marks the perspective view of the camera. The dimensions are not in scale

Fig. 7 (Color online) **a** Contours of a sample $h(x, y)$ of the current body at $t - t_f = 234$ ms. **b** Time series of \bar{h} for the current body, where $t_f (= 152$ ms) is time duration of the current front passing the measurement domain. The gate is opened at $t \approx -40$ ms



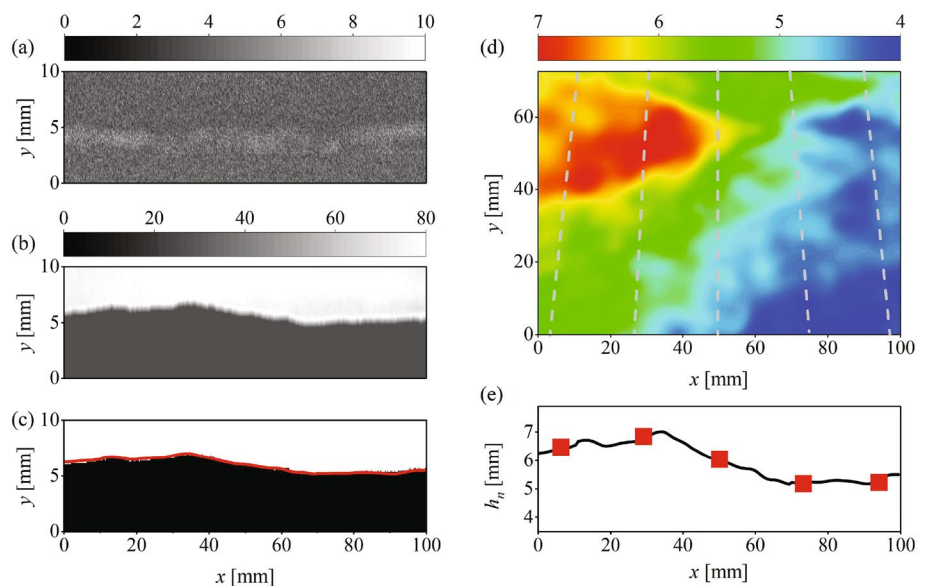
the background. The visualization was with illumination from the same LED as the synthetic Schlieren measurement. This visualization measurement approximately captures the surface height averaged in the y -direction (see details below). The two cameras were synchronized, and the sampling rate of the measurements was 500 frames per second. The measurement domain of the free surface was 110×70 mm in streamwise and spanwise direction, respectively. The upstream border of the measurement domain was about 390 mm downstream of the gate.

The height profile of the current body was obtained from the visualization measurements as follows. An image of the standard deviation (at each pixel) (see Fig. 8a) and an image of the averaged background were obtained from the background images captured before the experiments (i.e., without liquid in the measurement region). A visualization image of the current body is shown in Fig. 8b. For the visualization image with removal of the averaged background image, if the absolute value at the pixel is larger than three times of the standard deviation at the pixel, it indicates that water flows over the pixelwise region, and the pixel is marked as black (see Fig. 8c). Similarly, if the

absolute value at the pixel is smaller than three times of the standard deviation at the pixel, no water flow is expected and the pixel is marked as white (see Fig. 8c). The height profile of the current is given by the borders between the white and the black regions, and then, a 3-point moving average operation was used to smooth the profile to give $h_v(x)$ for the visualization measurements (see Fig. 8c).

The visualization camera has a perspective view of the flow, and a point of the height profile approximately corresponds to the maximum height along a perspective line (see, e.g., the dash line in Fig. 6b). Thus, for better comparison with the height profile of the visualization measurement, $h(x, y)$ from our synthetic Schlieren method is post-processed. 100 points are defined with equidistance in the streamwise length of the field-of-view of the visualization camera, and they are connected to the position point of the visualization camera to define 100 perspective lines (see examples of 5 perspective lines, the dashed lines in Fig. 8d). The surface height $h(x, y)$ is interpolated along the 100 perspective lines, and the maximum value along each perspective line is obtained to give $h_n(x)$.

Fig. 8 (Color online) Illustration of the image processing in the visualization measurement: **a** The image of the standard deviation of the background images; **b** A sample of visualization image; **c** The tuned image with marking the pixel with grayscale larger (or smaller) than three times of the standard deviation as black (or white), where a red line marks the height profile. Illustration of obtaining $h_n(x)$ (**e**) from $h(x, y)$ (**d**), where dashed lines indicate the perspective direction, and five red squares denote the data from the five dashed lines



We then compare $h_n(x)$ and $h_v(x)$ in Fig. 9a, and they approximately collapse. About 24, 000 points of the current profiles from about 180 snapshots are shown as gray dots in Fig. 9b, where the red squares show the bin-averaged data. The two measurement results agree with small differences, which are mainly from two aspects: (1) The data of two measurements are from different sizes of the measurement domain. Although a black mask was used to block the area outside the gray rectangular in Fig. 6b, the perspective illumination made the visualization measurement domain slightly larger than the measurement domain from the present method. (2) The visualization measurements are affected by the light reflection at the free surface and the spatially inhomogeneous illumination intensity which varies in time. Consequently, the current profiles from the visualization measurements have larger uncertainties.

4.3 Dynamics of current front

The sharp fronts where the height changes abruptly ($\nabla h \gg 1$) in a short streamwise distance (see Fig. 10a) results in an ill-conditioned Jacobian matrix of the Newton–Raphson method. This leads to height errors at the downstream region of the front edge, where the height should be zero. We found that imposing a Dirichlet boundary condition at the downstream border of the computation domain (i.e., $h(\max(x), y) = 0$) can remedy this issue. In addition, occasionally in some snapshots, downstream of the fronts, there are spike-like structures possibly given by numerical noise (height of about 0.5 mm, see Fig. 10b), the cause of which is unclear. The dynamics of the current front (after removing the noise downstream of the front) and the body can be seen in a supplemental movie.

In Fig. 10(a), a region of large height $h \approx 5$ mm can be seen close to the front, downstream of which the height decreases to about 3 mm with large spatial fluctuations.

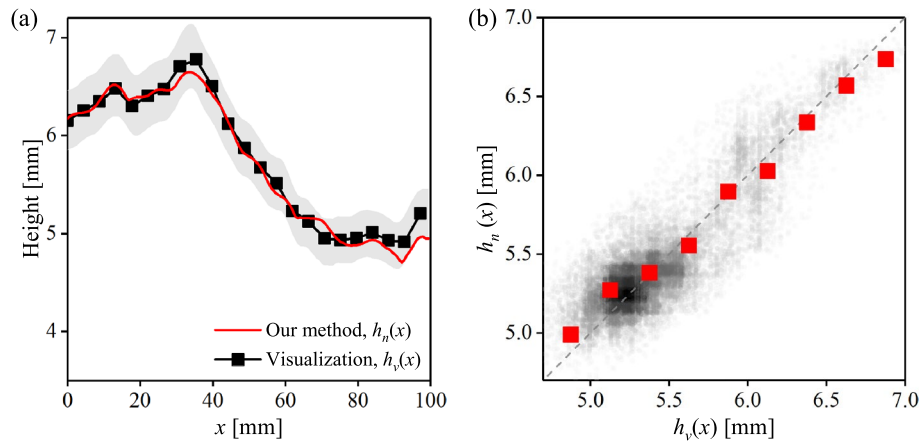


Fig. 9 (Color online) Comparison of streamwise profiles of current body between the visualization and our method. **a** The streamwise profiles $h_v(x)$ (visualization) and $h_n(x)$ (present method) along the streamwise direction x . Gray band shows the error bar of visualization

measurement. **b** $h_n(x)$ versus $h_v(x)$, where individual data are shown in points, and the grayscale level indicates the probability density function (pdf) (the darker the larger pdf), and red squares show the bin-averaged data

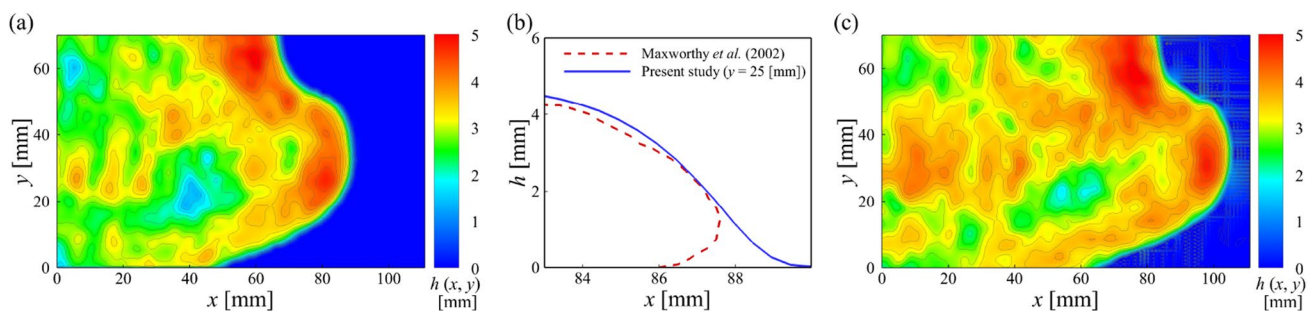


Fig. 10 **a** Contours of a sample of the current front with no numerical noise. **b** The height profile of the current front, where the blue solid line shows the profile at $y = 25$ mm in (a). The red dashed line shows a typical profile of a gravity current, which is extracted from

the top panel of Fig. 4 in Maxworthy et al. (2002), and the values are adjusted to shown in the same figure. **c** Contours of a sample of the current front with spike-like numerical noise downstream of the front

This experimental scenario agrees with the simulations of Shin et al. (2004); Borden and Meiburg (2013). However, a difference on the structures of the front interface can be observed (and is expected) in Fig. 10(b). This difference is due to the breakdown of the working principle in this region (see the discussion in Sect. 5.4). We then show the spatio-temporal diagram of the current front in Fig. 11(a). The dash line marks the position of the current front at times, and a constant propagation speed can be seen. The front speed in our experiment is approximately 0.82 m/s. This agrees well with the result of Lowe et al. (2005) (about 0.83 m/s) and Adegok et al. (2014); Lauber and Hager (1998) (about 0.9 m/s). In the measurement domain, the averaged height changes in time, i.e., it increases from 0 to approximately 2.4 mm (see Fig. 11b).

5 Discussion

Equation (2) has a simple form and can be solved fast for a case using our MATLAB code. This is ideal for performing the uncertainty and error analysis, as well as deriving the critical condition when the working principle is broken, and thus, Eq. (2) is used for solving the surface height in this section.

5.1 Measurement uncertainty

The measurement uncertainties of the height surface stem from those of the displacement $\delta \mathbf{r}$ and the calibrated angle β , as well as the refractive indices n_a and n_l .

For the uncertainty of the displacement, the presented method relies on computing the displacements of dots of a pattern via a cross-correlation method. In the present study, the configuration of the interrogation windows of size decreasing down to 24×24 pixel² with 75% overlap (a suggested setting in Lavisio Davis), and a sub-pixel Gaussian interpolation was used Raffel et al. (2018). The uncertainty is expected to be around 0.1 pixel and smaller, because the random-dot pattern was generated to satisfy an optimal PIV condition Moisy et al. (2009). In our problem, the uncertainty on each component of displacements of an instantaneous sample is up to about 0.05 pixel (from Lavisio Davis©, see Wieneke (2015)). To investigate the uncertainty of the cross-correlation method on the measured surface height, the uncertainty propagation in a linear system cannot be applied to Eqs. (1) (or 2). Instead, random noise was added to two components of displacement fields to evaluate the uncertainty. Two types of noises were examined: Gaussian distribution with a mean of 0.1 pixel and a standard deviation of 0.05 pixel, where signs (+ or -) of the values are

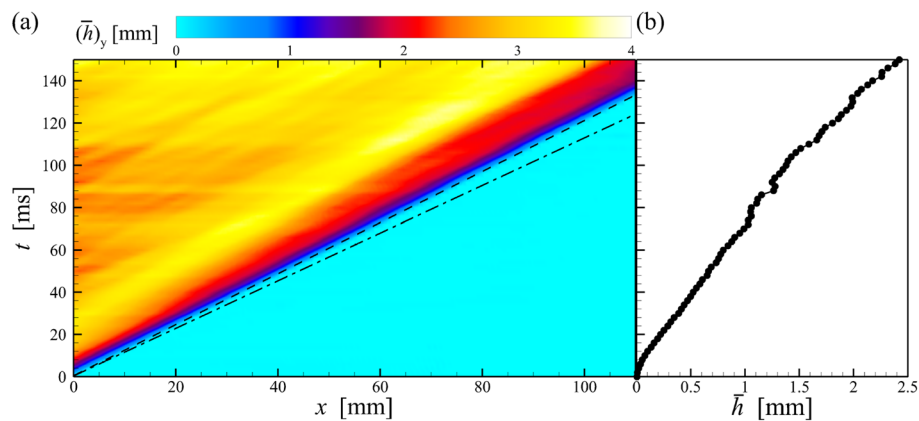


Fig. 11 (Color online) **a** Spatio-temporal diagram of the current fronts, where the color indicates the $h(x, y)$ averaged over y , $(\bar{h})_y(x) = \sum_{i=1}^N h(x, y_i)/N$. The dash line indicates the front speed in our experiments. The front speed of Lowe et al. (2005) is nearly the

same as our experiments and is also denoted by this dash line. The dash-dot line indicates the front speed of Adegok et al. (2014). **b** The corresponding spatial averaged height $\bar{h}(t)$, in sharing the same axis t

randomly chosen; white noise (ranging from -0.15 pixel and 0.15 pixel). The contaminated displacements were used to calculate the surface height $\hat{h}(x, y)$, and the resulted difference is $\hat{h}(x, y) - h(x, y)$. This procedure was repeated 100 (Gaussian) +100 (white) times, and for each time, the noise was randomly generated.

The uncertainty of the calibration angle $\Delta\beta$ can be obtained from uncertainty propagation as $\Delta\beta = [(\partial\beta/\partial\delta\mathbf{r}_c)^2 (\Delta\delta\mathbf{r}_c)^2 + (\partial\beta/\partial h_c)^2 (\Delta h_c)^2 + (\partial\beta/\partial n_a)^2 (\Delta n_a)^2 + (\partial\beta/\partial n_l)^2 (\Delta n_l)^2]^{1/2}$. Here, $\Delta(\cdot)$ refers the uncertainty of a quantity, i.e., $\Delta\delta\mathbf{r}_c = 0.1$ pixel ($\approx 8 \times 10^{-3}$ mm in our setup), $\Delta h_c = 0.01$ mm, $\Delta n_a = \Delta n_l = 5 \times 10^{-5}$ are considered. For the calibration data shown in Fig. 2, the mean and the standard deviation of $\Delta\beta$ are about 10^{-4} and 3×10^{-4} , respectively, taken statistics from the spatial distribution of $\delta\mathbf{r}_c$. For the configuration used in this study, the dominant term contributing to $\Delta\beta$ (approximately 90%) is from $(\partial\beta/\partial\delta\mathbf{r}_c)^2 (\Delta\delta\mathbf{r}_c)^2 = (-1/[h_c(1 - n_a/n_l)])^2 (\Delta\delta\mathbf{r}_c)^2$. Following the same procedure above, the Gaussian and the white random noises are imposed on β and repeated 100 times for each type of noise, to obtain the uncertainty of the surface height. The uncertainty of the refractive indices is evaluated by using 5 values for each of n_a and n_l (within the uncertainty bounds) to examine their influence on the solved surface height.

The above analysis was implemented on the data shown in Fig. 4(b) and a case with analytical solution, which was numerically generated in equation (2) with $h = 0.5 \sin(0.1x + 0.08y) + 10$ (simulating ripple stripes). For the former case, the relative uncertainty of h resulted from that of $\delta\mathbf{r}$, which is obtained from 100 noise samples, is approximately 0.2% in average (and a standard deviation of 1.2%) for the Gaussian noise, and 0.1% in average (and a standard deviation of 0.9%) for the white noise. The mean relative uncertainty of h from that of β is approximately 3.0% and 1.4% for the Gaussian and the white noises, respectively, and their corresponding standard deviation is approximately 0.3% and 0.1%, respectively. The mean relative uncertainty of h from that of n_a and n_l is approximately 10^{-5} % for the considered values. For the latter case, the mean relative error of h from that of $\delta\mathbf{r}$ is -0.3% (and a standard deviation of 2.0%) and nearly zero (and a standard deviation of 1.4%) for the Gaussian and white noises, respectively. The mean relative uncertainty of h from that of β is approximately 4.1% and 2.2% for the Gaussian and the white noises, respectively, and their corresponding standard deviation is approximately 0.5% and 0.3%, respectively. The mean relative uncertainty of h from that of n_a and n_l is approximately 10^{-4} % for the considered values.

For the testing results above, the overall uncertainty of the surface height seems to be from that of β . However, if in the calibration h_c is doubled, the dominant contribution term to $\Delta\beta$ is approximately reduced half, and the corresponding

uncertainty of the surface height is approximately 1/3 of the original value. An even larger h_c is expected to further reduce the influence of $\Delta\beta$ to the measured surface height. In summary, the uncertainty of the measured surface height is approximately 3% for this method, and it is determined by the uncertainty of the displacement, which is given by the uncertainty of the correlation algorithm.

5.2 Effect of camera vibration

The measurements using our method are sensitive to camera vibration. In our preliminary set-up, the camera vibration was particular evident, and it was possibly from the cooling fan of the high-speed camera, as reported elsewhere Moisy et al. (2009), or the vibration of the supporting profiles for the camera. The vibration produced approximately 0.2 pixel in images of a still target, and the vibration was mainly along y direction in the images. To evaluate the effect of the camera vibration on surface height, a noise taking the form $\delta\mathbf{r}_{\text{vibration}}(x, y) = c_r \cdot y + c_t$ is employed to mimic rotation (c_r) and translation (c_t) of the camera in vibration. $\delta\mathbf{r}_{\text{vibration}}$ is then added to the measured $\delta\mathbf{r}$ to compute $h(x, y)$. The combination of $-3 \times 10^{-4} \leq c_r \leq 3 \times 10^{-4}$ and $-0.2 \leq c_t \leq 0.2$ (in pixel) gives vibration displacement up to 0.4 (pixel). Taking a measurement sample of $\bar{h} \approx 10$ as an example (as shown in Table 3 in appendix), the mean height ranges from 9.3 to 13, producing the relative error up to approximately 30% (see Table 4 in appendix).

Particular care is therefore necessary to stabilize the camera against vibrations. If the vibration remains intolerable, measuring the vibration simultaneously with the surface height is necessary. For instance, the border or the corner region in the field-of-view of a camera can be used to record still objects without any light refraction, while the center region of the frame can be used to measure the marker displacements for the surface height. The shifts of the still targets can be then taken for evaluation of the camera vibration, e.g., through a two-dimensional linear interpolation (with assumed form $\delta\mathbf{r}_{\text{vibration}}(x, y) = c_r \cdot y + c_t$). The $\delta\mathbf{r}_{\text{vibration}}(x, y)$ needs to be removed from $\delta\mathbf{r}(x, y)$ before the Newton-Raphson computation.

The vibrations can also be decomposed to the motions within the camera sensor plane and the ones along the optical axis. The former brings in spatially uniform displacement to $\delta\mathbf{r}$. The latter gives extra magni-/mini-fication of the displacement field which is associated with the focal length of the camera lens, and it affects the mean surface height more rather than the surface topography (i.e., ∇h). For a small vibration of a camera along the optical axis, the magnitude of the measurement error of the surface height at a point \mathbf{x} equals $[-\delta\mathbf{r}(\mathbf{x})/(1 - n_a/n_l)/(\nabla h \cdot l_o - \mathbf{x})]^{-1}$, after ignoring the high-order small-magnitude terms. Here, l_o is

the object distance of the camera. Given the trivial influence of the vibration on ∇h , the magnitude of the measurement error of the surface height is reduced as l_o is increased if the magnification of the displacement is kept the same. This implies that if keeping the measurement field-of-view and imaging magnification the same, a camera lens with a longer focal length (used with longer object distance) is preferred to reduce the influence of the vibrations.

5.3 Effect of resolution

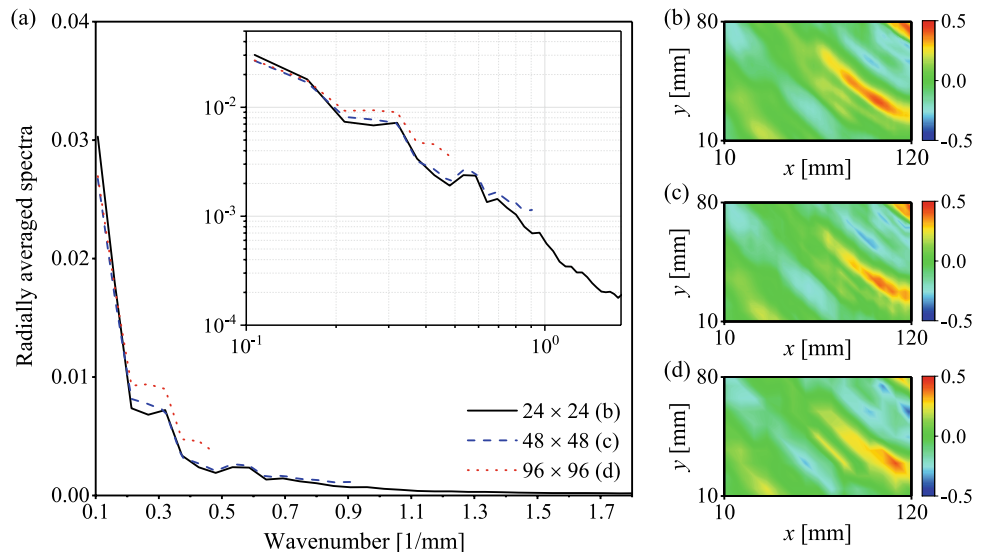
The spatial resolution of the displacement $\delta \mathbf{r}(x, y)$ plays a key role for measurements of $h(x, y)$. The dynamic range of surface topography wavelengths is determined approximately by the dynamic range of $\delta \mathbf{r}$. To examine the effect of resolution, a pair of images was processed with the interrogation window size of 24^2 pixels² up to 96^2 pixels² and 0% overlap was used. As shown in Table 2, as the resolution is decreased (increase in window size), the topographic structures are smoother and more flat, given by the decrease of $\max(|\nabla h|)$ and the decrease of $\max(h) - \min(h)$, respectively, and the averaged height is correspondingly changed.

The two-dimensional power spectrum was calculated for $h(x, y) - \bar{h}$ to examine the fluctuations of the heights, and

Table 2 The effect of interrogation window size on the surface height and topography of the snapshot in figure 4(c)

Interrogation window [pixel ²]	$\max(h) - \min(h)$ [mm]	$\max(\nabla h)$ [mm]	\bar{h} [mm]
24×24 (0% overlap)	0.921	0.165	9.666
48×48 (0% overlap)	0.882	0.116	10.214
96×96 (0% overlap)	0.852	0.063	10.189

Fig. 12 (Color online) **a** Radially averaged spectra of $h(x, y) - \bar{h}$ for three interrogation window sizes. The inset shows the same data in logarithm–logarithm axes. **b–d** Contours of the same region obtained with interrogation window 24^2 , 48^2 and 96^2 pixels², respectively.



the radially averaged power spectrum is shown in Fig. 12(a). The curve of 48^2 nearly collapses with that of 24^2 for moderate and large topography structures (see b and c). For an interrogation window of 96^2 pixels², the spectra curve deviates that of 24^2 , while the noticeable difference on the surface topography can be seen in (b) and (d).

5.4 Breakdown condition of working principle

In the present method, the governing equation is physically correct provided that the light is refracted *once* on the air–liquid interface. The working principle of the present method is violated when a light ray is refracted more than once or caustics is formed. The critical condition for the former case can be depicted: For a given sinusoidal plane wave $h(x) = h_p + \eta_0 \cos(2\pi x/\lambda)$ with wavelength λ and amplitude η_0 , the critical wave amplitude is $\eta_c = \lambda[1/4 - \beta_x/(2\pi)]$. At the same wavelength, when $\eta_0 > \eta_c$, light ray may be refracted more than once, see the dashed line in Fig. 13(a). For the snapshot in Fig. 4(b), $\lambda \approx 60$ (mm) and $\beta_x \approx 0.08$ (rad) give

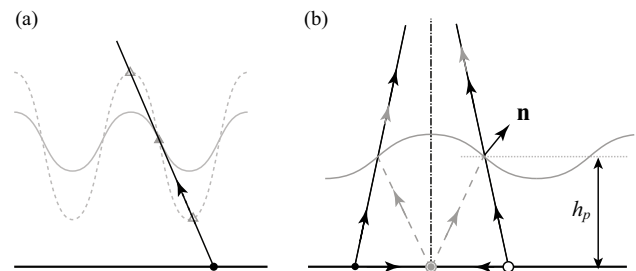


Fig. 13 The illustration of breakdown condition of the working principle: **a** Large oscillation amplitude of surface wave, where the light ray is refracted more than once as marked by hollow triangles. **b** Critical condition for formation of caustics.

$\eta_c \approx 15 \text{ (mm)} > 4 \text{ (mm)}$, and rays are refracted only once at the surface. The formation condition of the caustics depends on the viewing angle and the curvature of the surface Gomit et al. (2013). Taking the same sinusoidal plane wave as an example, substituting $h(x, y) = h_p + \eta_0 \cos(2\pi x/\lambda)$ into Eq. 2 with $\delta \mathbf{r}(\lambda/4) \approx h_p \beta_x + \lambda/4$, the critical height is $h_{p,c} = \lambda^2/[8\pi\eta_0(1 - n_a/n_l)]$ taken the minimum of h_p . When $h_p < h_{p,c}$, no caustics is expected to be formed. However, for the very front of the current in the dam-break flow, the light rays are refracted more than once.

An *a posteriori* method was introduced in Moisy et al. (2009) to evaluate the formation of the caustics. When the largest extensional strain of the displacement field is smaller than one, no formation of caustics is expected. We used this *a posteriori* method to examine all snapshots in this study. For instance, the maximum eigenvalue for the snapshot in Fig. 4(b) is 0.193 (< 1) and 0.025 (< 0.15) Moisy et al. (2009).

5.5 The effect of linearizing the governing equation

Equation (2) is obtained by linearizing the governing Eq. (1), e.g., taking $\tan \nabla h \approx \nabla h$ and $\tan \beta \approx \beta$, by truncating the second and higher order terms in their respective Taylor expansion. For example, at $\nabla h = 0.5$, $\tan(\nabla h) \approx 0.5463$ and the relative difference is about 8%. The influence of the approximation on the solution is examined in the following.

For the snapshot of the surface ripples shown in Fig. 4(b), the solution of Eq. (2) and Eq. (1) is shown in Fig. 14(a) and (b), respectively. The difference between (a) and (b) is shown in (c). The mean relative difference is about 0.02%, and the standard deviation is also about 0.02%. For the snapshot of the current body shown in Fig. 7(a), the solution of Eq. (2) and Eq. (1) is shown in Fig. 15(a) and (b), respectively. The difference between (a) and (b) is shown in (c). The mean relative difference is about $10^{-3}\%$, and the standard deviation is about 0.1%.

For the snapshot of the current front shown in Fig. 10(a), the solution of equation (2) and equation (1) has visual differences (see Fig. 16a and b). The difference between the two solutions, as shown in (c), gives the mean relative difference about 12% and the standard deviation of about 30%. Such difference is expected, since in the front $\nabla h \gg 1$ which violates the linearization approximation. Examination of the height profiles shows that the difference also exists downstream of the front (see Fig. 16d). Although the same Dirichlet boundary condition was employed for both solvers, the noise at downstream of the front for Eq. (1) remains noticeable, whereas that for the linearized equation is nearly removed.

The linearized equation is accurate for $\nabla h < 1$, whereas for $\nabla h \gg 1$ (the current front), solving equation (1) is expected to give better results. However, downstream of the front there is noticeable noise albeit the Dirichlet boundary

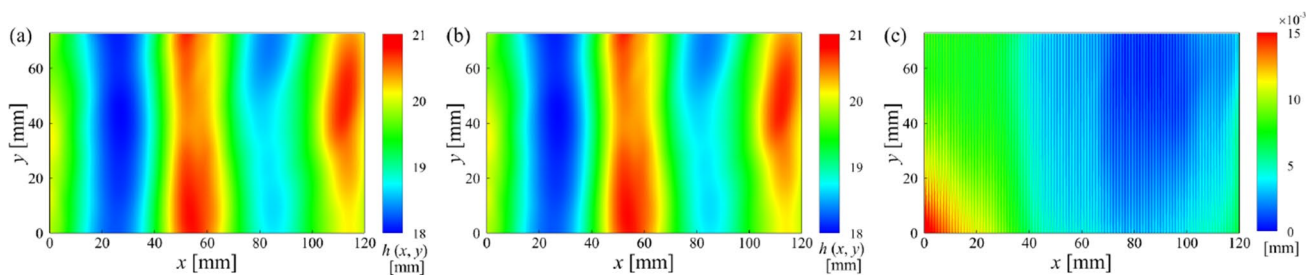


Fig. 14 (Color online) Contours of a sample of the surface ripples (the same snapshot as in Fig. 4 which is obtained by solving **a** linearized equation (2), **b** equation (1) and **c** their difference.

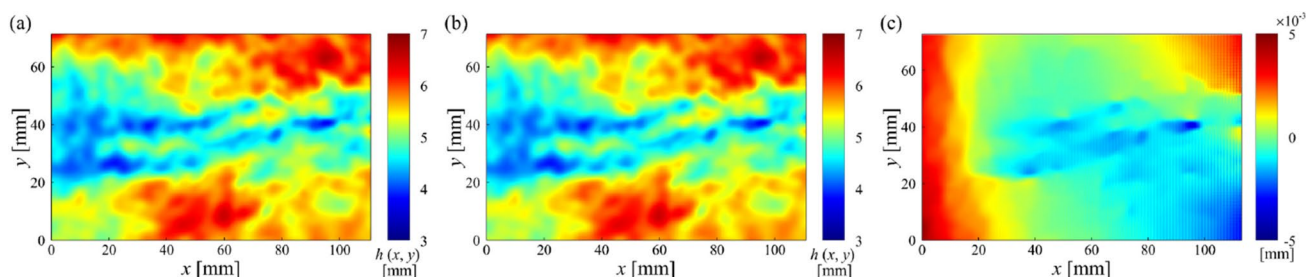


Fig. 15 (Color online) Contours of a sample of the current body (the same snapshot as in Fig. 7) obtained by solving **a** linearized equation (2) and **b** Eq. (1) and **c** their difference

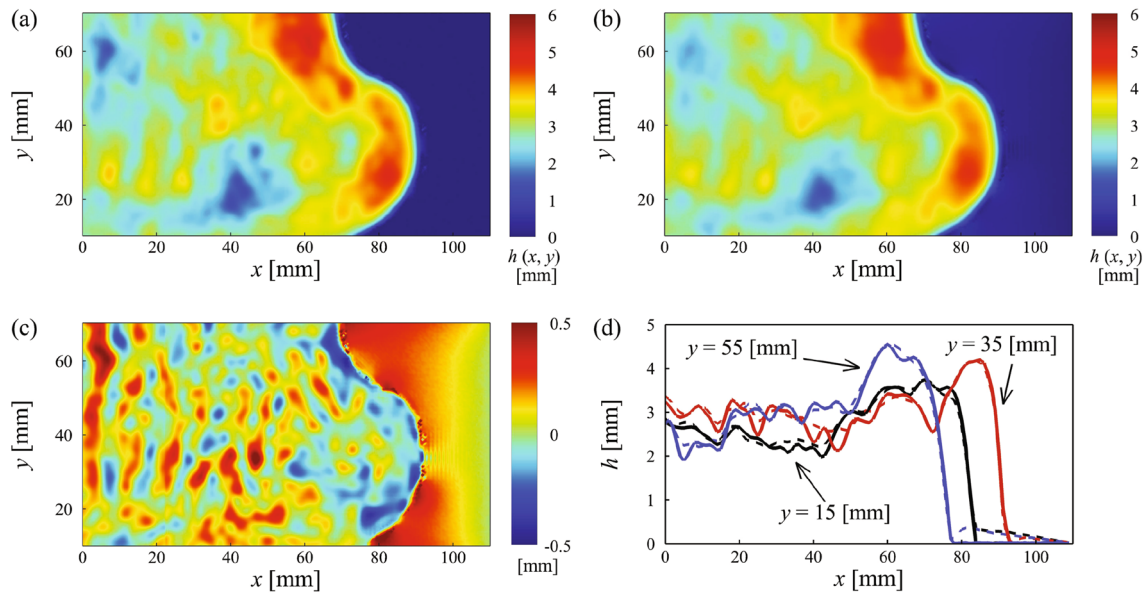


Fig. 16 (Color online) Contours of a sample of the current body (the same snapshot as in Fig. 10) obtained by solving **a** linearized equation (2) and **b** Eq. (1) and **c** their difference. **d** Three height profiles chosen from (a) (solid lines) and b (dashed lines) at $y = 15$ mm, 35 mm and 55 mm

condition is imposed. In consideration that solving Eq. (1) takes much more computation power, the linearized equation is therefore preferred.

6 Conclusion

In this paper, we introduce a single-camera synthetic Schlieren method to measure the topography and height of dynamic free surface. In this method, displacements of the dot patterns are attributed to both the surface height and its spatial gradients, while the reference height is not required. This method is thus applicable to flows with spatially averaged height changing in time. The dot displacements are obtained by the cross-correlation operation, while calibration is carried out to obtain the viewing angle of a camera toward the pattern object. The surface height, the only unknown in the governing equation, can be solved via a Newton–Raphson method with a proper initial guess.

We carried out two experiments to demonstrate the present method. In the experiments of surface ripples, the spatial averaged height is nearly constant. No particular treatment is performed on the boundary conditions at the borders of the computation domain. The measurement results of the present method and those of the FS-SS method of Moisy

et al. (2009) agree well. In another experiment, the averaged height of the current in a dam-break problem changes with time. For the current front, a Dirichlet boundary condition is applied to the downstream boundary condition of the computation domain to amend the ill-conditioned Jacobian matrix in the Newton–Raphson solver (possibly due to the sharp current front). The Dirichlet condition is not necessary for the current body, but only when the front passes the measurement section.

The measurement uncertainty was analyzed, and it is found that the uncertainty of the measured surface height (approximately 3% in this study) is dominated by the uncertainty of the displacements obtained using the correlation algorithm. In addition, the presented method is still susceptible to some limitations, when the working principle can be satisfied. Specifically, the measurement results are sensitive to the camera vibrations, a problem also reported in the FS-SS method. The vibrations change the spatial relation between the camera and the dot pattern; consequently, the solutions are affected particularly on the averaged surface height. Thus, particular care is required to improve the stability of the camera as well as other components of the experimental setup. When further improvement on the setup stability is impossible, a camera with a lens of long focal length is suggested while keeping the same field-of-view being viewed, the solution is found to be less sensitive to the vibrations in our practical tests. An

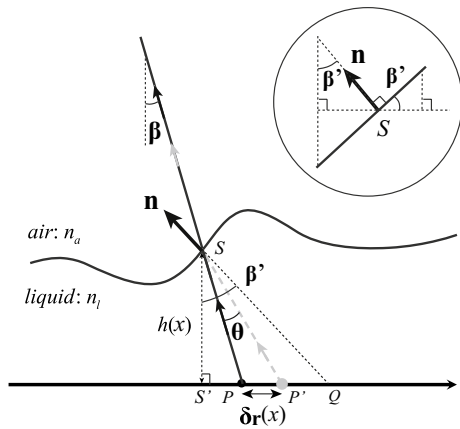


Fig. 17 Schematic of the derivation of working principle. The inset shows the zoom-in around the point S .

alternative suggested solution is to take the measurements of free surface using a portion of the camera sensor, in the meanwhile use the other portion of the sensor to record the still object which is not affected by light refraction on imaging.

Table 3 The averaged surface height \bar{h} computed from the displacements including the camera vibration $\delta \mathbf{r} + \delta \mathbf{r}_{\text{vibration}}$ for the measurement sample of $\bar{h} \approx 10$ from the experiments of liquid ripples, and $\delta \mathbf{r}_{\text{vibration}}(x, y) = c_r y + c_t$.

c_r	-3×10^{-4}	-2×10^{-4}	-1×10^{-4}	0	1×10^{-4}	2×10^{-4}	3×10^{-4}
c_t							
-0.20	10.280	9.917	9.629	9.480	9.538	9.817	10.257
-0.15	9.995	9.672	9.467	9.452	9.668	10.071	10.577
-0.10	9.739	9.485	9.401	9.546	9.902	10.387	10.925
-0.05	9.534	9.387	9.457	9.754	10.209	10.737	11.285
0.00	9.406	9.403	9.632	10.046	10.557	11.101	11.647
0.05	9.386	9.542	9.903	10.388	10.924	11.469	12.006
0.10	9.486	9.785	10.234	10.755	11.296	11.835	12.359
0.15	9.697	10.100	10.597	11.130	11.667	12.193	12.704
0.20	9.988	10.454	10.972	11.505	12.031	12.544	13.040

Table 4 The relative errors (%) of $\bar{h}(x, y)$ referring to the case $c_r = 0$ and $c_t = 0$ in table 3

c_r	-3×10^{-4}	-2×10^{-4}	-1×10^{-4}	0	1×10^{-4}	2×10^{-4}	3×10^{-4}
c_t							
-0.20	2.33	-1.28	-4.15	-5.63	-5.06	-2.28	2.10
-0.15	-0.51	-3.72	-5.77	-5.91	-3.76	0.25	5.29
-0.10	-3.05	-5.58	-6.42	-4.97	-1.43	3.40	8.75
-0.05	-5.10	-6.56	-5.86	-2.90	1.62	6.88	12.34
0.00	-6.37	-6.40	-4.12	0.00	5.08	10.51	15.94
0.05	-6.57	-5.02	-1.42	3.41	8.74	14.17	19.51
0.10	-5.57	-2.59	1.88	7.06	12.45	17.81	23.02
0.15	-3.47	0.54	5.49	10.79	16.14	21.38	26.46
0.20	-0.57	4.06	9.22	14.52	19.76	24.87	29.81

Appendix

The derivation of governing Eq. (1) is shown below. From the sketch in Fig. 17, we have

$$\begin{cases} S'P = h \tan(\beta), \\ S'P' = h \tan(\beta + \theta), \\ \delta r = S'P' - S'P, \\ \tan(\beta') = \nabla h, \\ \sin(\beta' - \beta - \theta) n_l = \sin(\beta' - \beta) n_a, \end{cases}$$

where the first four equations are obtained from geometry and the last one is from the Snell's law. Combining the above equations to eliminate θ gives the equation (1),

$$\delta r = h \cdot \tan(\tan^{-1}(\nabla h) + \sin^{-1}[(n_a/n_l) \cdot \sin(\beta - \tan^{-1}(\nabla h))]) - h \cdot \tan(\beta).$$

Tables 3 and 4.

Supplementary Information The online version contains supplementary material available at <https://doi.org/10.1007/s00348-021-03326-0>.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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