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LETTER

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Fatigue damage modeling of ceramic-matrix composites: A short review

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Abstract
Prediction of fatigue damage and fatigue life in fiber-reinforced ceramic-matrix composites under thermomechanical loading is of critical importance in the reliability and safety design of advanced aerospace structures. This article presents a short review of the state of the art for the primary fatigue models in the fatigue life prediction of ceramic-matrix composites. Two kinds of fatigue models have been introduced: (a) phenomenological models, including fatigue life models, residual strength model, and residual stiffness model, and (b) progressive damage models correlated with macroscopic residual mechanical properties and mesoscopic damage mechanisms. Based on the literature review, the writer recommends that the most effective method for ceramic-matrix composites fatigue modeling is progressive damage models. Finally, the writer proposes to establish a progressive fatigue damage model associated with the service environment.

KEYWORDS
ceramic-matrix composites (CMCs), fatigue damage modeling, progressive damage models (PDM), thermomechanical fatigue

1 | INTRODUCTION

Fiber-reinforced ceramicmatrix composites (CMCs) are excellent composites due to its outstanding properties at high temperatures, such as high stiffness, high strength, and stiffness-to-density ratios as well as good thermal stability. The high-temperature applications for CMCs in aerospace propulsion system components (combustors, nozzle guide vanes, flame stabilizers, etc.) and thermal protection systems in reentry vehicles[1,2] are shown in Figure 1, referring to previous stuides.[3,4] More especially, the aero-engine applications require the long-term thermal durability of the structural materials under a variety of conditions involving loadings and temperature, even are expected to undergo loading fluctuations during operation.[5-8] The fatigue damage occurs in the composites, reducing the load-carrying capacity, and sometimes leading to catastrophic failure under combined thermal and mechanical loads. In addition, the current fatigue life prediction methods for those composites often require large safety factors, and extensive prototype-testing allowing for an acceptable life prediction.[9,10] Composite structures are often overdesigned. Consequently, a clear understanding of the fatigue behavior of CMCs is of the essence for the design of components with high structural integrity.

The present understanding of fatigue behavior for fiber-reinforced CMCs is as follows. So far, the research on CMC fatigue mainly focuses on the following aspects: (a) purely fatigue experimental, often without a detailed discussion of the underlying mechanisms for the observed particular trends, and (b) fatigue theoretical, these models often assume or emphasize incorrect fatigue damage mechanisms. The lack of extensive research for CMCs can be mainly attributed to the fact that fatigue experiments are hard to perform, which require enormous patience, and the results are often

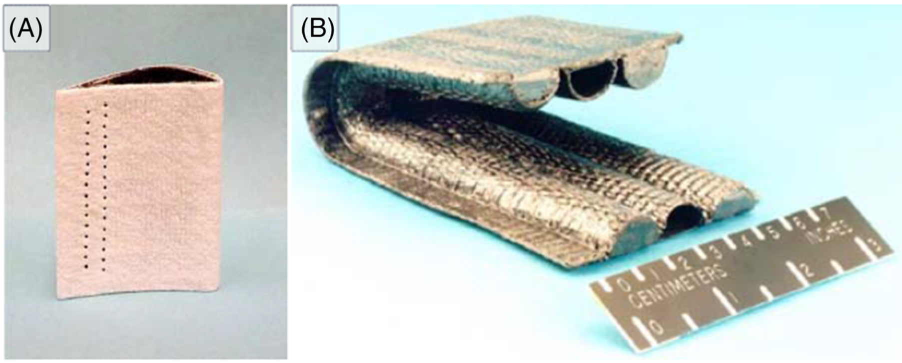

FIGURE 1 Applications for ceramic-matrix composites (CMCs) in gas turbine engines and thermal protection systems (TPS) for hypersonic vehicles. (A) Blade and (B) Mo-Re tube embedded in C/C CMC, used in NASP heat-pipe-cooled wing leading edge

complex to interpret.[11,12] The research in fatigue damage modeling of CMCs is relatively limited compared with research on other mechanical properties of CMCs.

CMCs are anisotropic and heterogeneous, and their behavior is more complicated than that of isotropic and homogeneous materials such as metals. The mesoscopic stress distribution is complex and presents multiaxial characteristics, resulting in a complicated failure mechanism in CMCs. CMC fatigue damage involves many different types of damage mechanisms.[13,14] Moreover, there are many kinds of CMCs, and their fatigue characteristics are affected by the reinforce fiber type, matrix type, environmental conditions (mainly high temperature, thermal oxidation, and steam environment), loading conditions and boundary conditions, etc. It is impossible to accurately predict the fatigue damage or residual life of all composite materials using the same model. Therefore, researchers carried out a large number of fatigue tests and established corresponding fatigue damage models, and these models have also been applied for CMCs. It is also an opportunity to review various fatigue models for CMCs.

In the present paper, we briefly review the classic fatigue damage models in composites firstly, to make readers better understand the concrete situation of CMCs. Then, the progressive damage model (PDM) is introduced and described in detail, taking into account the fatigue damage mechanism. Finally, the writer suggests to establish a progressive fatigue damage model associated with the service environment, which enable one to come even closer to aerospace industrial needs. In this model, the individual contribution of thermal shock, long-term thermal aging, and other factors to the fatigue damage of materials can be described quantitatively, and the quantitative factors should be incorporated. This approach leads to a good understanding of factors related to service environment that is most likely to be a decisive factor in predicting the lifetime of the material.

2 | PHENOMENOLOGICAL FATIGUE MODEL

According to the different mechanisms of quantitatively characterizing fatigue damage, the fatigue life prediction model of existing composites can be classified into two types: macrophenomenological model and micromechanical model.[15,16] The phenomenological fatigue model of composites can be divided into three categories[17,18]: (a) fatigue life model based on *S-N* curve or equivalent life map, (b) residual strength model, and (c) residual stiffness model.

2.1 | Fatigue life model

The fatigue life model of composites uses the existing test results to fit the model formula, mainly the *S-N* curve and the equivalent life map.[6,19] Studies have shown that the classical Goodman method for metal fatigue performance prediction is no longer suitable for composite materials. Current fatigue life models generally adopt a failure criterion as the basis and take the empirical *S-N* curve as the input. The *S-N* curve be used to predict the number of cycles to failure, which has the form:

$$\Delta\sigma = \sigma_{\mathrm{UTS}}(N_{\mathrm{f}})^{B}, \tag{1}$$

where $\Delta\sigma$ is the applied stress amplitude, σ_{UTS} is the monotonic failure stress, N_{f} is the number of fatigue life cycles, and B is the fitting parameter determined by experiments. Equation (1) does not take into account the damage accumulation.

A representative example of the fatigue life model is developed by Philippidis,[20] based on the Tsai-Hill criterion:

$$\left(\frac{\sigma_{11}}{\sigma_{11f}}\right)^2+\left(\frac{\sigma_{22}}{\sigma_{22f}}\right)^2+\left(\frac{\tau_{12}}{\tau_{12f}}\right)^2-\left(\frac{\sigma_{11}}{\sigma_{11f}}\right)\left(\frac{\sigma_{22}}{\sigma_{22f}}\right)<1 \tag{2}$$

where σ_{11f},σ_{22f} and τ_{12f} denote the fatigue failure stress of the *S-N* curves. This criterion can be applied to model fatigue evolution at any stress ratio and frequency as long as the *S-N* curves are available.

The fatigue life model method requires a lot of experimental work and does not consider the actual damage mechanism, such as fiber fracture and matrix crack.

2.2 | Residual strength model

The residual strength model assumes that the residual strength decreases monotonically with the number of fatigue cycles of the structure increases, and the initial residual strength is defined equal to the static strength.

The general expression of the residual strength model is[21]

$$\frac{\mathrm{d}\sigma_{\mathrm{r}}}{\mathrm{d}n}=-B\frac{An^{A-1}}{C\sigma_{\mathrm{r}}^{C-1}}, \tag{3}$$

where σ_{r} is the residual strength, n is the number of cycles, and A,B, and C are fitting parameters depending on the loading level and are constant for the same load level. At the same load level, it can be integrated as

$$\sigma_{\mathrm{r}}^C=\sigma_{\mathrm{ult}}^C-\left(\sigma_{\mathrm{ult}}^C-\sigma_{\mathrm{p}}^C\right)\left(\frac{n}{N_{\mathrm{f}}}\right)^A, \tag{4}$$

where σ_{ult}^C and σ_{p}^C denote the static strength and the peak value of the cyclic stress at the load level, respectively. There are also many types of residual strength models proposed by researchers, which can be summarized as special forms of Equation (4).

2.3 | Residual stiffness model

The residual stiffness model connects the gradual deterioration of the stiffness of the composite structure with the fatigue life in terms of macroscopically observable properties and analyzes the deformation and stress redistribution behavior of structures.[22] In the fatigue test, the residual stiffness can be tested repeatedly as required, and even can be performed in situ testing without introducing addition damage. Based on continuous damage mechanics, the internal variable D is used to represent the amount of stiffness degradation, and $\mathrm{d}D/\mathrm{d}N$ is the rate at which the stiffness decreases with the number of cycles. One such model is proposed by Whitworth,[23]

$$E(n)=E(0)\left(\frac{\sigma_{\mathrm{a}}}{c_1\sigma_{\mathrm{u}}}\right)^{\frac{1}{c_2}}\left[-h\ln(n+1)+\left(c_1\frac{\sigma_u}{\sigma_a}\right)^{\frac{m}{c_2}}\right]^{\frac{1}{m}}, \tag{5}$$

where $E(0)$ and $E(n)$ are the initial stiffness, stiffness at n cycles, and σ_{u} and σ_a are the ultimate strength and applied stress, respectively. The parameters c_1,c_2,h, and m can be obtained from experiments.

In summary, the phenomenological fatigue model is directly used in the industry because of its simple form, which directly links the number of fatigue cycles with life. However, the phenomenological fatigue models can only predict the fatigue behavior of composite laminates under uniaxial fatigue loading and cannot reflect the complex stress state of the actual structure. To obtain the precise parameters of the fatigue model, only the material level testing can simulate the complex stress state of the actual structure to fully characterize the fatigue behavior of materials. In fact, the fatigue damage mechanisms observed under fatigue loading including matrix crazing, matrix cracking, fiber/matrix interfacial debonding, delaminations, fiber buckling, fiber breaking/pull out, and finally global fracture,[24,25] as shown in Figure 2. More essentially, the fatigue damage process in CMCs is accompanied by progressive damage propagation and stress redistribution behavior. Based on the above consideration, the fatigue damage model itself needs to be improved, and it needs to perform multiscale simulations based on fatigue damage mechanisms and analyze the whole fatigue evolution process in order to reveal the behavior of actual CMCs and accurately predict the fatigue life of the composites.

3 | PROGRESSIVE DAMAGE MODELS

The fatigue failure of composite structures is a progressive damage process. Firstly, the damage accumulation is gradually formed at the maximum stress level. The local damage and stiffness deterioration cause stress redistribution. The position of the maximum stress value shifts, and the damage gradually spreads to other parts. Therefore, it is necessary to determine the local damage model of the materials, and an intensive understanding of the fatigue progressive damage mechanisms is necessary for an optimally designed CMC structure. In the PDM, one or more properly damage variables are introduced to describe the deterioration of the composite components.

The progressive damage models are based on the underlying micromechanical damage mechanisms, such as frictional sliding of the bridging ligaments/fibers and microcracking, which result in a decrease in macroscopic observable mechanical properties.[18,26] In other words, the damage models can not only predict the growth of damage (such as the size of the delaminated area and number of transverse matrix cracks per unit length) but also correlate the damage growth with the residual mechanical properties (stiffness/strength).

More essentially, the damage mechanism model usually needs to be combined with multiscale simulation to study the relationship between mesoscopic damage mechanism and macroscopic fatigue behavior and the fatigue response under complex stress conditions. Based on the failure mechanism of CMCs, the key factors such as interface debonding, matrix cracking, and fiber fracture are studied. Rouby et al.[27] proposed a micromechanical model describing the fatigue effects based on the degradation of shear stress at the fiber/matrix interfaces, due to the interfacial wear caused by see-saw sliding. Evans et al.[28] emphasized that fiber and interface degradation are the dominant fatigue mechanism for many CMCs, and the matrix itself has no fatigue mechanism operated, because the matrix is low toughness, the basic model proposed is shown in Figure 3. Shokrieh and Lessard[29,30] established progressive fatigue damage modeling technique for simulating the fatigue behavior of laminated composite materials, with or without stress concentrations, which integrate of three major components: material property degradation rules, failure analysis, and stress analysis. The model is capable of simulating the residual stiffness, residual strength, and fatigue life of composite laminates with arbitrary geometry and stacking sequence under complicated fatigue loading conditions. Montesano et al.[31] characterized and modeled the high-temperature fatigue behavior of a triaxial braided carbon fiber-reinforced polymer matrix composite. Based on quantified parameters of observed damage mechanisms, the developed fatigue damage model can accurately predict the response of materials under various stresses and

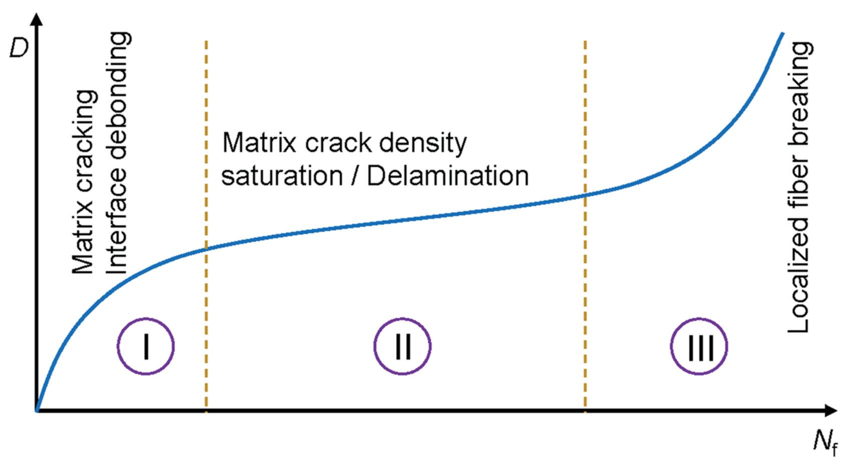

FIGURE 2 Schematic representation of fatigue damage initiation and accumulation process with cycles in ceramic-matrix composites. The fatigue damage evolution is a nonlinear function of number of cycles and has three characterizing patterns: Stage (I) is characterized by matrix microcracking and fiber/matrix debonding; Stage (II) is characterized by the initiation and propagation of delamination, matrix cracks has been up to saturation state; Stage (III) revealed that all the damage modes would be developed rapidly in a fast-decreasing stiffness of the composites

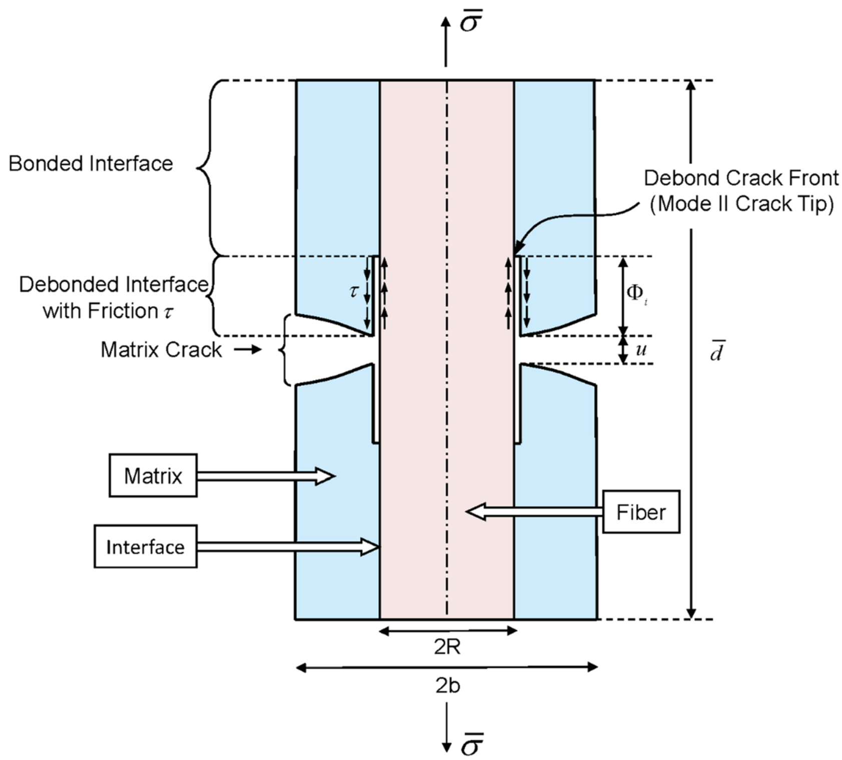

FIGURE 3 The cell model relating macroscopic strains to constituent properties

temperatures and captures the unique characteristics of fatigue damage evolution of braided composite. Shojaei et al.[32] investigated the microscale damage mechanisms in CMCs in detail and a continuum damage mechanic (CDM) model is developed within a physically consistent framework to study two common failure modes in CMCs, ie, matrix/interphase fracture and fiber sliding. Min et al[33] developed a micromechanic analysis modeling method to analyze the damage process and fatigue failure of CMCs. The concept of the repeating unit of fiber-reinforced composite material is used to represent the whole composite structure. Based on hysteresis energy dissipation, Li et al.[34-36] developed a damage model for fiber-reinforced CMCs under multiple loading stress levels. Considering the interaction of multiple loading sequence and fatigue damage mechanism, the variation of matrix cracking, fiber/matrix interface debonding and slip, fatigue hysteresis loop, fatigue hysteresis modulus, and fatigue hysteresis dissipationenergy with the number of cycles were analyzed.

Based on the existing research results of the mesomechanical model of CMCs, a multiscale simulation technique for fatigue damage and a fatigue life prediction method based on the whole process of fatigue damage evolution were established for braided CMCs. The RVE model of the process is used to study the failure behavior of the CMC fiber bundle under complex stress state, and then the parameters of the fiber bundle anisotropy damage constitutive model based on continuous damage mechanics are obtained.

4 | PDM ASSOCIATED WITH THE SERVICE ENVIRONMENT FACTORS

The thermomechanical fatigue is of importance for the understanding of thermal and thermomechanical fatigue phenomena in CMCs but not considered in this paper. Thermal mechanical fatigue is always caused by fluctuations in temperature and mechanical load in service. For the fail-safe design of components, understanding followed by modeling of these effects is extremely critical.

(a) The service environment will be critical in determining the lifetime of any component. In the case of a CMCs in combustion, the desired service environment could be described as being very harsh.[37,38] As the service environment of CMCs being very harsh as the components will be operating at temperatures in excess of 1000°C with varying loads as well as varying water vapor and impurities corrosion. The factors present in a combustion environment can be seen on the outer circle in Figure 4, which are the critical elements to establish a suitable and reliable fatigue damage model. In this respect, consider the factors related to the service environment, such as thermal shock,[39] long-term thermal

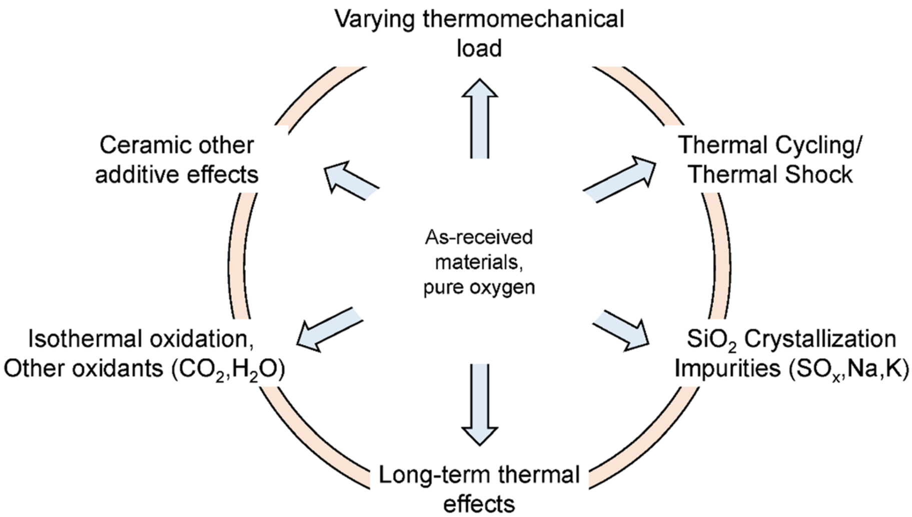

FIGURE 4 Factors related to service environment which need to be considered when researching ceramic-matrix composites (in gas turbines)

aging,[40] water vapor,[41,42] and corrosion, which are the critical elements to establish a suitable and reliable fatigue damage model, and also need to carry out systematic research.

Recently, Yang et al.[43,44] proposed a cyclic thermal shock-induced thermomechanical damage evolution model for woven oxide/oxide CMCs and developed a nonlinear damage evolution model for the thermal shocked-CMCs under plane stress assumption based on the framework of CDM. It provides an in-depth understanding of the mechanical behavior and thermomechanical damage evolution in CMCs under extreme transient thermal loads and provides theoretical and experimental basis for the fatigue life prediction of CMCs under thermomechanical loading conditions. Carelli et al.[45] investigated the effect of long-term thermal aging (1000 h) at temperatures of 1000–1200°C in air on an all-oxide fiber-reinforced composite. These studies are essentially designed to develop a comprehensive and reliable fatigue life model.

(b) Differences in thermal expansion coefficients of the matrix and the fiber-reinforced result in the residual stresses exist in most CMCs, which play a key role in the mechanical behavior, including but not limited to fatigue. As residual stress changes with temperature under operating conditions, some microstructure parameters also change, leading to thermal fatigue. Consequently, there is still a lot of research work in this area that needs to be carried out systematically. Understanding such effects is significant for improving CMCs design such as the selection of the matrix and reinforcement type, processing parameters, and content.

5 | CONCLUSIONS

Fatigue in CMCs is an important area with direct implications for the structural integrity of components. The survey presented illustrates that although fatigue behavior of CMCs has been intensively studied, an essential research thrust is needed to achieve reliable fatigue damage modeling of the processes that take place under thermomechanical loading. Due to the space limitations, this article only introduces some of the main overviews of the fatigue modeling for CMCs.

Compared with the fatigue life model, the residual strength model has a clearer physical meaning, but a lot of destructive tests are needed. The residual stiffness model can reduce the destructive test, save the experimental cost, and have the advantage to predicate the deformation and stress redistribution. However, the three models are not strictly independent of each other, because the residual strength model and the residual stiffness model are also used in the fatigue life model. For the residual stiffness model, the failure criterion is additionally established based on the residual stiffness, or the fatigue life is predicted by combining the residual strength and other parameters. More essentially, these models are specific to a particular composite and do not consider mesocomposition and damage mechanisms and cannot be generalized when component ratios or weaving parameters change. At the material level, the failure criteria used by various models to predict fatigue life are different. For the fatigue life model, the material fracture is used as the failure criterion, and for the residual strength model, the residual strength is degraded to be equal to the cyclic peak stress as the failure criterion.

The progressive damage models are the most promising tool, as the damage process of composite structures can be quantitatively reflected. A more generalized approach is microscopic damage evolution correlated with residual mechanical properties, integrating failure criteria for different damage mechanisms and constant-life analysis. The model needs to consider more factors, complex forms of expression, and difficulty in research, and there is still much research work to be done on life prediction. At present, it is mainly fatigue damage model and fatigue life prediction on the material level.

A modified version of the progressive damage model for the prediction of the fatigue lifetime of CMC structures was also presented. Its advantages lie in the possibility to perform lifetime predictions not only for the pure material's level but more generally for real engineering structures. In order to do that, several factors related to the service environment had to be added to the framework of PDM. This opens a very interesting perspective for the fatigue life of composite structure for lifetime prediction and eventually. Of course, it is necessary to additional developments and validations.

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