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Periodic Response and Stability of a Maglev System with Delayed Feedback Control Under Aerodynamic Lift

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Abstract:

In this research, the periodic response and stability of a nonlinear maglev system under the combined effects of steady and unsteady aerodynamic lifts is investigate, considering time delay in the feedback control loop. Firstly, a nonlinear maglev system with a single levitation point that accounts for the nonlinearity of the electromagnetic force, time delay in the feedback control loop, and effect of aerodynamic lift is established. Then, the periodic solutions of the maglev system with aerodynamic lift and time delays are obtained by an incremental harmonic balance analysis, in which the explicit time-delay action matrices used indicate that the effect of time delay on the response of the maglev system is periodic. The stability of the periodic solutions based on a finite difference continuous time approximation method and Floquet theory is studied, from which the critical time delay is obtained. Also examined is the relationship between the periodic vibration amplitudeand the time delay, steady aerodynamic lift coefficient, and frequency of the unsteady aerodynamic lift, as well as the variation of critical delay with respect to the position feedback and velocity feedback with the control gain parameters. In addition, the stability boundary for the simultaneous time-delayed position and velocity feedback is obtained.

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KeyWords: Maglevvehicle,Aerodynamiclift,Timedelay,Incrementalharmonicbalance,Periodicresponse,Stability.

1Introduction

Electromagneticsuspension(EMS)trainshavetheadvantagesoflowenergyconsumption,low environmentalimpacts,lownoise,lowmaintenance,andstrongclimbingabilities,andtheyhaveundergonetremendousdevelopmentinrecentyears.The dynamic characteristics of maglev train is important to its application, and many scholars have studied it in recent years[1, 2].Thenewgenerationofhigh-speedmaglevtrainswillhavedesignspeedsreaching600km/h,whichexceedsthecruisingspeedoflow-speedaircraft,andtheiraerodynamicloadswillhaveanon-negligibleeffectontheirdynamicbehaviorsduringhigh-speedmaglevoperation.Inrecentyears,moreandmoreresearchershavebegunfocusingontheaerodynamicloadsofhigh-speedmaglevtrainsandtheyhaveanalyzedtheaerodynamicloadsandpressurewavesofmaglevtrainsoperatinginopenair,passingothertrains,andothersituations[3–7].However,thedynamicresponsesofmaglevtrainssubjectedtoaerodynamicloadshaverarelybeenstudied.Kwonetal.[8]simulatedtheresponsesofmaglevvehicleswhentheypassedasuspensionbridgeandtheyweresubjectedtowindgusts.Yau[9]consideredtheaerodynamicloadcausedbyunstableairflowandcalculatedtheresponseofacoupledvehicle-railsystem.WuandShi[10]analyzedthedynamicresponseofamaglevvehiclebodyundertheactionofawindfield.Thecurrentresearchonthedynamicsofmaglevvehiclesunderaerodynamicconditionsmainlyusessimulationmethodstostudythemotionresponsesandstabilityundervariousoperatingconditions,butthesesimulationsdonotaccountforthevibrationcharacteristics,suchasthestabilityandnonlinearresponsecharacteristicsofmaglevvehicles.

ThelevitationstabilityofanEMS-typemaglevvehiclereliesonacontrolledverticalelectromagneticforce.Comparedtoaerodynamicforcesintheotherdirections,theverticalaerodynamicforce,oraerodynamiclift,hasthemostdirecteffectonthedynamiccharacteristicsofamaglevtrain.UsingatrainfortheShanghaiMaglevLineasanexample,numericalcalculationsshowedthatforarunningspeedof600km/h,theaerodynamicliftoftheleadcarcouldreach50%ofthevehicleweight[4].Therehasbeenanindustrialconsensusinthedesignofcontrolsystemsforhigh-speedmaglevtrainsthattheeffectofaerodynamicliftmustbetakenintoaccount.Asteadyaerodynamicliftchangestheequilibriumstateofamaglevtrain;asthevehiclespeedincreases,thetrainmaybecomeunstable.Therefore,thereisacriticalspeedduetotheaerodynamiclift.Wehavepreviouslyinvestigatedthedestabilizationmechanismofmaglevvehiclesundersteadyaerodynamiclift,proposedtheconceptofacriticalspeed,andsummarizedtwoinstabilitymodesforsteadyaerodynamiclift[11].

Inadditiontothesteadyaerodynamicliftduringthehigh-speedoperationofmaglevtrains,thedevelopmentofunstablevorticesaroundatraincausespronouncedunsteadyaerodynamiclift[4].Toensurethesafeandcomfortableoperationofmaglevtrainsinthepresenceofaerodynamicliftandtoprovideabasisforthedesignofacontrolsystem,inthisstudy,weinvestigatedthedynamiccharacteristicsofmaglevtrainsunderthecombinedactionofsteadyandunsteadyaerodynamicliftbasedontheresultsofpreviousstudies.

Amaglevtrainisanonlinearsystemthatintegratesmechanics,control,power,andelectronics,withelectromagneticnonlinearitybeingoneofthemostimportantnonlinearities.Furthermore,timedelaysareinevitableinthemeasurementandreceptionofsignalsandintheprocessing,output,andexecutionofsignals.Theexistenceoftimedelayswillcausechangesinthedynamicbehaviorofamaglevsystemanditmayevencauseasystemtobecomeunstable.Inrealvehicletests,therehavebeencasesofviolentvehicle-guidewayresonancecausedbyexcessivetimedelaysinacontrolloop[12,13].Toaddresstheeffectoft imedelaysonthestabilityofmaglevvehicles,Lietal.[14]analyzedtheeffectsoftimedelaysonthestabilityofacoupledvehicle-guidewaysystem.Wangetal.[15–17]consideredtheeffectsofdelayedpositionfeedbackcontrolandvelocityfeedbackcontrolontheHopfbifurcationsandresonanceproblemsofmaglevsystems.Zhangetal.[18–22]discussedtheeffectsoftimedelaysonthestabilityandHopfbifurcationsofrigid-guidewayandelastic-guidewaymaglevvehicles.Xuetal.[13]studiedtheHopfbifurcationproblemofcoupledvehicle-railsystemswithsimultaneoustimedelaysinpositionandspeedfeedback.Sunetal.[23]studiedtheinteractionandconstraintsoftime-delayparametersontheHopfbifurcationofalow-speedmaglevvehicle.Sunetal.[24]proposedanadaptiverobustcontrollerbasedontheRiccatimethodandsliding-modetechnologywhenatimedelaywasconsidered.

Previousresearchstudiesonmaglevvehiclesystemshaveeithernotconsideredtheinfluenceofaerodynamicliftortheyhaveonlyconsideredaerodynamicloadsinmultipledirectionstoanalyzetheridequality.Thesuspensionstabilityofamaglevtrainisthemostimportant,andtheinfluenceofaerodynamicliftonthesuspensionstabilityisthemostdirectofallaerodynamicloads.Todate,therehasnotbeenanyrelevantresearchonthesuspensionstabilitycharacteristicsofamaglevvehicleunderaerodynamiclift.Inaddition,previousresearchontime-delayedmaglevtrainsystemshasmainlyfocusedonHopfbifurcationsinsteadofthedynamicresponseunderexternalexcitation,especiallytheexcitationofaerodynamiclift.Whenatrainisrunningataveryhighspeed,itisanextremelyimportantproblemtodeterminewhethertheresponsecharacteristicsofthemaglevvehiclesystemwithtimedelayunderunsteadyaerodynamicliftcanmeetthecontrolrequirements.However,noresearchershavestudiedthenonlineardynamiccharacteristicsofmaglevsystemsassociatedwiththeaerodynamicliftedffectandtimedelaysinthefeedbackcontrolloop.Thispaperdescribesthefirstresearchtoexaminetheperiodicresponseanditsstabilityforamaglevsystemundertheactionofbothunsteadyaerodynamicliftandtimedelay.Many kinds of loads during the operation of train can be regarded as harmonic excitation to some extent, and the periodic response is important content of vehicle dynamics [25].Inthisstudy,wetookthenonlinearelectromagneticforcesandtimedelaysinthepositionandvelocityfeedbackcontrolloopintoaccountandweestablishedasingle-magnetsuspensionnonlinearmaglevsystemthatconsideredthesteadyandunsteadyaerodynamicliftwithsimpleharmonicvariations.Basedontheincrementalharmonicbalance(IHB)method,weanalyzedtheperiodicsolutionsofmaglevsystemsandweinvestigatedthestabilityoftheperiodicsolutionsbasedonafinitedifferencecontinuoustimeapproximationmethodandtheFloquettheory.Weanalyzedtherelationshipbetweentheperiodicsolutionamplitudeandthetimedelayaswellastheaerodynamicliftfluctuationfrequency.Wealsoconsideredtherelationshipbetweenthecriticalvelocityfeedback,thepositionfeedbacktimedelays,andthecontrolgainparameters.Inaddition,thedelayedstabilityboundarieswhenthepositionfeedbackandvelocityfeedbacktimedelayswereconsideredsimultaneouslywereinvestigatedinthisstudy.Thesefindingsareimportantforrevealingthesuspensionstabilitycharacteristicsofahigh-speedmaglevvehiclesystem.

2SystemModelandDynamicEquations

Toclearlydescribethemechanismfortheinfluenceofaerodynamicliftonthesuspensiondynamicstabilityofamaglevsystem,webegantheinvestigationusingthesingle-magnetsuspensioncontrolmodelshowninFigure1.Inthismodel,themaglevwassimplifiedtooneelectromagnet,onlytheverticalvibrationandverticalelectromagneticforceoftheelectromagnetweretakenintoaccount,andtherotationmotionandelectromagnettorquewereignored.Theguide-waywasconsideredtoberigid.Inthefigure,Frepresentstheelectromagneticforce,F_{Lift}representstheaerodynamiclift,δrepresentsthesuspensionclearanceoftheelectromagnet,and$v$representsthevehiclespeed.

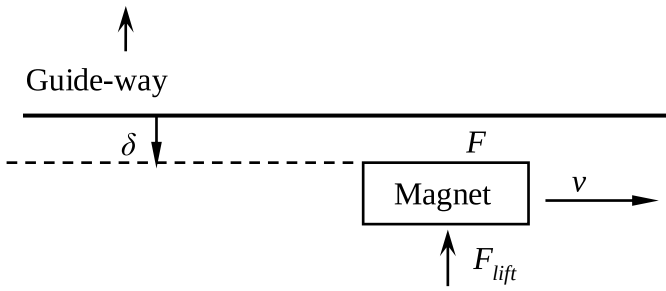


Figure1Schematicdiagramofasingle-magnetsuspensionsystem

Sincetheunstablevortexofthetrainwasperiodic,weregardedtheunsteadyaerodynamicliftint hisstudyasharmonic.Consideringtheoncomingairflowofthemaglevtrain,thesteadyandunsteadya erodynamicliftforcesactingonthevehiclecouldbeexpressedas

$$\begin{aligned} F_L &= F_{L0} + F_{L\Delta} \\ F_{L0} &= \frac{1}{2} C_L \rho A_v v^2 \\ F_{L\Delta} &= H \sin\left(2\pi f \cdot t\right) \end{aligned} \tag{1}$$

where$F_L$istheaerodynamiclift,F_{L0}isthesteadyaerodynamiclift,$F_{L\Delta}$istheunsteadyaerodynam iclift,$C_L$isthesteadyaerodynamicliftcoefficient,ρisthedensityofair(1.225kg/m^3),$A_v$isthevehiclet opsurfaceareaofthevehicle,$v$istheheadwindvelocity(i.e.,thevehiclespeed),$f$isthefrequencyofthe unsteadyaerodynamiclift,and$H$istheamplitudeoftheunsteadyaerodynamiclift.

Themaglevsystemhadanequilibriumstatethatwasdenotedas(I_0,δ_0),wheretheelectromagneti cforcewasequaltothesumofgravityandthesteadyaerodynamiclift.I_0wastheequilibriumcurrentan d$\delta_0$wastheequilibriumclearance.Furthermore,$i$denotedthefluctuationofthecurrentwithrespecttot heequilibriumcurrent,and$s$denotedthefluctuationofthesuspensionclearanceoftheelectromagnet withrespecttotheequilibriumclearance.Thenthesuspensionforcebetweentheelectromagnetandth eguide-waycouldbewrittenas

$$F = \frac{\mu_0 A N^2 \left(I_0 + i\right)^2}{4\left(\delta_0 + s\right)^2} \tag{2}$$

where$\mu_0$isthemagneticpermeabilityofair,Aistheeffectiveareaoftheelectromagnet,and$N$isthenum berofturnsinthecoil.

ExpandingtheaboveformulaasaTaylorseries,weobtained

$$F=\frac{\mu_0 AN^2}{4\delta_0^{\,2}}\left(I_0^{\,2}+2I_0 i+i^2\right)\left[1+2\left(-\frac{s}{\delta_0}\right)^1+3\left(-\frac{s}{\delta_0}\right)^2+4\left(-\frac{s}{\delta_0}\right)^3\cdots\right] \tag{3}$$

Thecurrentwascontrolledbyaproportional-differential(PD)controller,andthetime-delayinthepositionandvelocityfeedbackcontrolloopwastakenintoaccount.Then,thefluctuationofthecurrent$i$wasexpressedas

$$i=k_p\cdot s\left(t-t_p\right)+k_d\cdot\dot{s}\left(t-t_d\right), \tag{4}$$

where$k_p$and$k_d$arethegainsofthepositionfeedbackcontrolandthevelocityfeedbackcontrol,respectively,and$t_p$and$t_d$arethetimedelaysofthepositionfeedbackandthevelocityfeedback,respectively.

SubstitutingEq.(4)intoEq.(3)andkeepingtermswithsecond-orderprecision,wehad

$$\begin{aligned}F=&\frac{\mu_0 AN^2}{4\delta_0^{\,2}}\left(I_0^{\,2}+2I_0\left[k_p\cdot s_p+k_d\cdot\dot{s}_d\right]+\left[k_p\cdot s_p+k_d\cdot\dot{s}_d\right]^2\right)\\&\cdot\left[1+2\left(-\frac{s}{\delta_0}\right)^1+3\left(-\frac{s}{\delta_0}\right)^2+4\left(-\frac{s}{\delta_0}\right)^3\cdots\right]\end{aligned}. \tag{5}$$

Theelectromagneticforceattheequilibriumstatewasexpressedas

$F_0=\frac{\mu_0 AN^2}{4\delta_0^{\,2}}I_0^{\,2}$ .Attheequilibriumstate,wehad $F_0=mg+F_{L0}$,then

$I_0=\frac{2\delta_0}{N}\sqrt{\frac{mg+F_{L0}}{\mu_0 A}}$ .Thefluctuationoftheelectromagneticforcewithrespecttotheequilibriumstatecouldbederivedas

$$\begin{aligned}\Delta F=F-F_0=&k_1\left(k_p s_p-\frac{I_0}{\delta_0}s\right)+k_1 k_d\dot{s}_d\\&+k_2\left(k_p^{\,2}s_p^{\,2}-\frac{4I_0 k_p}{\delta_0}ss_p+\frac{3I_0^{\,2}}{\delta_0^{\,2}}s^2\right)+k_2\left(2k_p k_d s_p\dot{s}_d-\frac{4I_0 k_d}{\delta_0}s\dot{s}_d\right)+k_2\left(k_d^{\,2}\right)\dot{s}_d^2\end{aligned} \tag{6}$$

Intheaboveformula, $k_1=\frac{2\mu_0 AN^2 I_0}{4\delta_0^{\,2}}$, $k_2=\frac{\mu_0 AN^2}{4\delta_0^{\,2}}$, $s\left(t-t_p\right)=s_p$, $\dot{s}\left(t-t_d\right)=\dot{s}_d$.

Takingthedownwardverticaldirectionasthepositivedirection,theequationofmotionofthemagneticsuspensionsystemintheverticaldirectionwithrespecttotheequilibriumstatecouldbewritten as

$$m\ddot{s} = F_{L\Delta} - \Delta F \,, \tag{7}$$

where$m$isthemassoftheelectromagnet.

BysubstitutingEqs.(1)and(6) intoEq.(7),weobtainedanonlinearmagneticlevitationmodelunderthecombinedeffectofsteadyandunsteadyaerodynamicliftandtheinfluenceofthetimedelaysinthefeedbackcontrolloops:

$$\begin{aligned} &m\ddot{s} + \mu_{11}s_p + \mu_{12}s + \mu_{21}\dot{s}_d + \mu_{31}s_p{}^2 + \mu_{32}ss_p + \mu_{33}s^2 \\ &+\mu_{41}s_p\dot{s}_d + \mu_{42}s\dot{s}_d + \mu_{51}\dot{s}_d{}^2 = H\sin\left(2\pi ft\right) \end{aligned} \tag{8}$$

Intheaboveformula,

$$\begin{aligned} &\mu_{11} = k_1k_p, \mu_{12} = -k_1\frac{I_0}{\delta_0}, \mu_{21} = k_1k_d, \\ &\mu_{31} = k_2k_p{}^2, \mu_{32} = -k_2\frac{4I_0k_p}{\delta_0}, \mu_{33} = k_2\frac{3I_0{}^2}{\delta_0{}^2}, \\ &\mu_{41} = 2k_2k_pk_d, \mu_{42} = -k_2\frac{4I_0k_d}{\delta_0}, \mu_{51} = k_2k_d{}^2, \\ &\mu_{61} = -k_3k_p{}^2, \mu_{62} = k_3\frac{3I_0k_p}{\delta_0}, \mu_{63} = -k_3\frac{2I_0{}^2}{\delta_0{}^2}, \\ &\mu_{71} = k_3\frac{3I_0k_d}{\delta_0}, \mu_{72} = -k_32k_pk_d, \mu_{81} = -k_3k_d{}^2. \end{aligned}$$

3IHBMethod

Forthenonlinearmaglevsystemshownin Eq.(8),theperiodicsolutionswerefoundusingtheIHBmethod.TheIHBmethodwasproposedbyLauandCheung[26]andithasbeenwidelyusedtosolvevariousnonlinearvibrationproblems.Thismethodcombinestheincrementalmethodusedinnumericalcalculationsandtheharmonicbalancemethod.Ithasbeenusedsuccessfullyinthestudyofstronglynonlinearapplications[27-33].

Letting $\tau = 2\pi ft = \omega t$, $s_p = s\left(\tau - \omega t_p\right)$, and $s'_d = s'\left(\tau - \omega t_d\right)$, Eq.(8) could be written as

$$\begin{aligned}&\omega^2 m\cdot s'' + \mu_{11}\cdot s_p + \mu_{12}\cdot s + \omega\mu_{21}\cdot s'_d + \mu_{31}\cdot {s_p}^2 + \mu_{32}\cdot ss_p + \mu_{33}\cdot s^2\\&+\omega\mu_{41}\cdot s_p s'_d + \omega\mu_{42}\cdot ss'_d + \omega^2\mu_{51}\cdot {s'_d}^2 = H\sin\tau\end{aligned} \tag{9}$$

Letting s_0 and ω_0 denote a certain state in the vibration process, the critical state could be expressed in incremental form, as follows:

$$\begin{aligned}s &= s_0 + \Delta s\\ s_p &= s_{0p} + \Delta s_p\\ {s_d}' &= {s_{0d}}' + \Delta {s_d}'\end{aligned} \tag{10}$$

$$H = H_0 + \Delta H \tag{11}$$

$$\omega = \omega_0 + \Delta\omega \tag{12}$$

By substituting the above three equations into Eq.(9) and omitting higher-order terms, we obtained the following:

$$\begin{aligned}&{\omega_0}^2 m\cdot\Delta s'' + \mu_{11}\cdot\Delta s_p + \mu_{12}\cdot\Delta s + \omega_0\mu_{21}\cdot\Delta {s_d}' + 2\mu_{31}s_{0p}\cdot\Delta s_p\\&+\left(\mu_{32}s_{0p}\cdot\Delta s + \mu_{32}s_0\cdot\Delta s_p\right) + 2\mu_{33}s_0\cdot\Delta s + \left(\omega_0\mu_{41}{s_{0d}}'\cdot\Delta s_p + \omega_0\mu_{41}s_{0p}\cdot\Delta {s_d}'\right)\\&+\left(\omega_0\mu_{42}{s_{0d}}'\cdot\Delta s + \omega_0\mu_{42}s_0\cdot\Delta {s_d}'\right) + 2{\omega_0}^2\mu_{51}{s_{0d}}'\cdot\Delta {s_d}' =\\&R - \left[2\omega_0 m{s_0}'' + \mu_{21}\cdot {s_{0d}}' + \mu_{41}\cdot s_{0p}{s_{0d}}' + \mu_{42}\cdot s_0{s_{0d}}' + 2\omega_0\mu_{51}\cdot {s_{0d}}'^2\right]\Delta\omega\\&+\Delta H\sin\tau\end{aligned} \tag{13}$$

$$\begin{aligned}R = {}&H_0\sin\tau - {\omega_0}^2 m\cdot {s_0}'' - \mu_{11}\cdot s_{0p} - \mu_{12}\cdot s_0 - \omega_0\mu_{21}\cdot {s_{0d}}' - \mu_{31}\cdot {s_{0p}}^2 - \mu_{32}\cdot s_0 s_{0p}\\&-\mu_{33}\cdot {s_0}^2 - \omega_0\mu_{41}\cdot s_{0p}s_{0d}' - \omega_0\mu_{42}\cdot s_0{s_{0d}}' - {\omega_0}^2\mu_{51}\cdot {s_{0d}}'^2\end{aligned} \tag{14}$$

When s_0, ω_0, and H_0 were the exact solution of the equation; then R=0.

The periodic steady-state solution was expressed as follows:

$$\begin{aligned}s_0(\tau) &= a_0 + \sum_{k=1}^{N_c} a_k\cos k\tau + \sum_{k=1}^{N_s} b_k\sin k\tau = \boldsymbol{CA}\\ \Delta s(\tau) &= \Delta a_0 + \sum_{k=1}^{N_c}\Delta a_k\cos k\tau + \sum_{k=1}^{N_s}\Delta b_k\sin k\tau = \boldsymbol{C}\Delta\boldsymbol{A}\end{aligned} \tag{15}$$

Where

$$\begin{aligned} \boldsymbol{C} &= \left[1, \cos\tau, \cos 2\tau, \cdots, \cos N_c\tau, \sin\tau, \sin 2\tau, \cdots, \sin N_s\tau\right] \\ \boldsymbol{A} &= \left[a_0, a_1, a_2, \cdots, a_{N_c}, b_1, b_2, \cdots, b_{N_s}\right]^T \\ \Delta\boldsymbol{A} &= \left[\Delta a_0, \Delta a_1, \Delta a_2, \cdots, \Delta a_{N_c}, \Delta b_1, \Delta b_2, \cdots, \Delta b_{N_s}\right]^T \end{aligned} \tag{16}$$

BasedonEq.(15),theharmonicexpansionsof$s_p$ands'_dwereasfollows:

$$\begin{aligned} s_{0p} &= a_0 + \sum_{k=1}^{N_c} a_k \cos\left(k\tau - k\omega t_p\right) + \sum_{k=1}^{N_s} b_k \sin\left(k\tau - k\omega t_p\right) \\ \Delta s_p &= \Delta a_0 + \sum_{k=1}^{N_c} \Delta a_k \cos\left(k\tau - k\omega t_p\right) + \sum_{k=1}^{N_s} \Delta b_k \sin\left(k\tau - k\omega t_p\right) \end{aligned} \tag{17}$$

$$\begin{aligned} s'_{0d} &= \sum_{k=1}^{N_c}\left[-a_k k \sin\left(k\tau - k\omega t_d\right)\right] + \sum_{k=1}^{N_s} b_k k \cos\left(k\tau - k\omega t_d\right) \\ \Delta s'_d &= \sum_{k=1}^{N_c}\left[-\Delta a_k k \sin\left(k\tau - k\omega t_d\right)\right] + \sum_{k=1}^{N_s} \Delta b_k k \cos\left(k\tau - k\omega t_d\right) \end{aligned} \tag{18}$$

LettingN_c=N_s=N,Eq.(17)and(18)couldbewritteninthefollowingform:

$$\begin{aligned} s_{0p} &= \boldsymbol{C}\boldsymbol{\Gamma}_p\boldsymbol{A}, \Delta s_p = \boldsymbol{C}\boldsymbol{\Gamma}_p\Delta\boldsymbol{A} \\ s'_{0d} &= \boldsymbol{C}'\boldsymbol{\Gamma}_d\boldsymbol{A}, \Delta s'_d = \boldsymbol{C}'\boldsymbol{\Gamma}_d\Delta\boldsymbol{A} \end{aligned} \tag{19}$$

AsshowninEq.(19),wecalled$\boldsymbol{\Gamma}_p$and$\boldsymbol{\Gamma}_d$explicittime-

delayactionmatrices.Theywereexpressedasfollows:

$$\boldsymbol{\Gamma}_p = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cos\left(\omega t_p\right) & 0 & \cdots & 0 & -\sin\left(\omega t_p\right) & 0 & \cdots & 0 \\ 0 & 0 & \cos\left(2\omega t_p\right) & \cdots & 0 & 0 & -\sin\left(2\omega t_p\right) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \cos\left(N\omega t_p\right) & 0 & 0 & \cdots & -\sin\left(N\omega t_p\right) \\ 0 & \sin\left(\omega t_p\right) & 0 & \cdots & 0 & \cos\left(\omega t_p\right) & 0 & \cdots & 0 \\ 0 & 0 & \sin\left(2\omega t_p\right) & \cdots & 0 & 0 & \cos\left(2\omega t_p\right) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sin\left(N\omega t_p\right) & 0 & 0 & \cdots & \cos\left(N\omega t_p\right) \end{bmatrix},$$

$$
\boldsymbol{\Gamma}_d = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & \cos(\omega t_d) & 0 & \cdots & 0 & -\sin(\omega t_d) & 0 & \cdots & 0 \\
0 & 0 & \cos(2\omega t_d) & \cdots & 0 & 0 & -\sin(2\omega t_d) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \cos(N\omega t_d) & 0 & 0 & \cdots & -\sin(N\omega t_d) \\
0 & \sin(\omega t_d) & 0 & \cdots & 0 & \cos(\omega t_d) & 0 & \cdots & 0 \\
0 & 0 & \sin(2\omega t_d) & \cdots & 0 & 0 & \cos(2\omega t_d) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & \sin(N\omega t_d) & 0 & 0 & \cdots & \cos(N\omega t_d)
\end{bmatrix}.
$$

Itwasfoundthatthetime-delayactionmatrices$\boldsymbol{\Gamma}_p$and$\boldsymbol{\Gamma}_d$wereperiodicfunctionsof$t_p$andt_d,respectively.Theirperiodswere2π/ωor1/f.Hence,wededucedthattheirimpactonthemaglevsystemwasalsoperiodic.Importantly,thesolutionwouldvaryperiodicallywiththechangingofthetimedelay.

AftersubstitutingEqs.(15)and(19)intoEq.(13) andthenapplyingtheGalerkinprocess,weobtainedasetofalgebraicequationswiththeunknownsΔA,$\Delta\omega$,andΔH,whichcouldbeexpressedasfollows:

$$\boldsymbol{K}_{mc}\Delta\boldsymbol{A} = \boldsymbol{R} + \boldsymbol{R}_{mc}\Delta\omega + \boldsymbol{R}_h\Delta H \ , \tag{20}$$

$$\begin{aligned}\boldsymbol{K}_{mc} &= \omega_0^{\ 2}\boldsymbol{M} + \boldsymbol{K}_{11} + \boldsymbol{K}_{12} + \omega_0\boldsymbol{C}_{21} + 2\boldsymbol{K}_{31} \\ &\quad + \boldsymbol{K}_{32}^{\ \ 1} + \boldsymbol{K}_{32}^{\ \ 2} + 2\boldsymbol{K}_{33} + \omega_0\boldsymbol{C}_{41}^{\ \ 1} + \omega_0\boldsymbol{C}_{41}^{\ \ 2} \ , \\ &\quad + \omega_0\boldsymbol{C}_{42}^{\ \ 1} + \omega_0\boldsymbol{C}_{42}^{\ \ 2} + 2\omega_0^{\ 2}\boldsymbol{C}_{51}\end{aligned} \tag{21}$$

$$\begin{aligned}\boldsymbol{R} &= \boldsymbol{H} - \Big[\omega_0^{\ 2}\boldsymbol{M} + \boldsymbol{K}_{11} + \boldsymbol{K}_{12} + \omega_0\boldsymbol{C}_{21} + \boldsymbol{K}_{31} \\ &\quad + \boldsymbol{K}_{32}^{\ \ 1} + \boldsymbol{K}_{33} + \omega_0\boldsymbol{C}_{41}^{\ \ 2} + \omega_0\boldsymbol{C}_{42}^{\ \ 2} + \omega_0^{\ 2}\boldsymbol{C}_{51}\Big]\boldsymbol{A}\end{aligned}, \tag{22}$$

$$\boldsymbol{R}_{mc} = -\Big[2\omega_0\boldsymbol{M} + \boldsymbol{C}_{21} + \boldsymbol{C}_{41}^{\ \ 2} + \boldsymbol{C}_{42}^{\ \ 2} + 2\omega_0\boldsymbol{C}_{51}\Big]\boldsymbol{A}, \tag{23}$$

where

$$
\begin{aligned}
&\boldsymbol{M}=\int_0^{2\pi}\boldsymbol{C}^T m\boldsymbol{C}''d\tau,\\
&\boldsymbol{K}_{11}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{11}\boldsymbol{C}\boldsymbol{\Gamma}_p d\tau,\boldsymbol{K}_{12}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{12}\boldsymbol{C}d\tau\\
&\boldsymbol{C}_{21}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{21}\boldsymbol{C}'\boldsymbol{\Gamma}_d d\tau\\
&\boldsymbol{K}_{31}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{31}s_{0p}\boldsymbol{C}\boldsymbol{\Gamma}_p d\tau,\boldsymbol{K}_{32}{}^1=\int_0^{2\pi}\boldsymbol{C}^T\mu_{32}s_{0p}\boldsymbol{C}d\tau\\
&\boldsymbol{K}_{32}{}^2=\int_0^{2\pi}\boldsymbol{C}^T\mu_{32}s_0\boldsymbol{C}\boldsymbol{\Gamma}_p d\tau,\boldsymbol{K}_{33}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{33}s_0\boldsymbol{C}d\tau\\
&\boldsymbol{C}_{41}{}^1=\int_0^{2\pi}\boldsymbol{C}^T\mu_{41}s_{0d}{}'\boldsymbol{C}\boldsymbol{\Gamma}_p d\tau,\boldsymbol{C}_{41}{}^2=\int_0^{2\pi}\boldsymbol{C}^T\mu_{41}s_{0p}\boldsymbol{C}'\boldsymbol{\Gamma}_d d\tau\\
&\boldsymbol{C}_{42}{}^1=\int_0^{2\pi}\boldsymbol{C}^T\mu_{42}s_{0d}{}'\boldsymbol{C}d\tau,\boldsymbol{C}_{42}{}^2=\int_0^{2\pi}\boldsymbol{C}^T\mu_{42}s_0\boldsymbol{C}'\boldsymbol{\Gamma}_d d\tau\\
&\boldsymbol{C}_{51}=\int_0^{2\pi}\boldsymbol{C}^T\mu_{51}s_{0d}{}'\boldsymbol{C}'\boldsymbol{\Gamma}_d d\tau\\
&\boldsymbol{H}=\int_0^{2\pi}\boldsymbol{C}^T H_0\cos\tau d\tau,\boldsymbol{R}_h=\int_0^{2\pi}\boldsymbol{C}^T\cos\tau d\tau
\end{aligned}
\tag{24}
$$

Ifwewereonlyconcernedwiththefrequency–amplituderesponsecurveatacertainaerodynamicamplitude,then$\boldsymbol{H}$hadafixedvalueandΔH=0.Thus,Eq.(20)became

$$\boldsymbol{K}_{mc}\Delta\boldsymbol{A}=\boldsymbol{R}+\boldsymbol{R}_{mc}\Delta\omega \tag{25}$$

Equation(25) representsasetoflinearequations.Itsnumberofunknownswasgreaterthanthenumberofequationsbyone,sointhesolutionprocess,oneoftheincrementswasselectedasaparameter.Wecouldchoosetheamplitude$a$ofacertainharmonicortheexcitationfrequencyasthecontrolincrement.TheNewton-Raphsoniterativemethodwasadoptedtocalculatethesolutionwhenthefrequencyωoronecomponentoftheamplitude$\boldsymbol{A}$wasgiven.Thecriterionforstoppinganiterationwasthatthecorrectivevector$\boldsymbol{R}$wassmallenough.Toefficientlycalculatetheamplitude–frequencyresponsecurve,thesamplingarclengthincrementmethodusedinthisresearchautomaticallytrackedtheresponsecurve.ReadersarereferredtotheworkofCheung[34]fortheassociatedtheoryofthearclengthincrementmethod.

4Stability

Welets$_0$betheobtainedsteady-statperiodicsolutionandletΔsbeaperturbation.Bysubstituting$s=s_0+\Delta s$intoEq.(9) andomittingthehigh-ordersmallquantities,weobtainedtheperturbationequations:

$$\begin{aligned}&\omega_0^{\ 2} m\Delta s'' + \mu_{11}\cdot\Delta s_p + \mu_{12}\cdot\Delta s + \omega_0\mu_{21}\cdot\Delta s_d{}' \\ &+\mu_{31} 2 s_{0p}\cdot\Delta s_p + \left(\mu_{32}s_{0p}\cdot\Delta s + \mu_{32}s_0\cdot\Delta s_p\right) + 2\mu_{33}s_0\cdot\Delta s \\ &+\left(\omega_0\mu_{41}s_{0d}{}'\cdot\Delta s_p + \omega_0\mu_{41}s_{0p}\cdot\Delta s_d{}'\right) \\ &+\left(\omega_0\mu_{42}s_{0d}{}'\cdot\Delta s + \omega_0\mu_{42}s_0\cdot\Delta s_d{}'\right) \\ &+2\omega_0^{\ 2}\mu_{51}s_{0d}{}'\cdot\Delta s_d{}' = 0\end{aligned} \tag{26}$$

Thestabilityofthesolutionoftheoriginalequationcorrespondedtothestabilityofthesolutionof theordinarydifferentialequations(Eq.(26))withperiodiccoefficientsandatimedelay.

Foratime-delaysystemcontainingperiodiccoefficients,wefirstconvertedthedifferentialequationwithatime delaytoadifferentialequationwithoutatimedelayusingthefinitedifferencecontinuoustimeapproximationmethod,afterwhichwestudiedthestabilityofthetime-delayedsystemwithperiodiccoefficientsusingFloquettheory.

Letting

$$\boldsymbol{q} = \left[q_1, q_2\right]^{\mathrm{T}} = \left[\Delta s', \Delta s\right]^{\mathrm{T}} \tag{27}$$

Eq.(26)couldbewritteninmatrixformasfollows:

$$\begin{aligned}&\dot{\boldsymbol{q}} = \boldsymbol{f}\left[\boldsymbol{q}(\tau), \boldsymbol{q}\left(\tau-\omega t_p\right), \boldsymbol{q}\left(\tau-\omega t_d\right)\right] = \boldsymbol{G}\boldsymbol{q}(\tau) + \boldsymbol{G}_p\boldsymbol{q}\left(\tau-\omega t_p\right) + \boldsymbol{G}_d\boldsymbol{q}\left(\tau-\omega t_d\right) \\ &\boldsymbol{G} = \begin{bmatrix} 0 & -\dfrac{1}{\omega_0^{\ 2}m}\left(\mu_{12} + \mu_{32}s_{0p} + 2\mu_{33}s_0 + \omega_0\mu_{42}s_{0d}{}'\right) \\ 1 & 0 \end{bmatrix} \\ &\boldsymbol{G}_p = \begin{bmatrix} 0 & -\dfrac{1}{\omega_0^{\ 2}m}\left(\mu_{11} + 2\mu_{31}s_{0p} + \mu_{32}s_0 + \omega_0\mu_{41}s_{0d}{}'\right) \\ 0 & 0 \end{bmatrix} \\ &\boldsymbol{G}_d = \begin{bmatrix} -\dfrac{1}{\omega_0^{\ 2}m}\left(\omega_0\mu_{21} + \omega_0\mu_{41}s_{0p} + \omega_0\mu_{42}s_0 + 2\omega_0^{\ 2}\mu_{51}s_{0d}{}'\right) & 0 \\ 0 & 0 \end{bmatrix}\end{aligned} \tag{28}$$

The state variables was expressed in terms of $\boldsymbol{q}(\tau)$, $\boldsymbol{q}(\tau-\tau_1)$, and $\boldsymbol{q}(\tau-\tau_2)$, (where $0<\tau_1\le\omega t_p$, $0<\tau_2\le\omega t_d$) had infinite dimensions. The state variables with a time delay $\boldsymbol{q}(\tau-\tau_1), 0<\tau_1\le\omega t_p$, $\boldsymbol{q}(\tau-\tau_2), 0<\tau_2\le\omega t_d$ could be discretized. Assuming that N_p and N_d are integers, we obtained $\Delta\tau_p=\omega t_p/N_p$ and $\Delta\tau_d=\omega t_d/N_d$. The derivatives of $\dot{\boldsymbol{q}}(\tau-i\cdot\Delta\tau_p)$ and $\dot{\boldsymbol{q}}(\tau-i\cdot\Delta\tau_d)$ were approximated with the following differences:

$$
\begin{aligned}
\dot{\boldsymbol{q}}(\tau-i\cdot\Delta\tau_p)&=\frac{1}{\Delta t_p}\left[\boldsymbol{q}\left(\tau-(i-1)\cdot\Delta\tau_p\right)-\boldsymbol{q}\left(\tau-i\cdot\Delta\tau_p\right)\right]\\
\dot{\boldsymbol{q}}(\tau-i\cdot\Delta\tau_d)&=\frac{1}{\Delta t_d}\left[\boldsymbol{q}\left(\tau-(i-1)\cdot\Delta\tau_d\right)-\boldsymbol{q}\left(\tau-i\cdot\Delta\tau_d\right)\right]
\end{aligned}
\tag{29}
$$

The following finite dimensional state variable was defined:

$$
\begin{aligned}
\boldsymbol{p}(\tau)=&\left[\boldsymbol{q}(\tau),\boldsymbol{q}(\tau-\Delta\tau_p),\boldsymbol{q}(\tau-2\Delta\tau_p),\cdots,\boldsymbol{q}(\tau-N_p\Delta\tau_p),\right.\\
&\left.\boldsymbol{q}(\tau-\Delta\tau_d),\boldsymbol{q}(\tau-2\Delta\tau_d),\cdots,\boldsymbol{q}(\tau-N_d\Delta\tau_d)\right]^{\mathrm{T}}\\
=&\left[\boldsymbol{p}_1(\tau),\boldsymbol{p}_2(\tau),\boldsymbol{p}_3(\tau),\cdots,\boldsymbol{p}_{N_p+N_d+1}(\tau)\right]^{\mathrm{T}}
\end{aligned}
\tag{30}
$$

The following system equation could be obtained based on the expansion state variables shown in Eq. (30):

$$
\dot{\boldsymbol{p}}(\tau)=\begin{bmatrix}
\boldsymbol{f}\left[\boldsymbol{q}(\tau),\boldsymbol{q}(\tau-\tau_p),\boldsymbol{q}(\tau-\tau_d)\right]\\
\frac{1}{\Delta\tau_p}\left[\boldsymbol{p}_1(\tau)-\boldsymbol{p}_2(\tau)\right]\\
\vdots\\
\frac{1}{\Delta\tau_p}\left[\boldsymbol{p}_{N_p}(\tau)-\boldsymbol{p}_{N_p+1}(\tau)\right]\\
\frac{1}{\Delta\tau_d}\left[\boldsymbol{p}_1(\tau)-\boldsymbol{p}_{N_p+2}(\tau)\right]\\
\frac{1}{\Delta\tau_d}\left[\boldsymbol{p}_{N_p+2}(\tau)-\boldsymbol{p}_{N_p+3}(\tau)\right]\\
\vdots\\
\frac{1}{\Delta\tau_d}\left[\boldsymbol{p}_{N_p+N_d}(\tau)-\boldsymbol{p}_{N_p+N_d+1}(\tau)\right]
\end{bmatrix}=\boldsymbol{\psi}\cdot\boldsymbol{p}(\tau)
\tag{31}
$$

where

$$\boldsymbol{\psi}=\begin{bmatrix} \boldsymbol{G} & 0 & \cdots & 0 & \boldsymbol{G}_p & \cdots & \cdots & 0 & \boldsymbol{G}_d \\ \frac{1}{\Delta\tau_p}\boldsymbol{I} & \frac{1}{\Delta\tau_p}\boldsymbol{I} & & & & & & & \\ & & \ddots & & & & & & \\ & & & \ddots & & & & & \\ & & & \frac{1}{\Delta\tau_p}\boldsymbol{I} & -\frac{1}{\Delta\tau_p}\boldsymbol{I} & & & & \\ \frac{1}{\Delta\tau_d}\boldsymbol{I} & & & & & -\frac{1}{\Delta\tau_d}\boldsymbol{I} & & & \\ & & & & & \frac{1}{\Delta\tau_d}\boldsymbol{I} & -\frac{1}{\Delta\tau_d}\boldsymbol{I} & & \\ & & & & & & & \ddots & \\ & & & & & & & \frac{1}{\Delta\tau_d}\boldsymbol{I} & -\frac{1}{\Delta\tau_d}\boldsymbol{I} \end{bmatrix} \tag{32}$$

IntheaboveEq.(32),$\boldsymbol{I}$=diag(1,1).Thestabilityofthecontrolequation(Eq.(26))couldbeassessedusingthestabilityofEq.(31).

Since$\boldsymbol{s}_0$,$\boldsymbol{s}_{0p}$,and$d\boldsymbol{s}_{0d}/d\tau$wereperiodicfunctionswithaperiodof$T=2\pi/\omega$,$\boldsymbol{\Psi}$wasalsoaperiodicfunctionwiththesameperiodas$\boldsymbol{s}_0$.Weassumedthatthematrix$\boldsymbol{\Phi}$wasthebasicsolutionmatrixofEq.(32)andthatitsatisfied

$$\begin{aligned} \dot{\boldsymbol{\Phi}}(\tau) &= \boldsymbol{\Psi}(\tau)\boldsymbol{\Phi}(\tau) \\ \dot{\boldsymbol{\Phi}}(\tau+T) &= \boldsymbol{\Psi}(\tau)\boldsymbol{\Phi}(\tau+T) \\ \boldsymbol{\Phi}(\tau+T) &= \boldsymbol{C}\boldsymbol{\Phi}(\tau) \end{aligned} \tag{33}$$

$\boldsymbol{C}$isthetransfermatrix.AccordingtoFloquettheory,thestabilityofasystemdependsontheeigenvaluesofthematrix$\boldsymbol{C}$.Ifthemoduliofalltheeigenvaluesof$\boldsymbol{C}$arelessthan1,thenthemotionofthesystemisboundedandthesolutionisstable;otherwise,themotionisunboundedandthesolutionisunstable.

Choosinganinitialconditionof$\boldsymbol{\Phi}(0)=\boldsymbol{I}$,thebasicsolutionofEq.(32)wassolvednumerically,andweobtainedthefollowing:

$$\boldsymbol{C}=\boldsymbol{\Phi}(T) \tag{34}$$

AccordingtothemethodgivenbyFriedmann[35],thetransitionmatrixcouldbeobtainedwiththefourth-orderRunge–Kuttanumericalintegrationmethod.

5AnalysisofNumericalExample

Basedontheaforementionedtheory,wewroteaprogramfortheperiodicsolutionandstabilityanalysis.Weexaminedanelectromagnetmoduleforanactualhigh-speedmaglevtrainandweselectedparametersbasedon1/8ofavehicle.Table1showstheparameterselected.

Table1Electromagnetparameters

m(kg)	μ_0(H/m)	δ_0(m)	N_m	R(Ω)	A_m(m^2)
6300	$4\pi\times10^{-7}$	0.01	290	0.61	0.622

TheaerodynamicamplitudewasA_0=25000Nandthereferenceareaofthevehiclewas11.86m^2. Sincetheparameterswereselectedbasedon1/8ofavehicle,thesteadyaerodynamicliftneededtobe1/8oftheaerodynamicliftthattheentirevehiclewassubjectedto, $F_{L0}=0.5C_L\rho A_v v^2/8$.

5.1Procedureverification

Toensurethereliabilityoftheanalysisprogram,wecalculatedtheperiodicsolutionsconsideringthetimedelaysinthepositionfeedbackandcomparedthemwiththestableperiodicsolutioncalculatedusingtheRunge–Kuttadirectintegrationmethod.

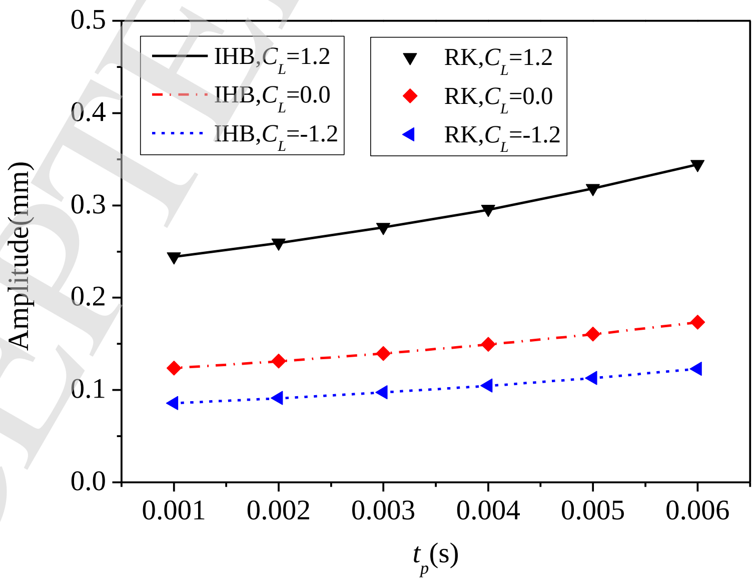


Figure2ComparisonofthecalculatedresultsusingtheIHBmethodandtheRunge–Kuttamethod

Figure2showsthevibrationamplitudesofperiodicsolutionswithdifferenttimedelaysforthepositionfeedbackt_p.

Theotherparameterswerek_p=2000,k_d=20,and$f$=10Hz.Theamplitudesoftheperiodicsolutionsobtainedbythetwomethodswereequal,whichverifiedthecorrectnessofanalysisprogram.

Basedontheverifiedanalysisprogram,weinvestigatedthenonlinearvibrationsofthemaglevsystemundertheactionofaerodynamicliftforthreetime-

delayscenarios.Thethreescenariosconsideredonlythetimedelayofthepositionfeedbackt_p,onlythetimedelayofthevelocityfeedbackt_d,andboththetimedelayofthepositionfeedback$t_p$andthetimedelayofthevelocityfeedbackt_d.Thesteadyaerodynamicliftthadanimportanteffectonthestabilityofthemaglevtrain.Foreachtime-

delayscenario,wecalculatedtheresponseandstabilityofthemaglevsystemforC_L=1.2(verticallyupwardsteadylift),C_L=0.0(zerosteadylift),andC_L=−1.2(verticallydownwardsteadylift).

5.2Timedelayofpositionfeedback

Inthissub-

section,wepresenttheanalysisresultsforthescenarioinwhichonlythetimedelayofthepositionfeedback$t_p$wasconsidered.Forthisscenario,wesetk_p=2000andk_d=20.Figure3presentstherelationshipcurvesbetweentheperiodicvibrationamplitudeandthetimedelayofthepositionfeedbackforthreesteadyliftconditions.TheanalysisresultsshowedthatthevibrationamplitudewasamaximumwhenthesteadyaerodynamicliftcoefficientwasC_L=1.2,followedbythatatC_L=0,anditwasaminimumatC_L=−1.2.Thisshowedthatwhenthesteadyaerodynamicliftwasverticallyupward,thevibrationamplitudeofthesystemexcitedbyunsteadyaerodynamicliftwouldbegreater.Inaddition,thefigureshowsthatwhentheexcitationfrequencywas15Hz,theamplitudewasamaximum.

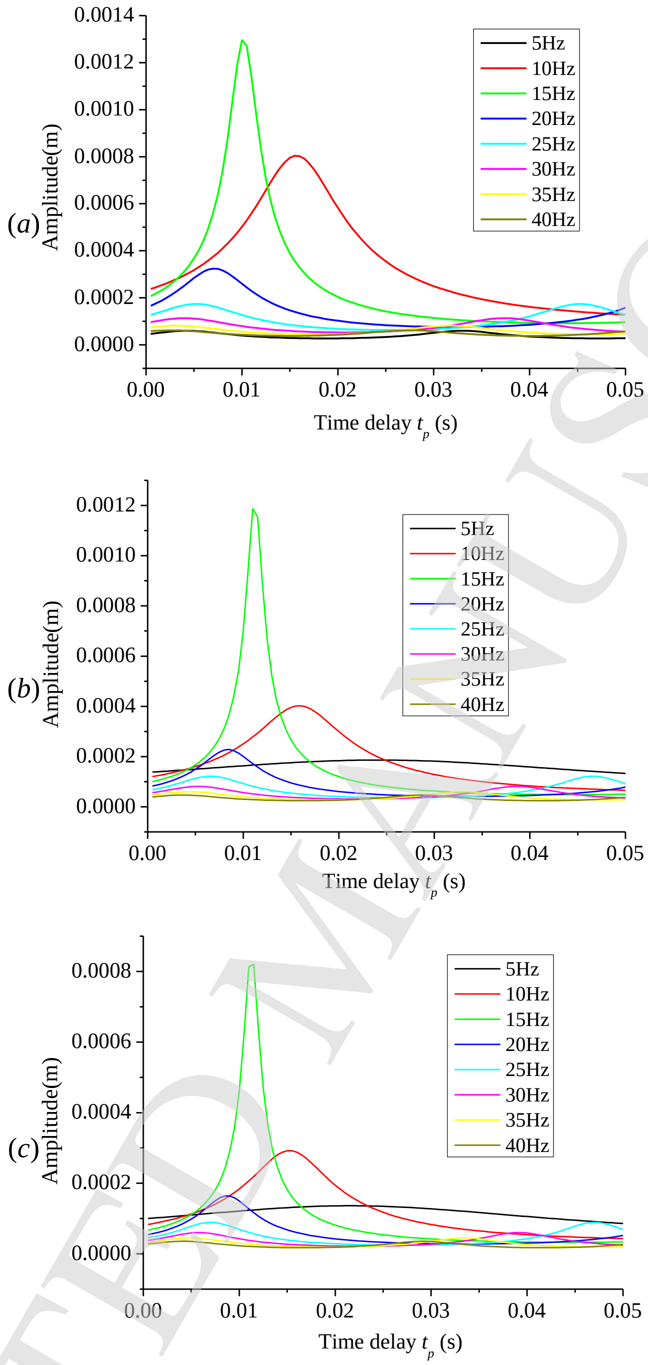


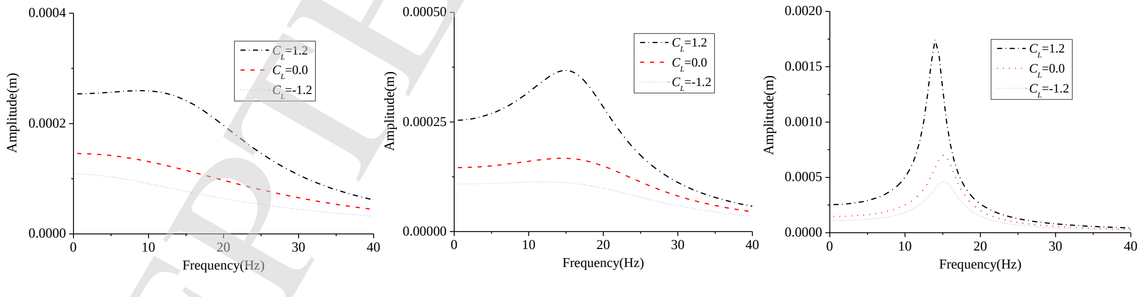
Figure3Relationshipbetweenperiodicvibrationamplitudeandpositiontimedelay:(a)C_L=1.2;(b)C_L=0;(c)C_L=−1.2

Itisworthnotingthatthetime-delayactionmatrixwasperiodicintermsof$t_d$withaperiodof$T$=1/f,justasshowninEq.(19).Thus,theperiodicsolutionhadtoexhibitperiodicvariationswithrespecttothetimedelay.TheresultsshowninFi gure3confirmedthisdeduction.Figure3showsthatforalow-frequencyexcitation,theperiodoftheamplitudechangewiththetimedelaywaslong.Forthetimedelayofthepositionfeedbackinthe0–0.05srange,theamplitudeincreasedmonotonically.Athighexcitationfrequencies,withthetimedelayofthepositionfeedbackinthe0–0.05srange,theamplitudevariationwiththetimedelayofthepositionfeedbackappearedperiodically.

Fromthestabilityanalysisoftheperiodicsolution,weobtainedthecriticaltimedelayofthesysteminstability.Table2showsthecriticaltimedelayofthepositionfeedbackfor$f$=10Hzand20Hz.Theresultsshowedthattheexcitationfrequencyoftheunsteadylifthadnoeffectonthecriticaltimedelay.ThecriticaltimedelaywassmallatC_L=1.2andgreateratC_L=0and−1.2,buttheoveralldifferencewassmall.

Table2Comparisonofcriticaltimedelaysfordifferentsteadyaerodynamicliftcoefficients

	***f*=10Hz**	***f*=20Hz**
C_L=1.2	0.01195s	0.01195s
C_L=0	0.01230s	0.0123s
C_L=−1.2	0.01225s	0.01225s



(a) (b) (c)

Figure4Amplitude–frequencyresponsecurvesfordifferenttimedelaysofthepositionfeedback:(a)t_p=0.002s,stable;(b)t_p=0.005s,stable;(c)t_p=0.01s,stable

Figure4showstheamplitude–frequencyresponsecurveforthreedifferenttimedelaysofthepositionfeedback.Theresultsshowmoreclearlythatthemaximumamplitudeswereallinthevicinityof15Hz.Comparedtothecasewithoutsteadyaerodynamiclift,theamplitudewasgreaterwhenthesteadyaerodynamicliftwasdirectedverticallyupwardandlesserwhenthesteadyaerodynamicliftwasdirectedverticallydownward.

Figure5andFigure6displaythedependenceofthecriticaltimedelayofthepositionfeedbackonthepositionfeedbackgain$k_p$andthevelocityfeedbackgaink_p,respectively.Theresultsshowedthatwhen$k_p$increased,thecriticaltimedelayofthepositionfeedbackdecreased.When$k_d$increased,thecriticaltimedelayofthepositionfeedbackincreased.Inaddition,thecriticaltimedelaysofthepositionfeedbackcorrespondingtoC_L=1.2,0,and−1.2weresimilar.

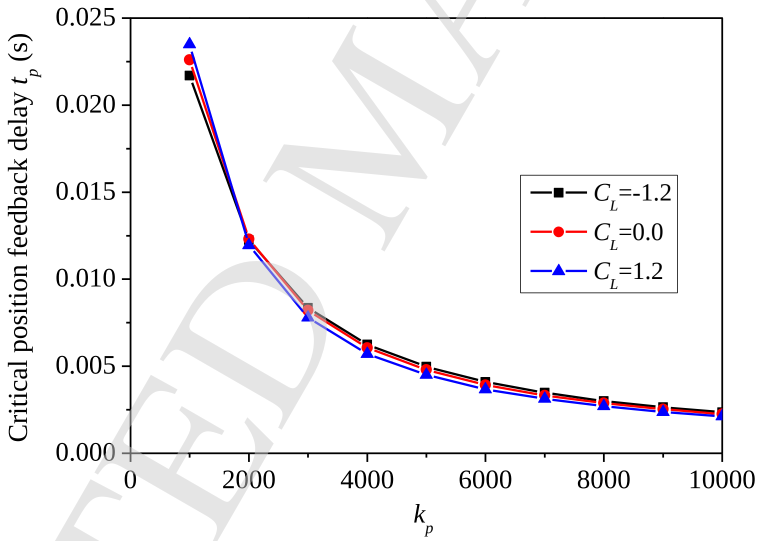


Figure5Relationshipbetweenthecriticaltimedelayofthepositionfeedbackandthepositionfeedbackgain(k_d=20,f=10Hz)

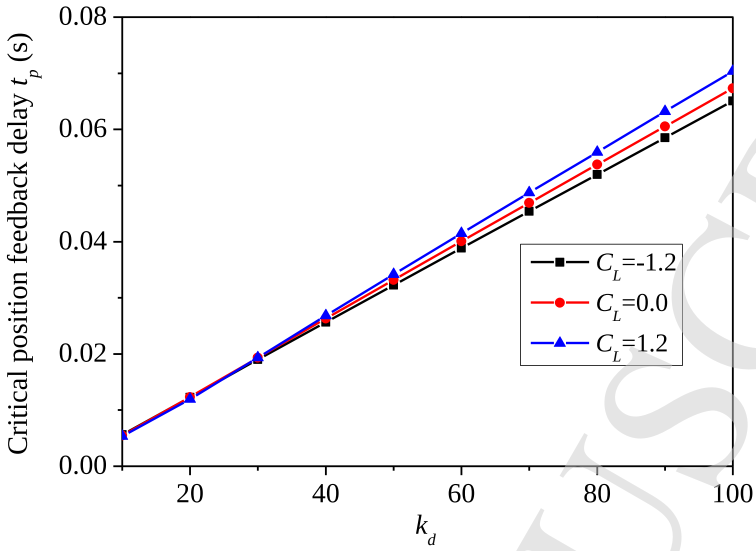


Figure6Relationshipcurvesforthecriticaltimedelayofthepositionfeedbackandthevelocityfeedbackgain(k_p=2000,f=10Hz)

5.3Timedelayofvelocityfeedback

Thissub-sectionshowsthenonlinearresponseofthemaglevsystemundertheactionofaerodynamicliftforthescenarioinwhichonlythetimedelayofthevelocityfeedback$t_d$wasconsidered.Intheanalysisdescribedinthissection,wechosethefollowingvalues:k_p=2000,k_d=20,H=25000N.

Figure7showstherelationshipcurvesfortheamplitudeoftheperiodicvibrationandthetimedelay$t_d$underthreesteadyaerodynamicliftconditions:C_L=1.2,0,and−1.2.Similarly,whenthesteadyaerodynamicliftwasverticallyupward,theresponseamplitudeofthesystemwasgreater.Equation(19)alsoshowsthattheeffectof$t_d$onthesystemwasperiodic,withaperiodof$T$=1/f,whichisalsoreflectedinFigure7.

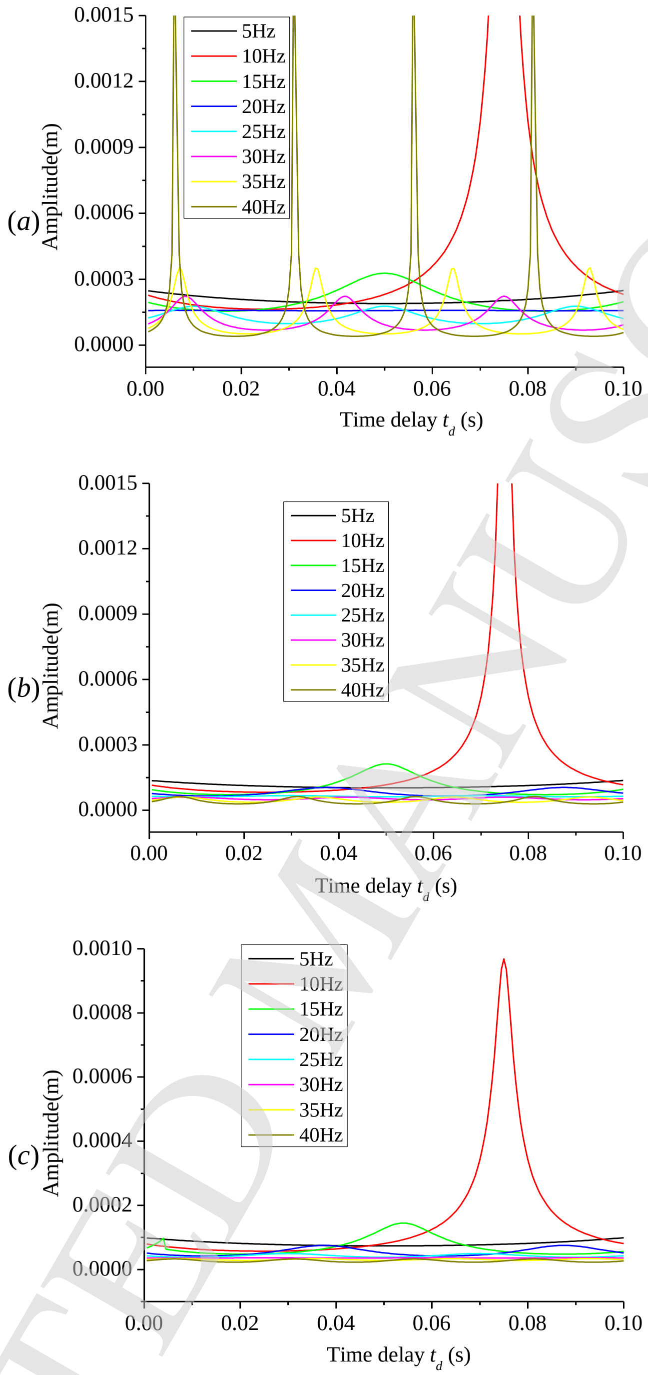
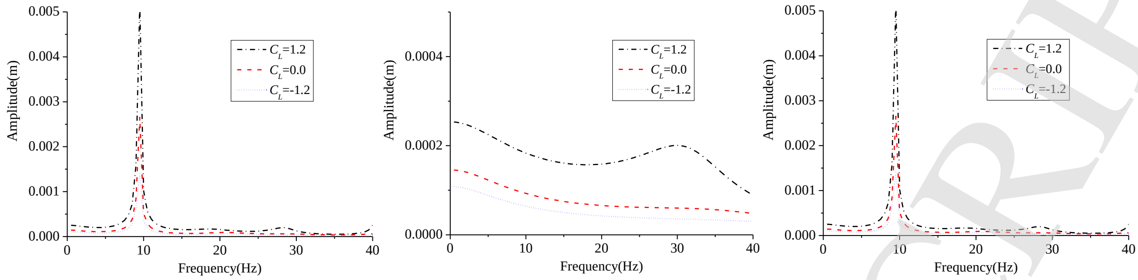


Figure7Relationshipcurvesfortheperiodicvibrationamplitudeandtimedelayofthepositionfeedback:(a)C_L=1.2;(b)C_L=0;(c)C_L=−1.2



(*a*) (*b*) (*c*)

Figure8Amplitude–frequencyresponsecurvefordifferenttimedelaysofthevelocityfeedback:(*a*)t_p=0.002s,stable;(*b*)t_p=0.01s,unstable;(*c*)t_p=0.08s,unstable

Inaddition,whenconsideringonlythetimedelayofthepositionfeedback,themaximumamplitudeoccurredatafrequencyof15Hz,andwhenconsideringonlythetimedelayofthevelocityfeedback,thefrequencywasabout10Hz,asshowninFigure8.

Figure9andFigure10showthedependenceofthecriticaltimedelayofthevelocityfeedbackonthepositionfeedbackgain$k_p$andthevelocityfeedbackgaink_d,respectively.Theresultsshowedthatthecriticaltimedelayofthevelocityfeedbackdecreasedas$k_p$and$k_d$increased.WhencomparedtothecaseofC_L=0,thecriticaltimedelayofthevelocityfeedbackwasgreaterwhenthesteadyaerodynamicliftwasintheupwarddirection(C_L=1.2),andthecriticaltimedelayofthevelocityfeedbackwassmallerwhenthesteadyaerodynamicliftwasverticallydownward(C_L=−1.2).

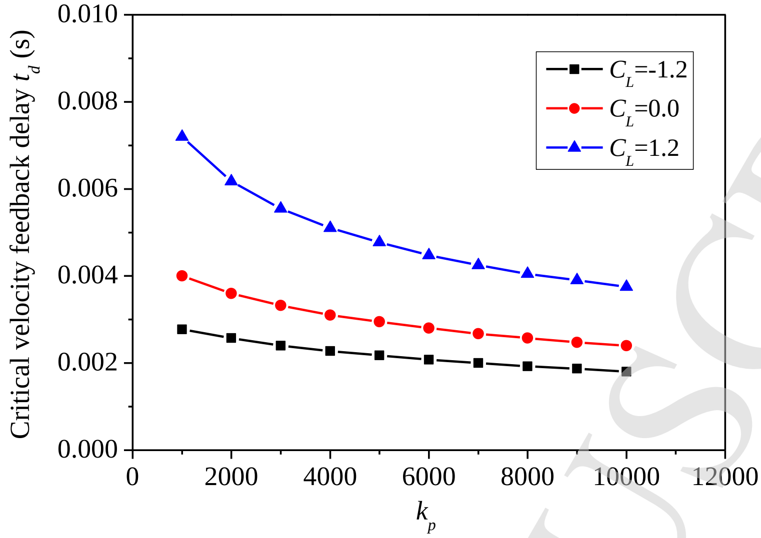


Figure9Relationshipbetweenthecriticaltimedelayofthevelocityfeedbackandthepositionfeedbackgaink_p(k_d=20,f=10Hz)

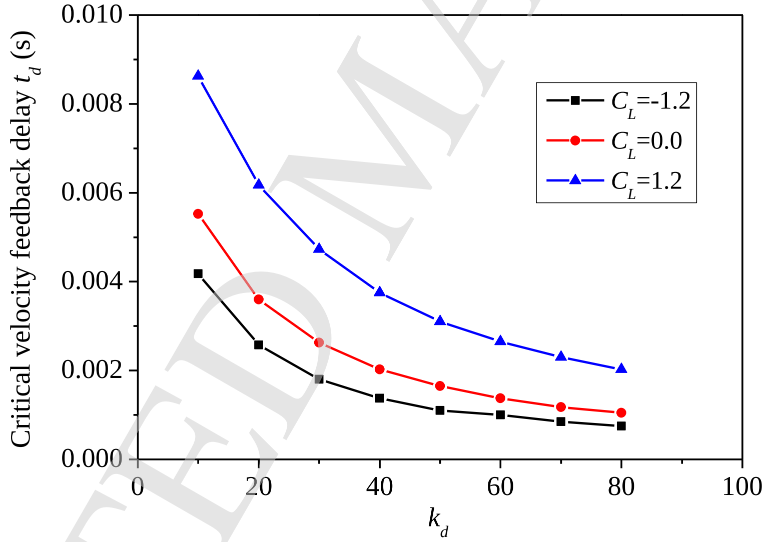


Figure10Relationshipbetweenthecriticaltimedelayofthevelocityfeedbackandthevelocityfeedbackgaink_d(k_p=2000,f=10Hz)

5.4Dualtimedelayinpositionandvelocityfeedback

Toaddressthecaseofadualtimedelayinthepositionandvelocityfeedbackcontrolloop,weanalyzedthestabilityoftheperiodicsolutionofthemaglevsystemundertheactionofaerodynamicliftandobtainedthestabilityboundaryforthetimedelay,asshowninFigure11.Thefigureshowsthetimedelaystabilityboundarycurveswhenthesteadyaerodynamicliftcoefficientwasequalto1.2,0,and−1.2.Whenthetimedelayofthepositionfeedbackandthetimedelayofthevelocityfeedbackwerelocatedonthelowerleftsideofacurve,theresponseofthemaglevsystemundertheaerodynamicliftwasstable.Whenthetimedelaywaslocatedontheupperright-handsideofacurve,theresponsewasunstable.

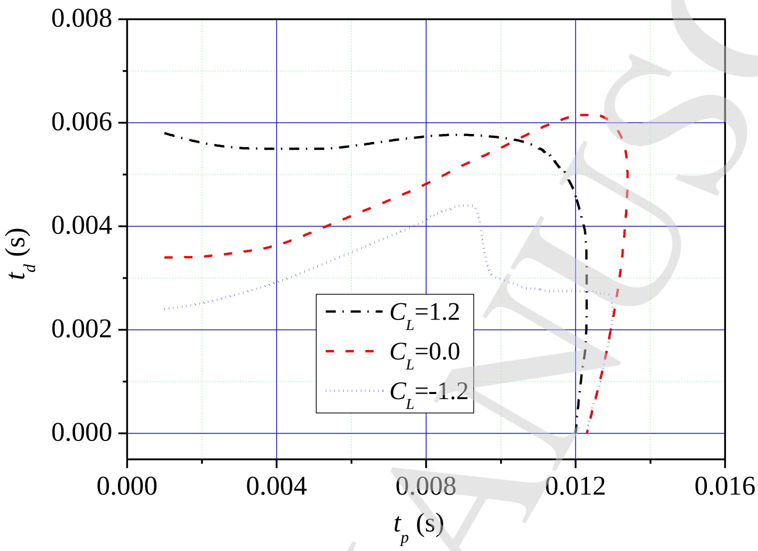


Figure11Stabilityboundarywithsimultaneoustimedelaysofpositionfeedbackandvelocityfeedback(k_p=2000,k_d=20,H=25000N,f=10Hz)

6Conclusion

Inthisstudy,weestablishedanonlinearmodelforamaglevsystemunderthecombinedeffectofsteadyandunsteadyaerodynamicliftandtime-delayedfeedbackcontrol.ThenonlinearperiodicsolutionofthesystemwascalculatedusingtheIHBmethod,andthestabilityoftheperiodicsolutionwasassessedusingthecontinuoustimeapproximationmethodandthemulti-variableFloquettheorytoobtainthecriticaltime-delay.Basedona1/8maglevvehicle,weconductedananalysisandobtainedtherelationshipbetweentheperiodicvibrationamplitudeofthemaglevsystemwithaerodynamicliftplusatimedelayandthefrequencyoftheaerodynamiclift.Wealsoobtainedtherelationshipcurveforthecriticaltimedelayofthepositionfeedback,thecriticaltimedelayofthevelocityfeedback,andthecontrolgainparameteraswe llasthestabilityboundarywhenthetimedelaysofboththepositionfeedbackandthevelocityfeedback wereconsidered.

Wearrivedatthefollowingmainconclusions:

(1)Comparedtothecasewherethesteadyaerodynamicliftwaszero,theresponsevibrationamplitudeofthesystemincreasedifthesteadyaerodynamicliftwasverticallyupwardandtheresponseamplitudeofthesystemwilldecreaseiftheaerodynamicliftwasverticallydownward.

(2)Thereweereexplicittime-delayactionmatrices.Theactionmatricesrevealedthattheeffectofthetimedelayontheresponseofthemaglevsystemwasperiodic,withtheperiodequalto1/f,where$f$isthefluctuationfrequencyoftheunsteadyaerodynamiclift.Thenumericalanalysisconfirmedthis.

(3)Thefluctuationfrequencyoftheunsteadyaerodynamicliftadnoeffectonthecriticaltimedelayofthesysteminstability.

(4)When$k_p$increased,thecriticaltimedelayofthepositionfeedbackdecreased.However,when $k_d$increased,thecriticaltimedelayofthepositionfeedbackincreased.Nevertheless,thecriticaltimedelayofthevelocityfeedbackdecreasedas$k_p$and$k_d$increased.

(5)Comparedtothecasewherethesteadyaerodynamicliftwaszero(C_L=0),thecriticaltimedelayofthevelocityfeedbackincreasedifthesteadyaerodynamicliftwasverticallyupward(C_L=1.2),and thecriticaltimedelayofthevelocityfeedbackdecreasedifthesteadyaerodynamicliftwasdownward(C_L=−1.2).However,forallthreeconditionsofthesteadyaerodynamiclift,thedifferencesinthecriticaltimedelayofthepositionfeedbackweresmall.

Thesefindingsandconclusionscouldbeextendedtoanentirevehicle,whichwouldbeofhelpfulinunderstandingthesuspensioncharacteristicsofamaglevvehicle.Forinstance,thevibrationamplitudeunderaerodynamicliftcouldguidetheaerodynamicshapedesignofamaglevvehicle,sothattheaerodynamicliftgeneratedbythevehicleshapecouldavoidtheareaoflargevibrationamplitude.Anotherexampleistherelationshipbetweenthecriticaltimedelayandthefeedbackcontrolgains,accordin gtowhichthecontrolgainparameterscouldbeselectedtoenablethevehiclesystemtohaveahighertolerancefortimedelayandtoimprovethecontrolperformance.

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