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Equivalent plane strain model of stone column reinforced foundation

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Abstract: Stone column replacement of soft soil foundation construction method is more and more used in water conservancy projects. On the one hand, the stone column has high permeability, which enhances the dissipation capacity of excess pore water pressure; on the other hand, it has a high stiffness, which reduces the additional stress in the soil, and therefore reduces the excess pore water pressure. These two mechanisms work together to improve the bearing capacity of soil. In the previous plane strain transformation model of column-soil, only the high permeability of stone column is simply considered, while the stiffness of stone column is ignored. According to the previous research results, and based on the principle of genetic algorithm, this paper puts forward two conversion models which only consider the permeability of stone column and one conversion model which considers two kinds of action mechanism of gravel pile at the same time. The results of finite element calculation show that the transformation method considering the two mechanisms of stone columns are more consistent with the axisymmetric model.

1. Introduction

Our country has complicated and changeable geological conditions and abundant water resources. Since the reform and opening up, dams have been built in almost all areas with suitable geological conditions. With the continuous development of hydropower, more and more cofferdam projects have to be constructed on soft soil foundation with poor permeable layer. Settlement and stability control of soft soil weir foundation in weak permeable layer has become an urgent problem to be solved. In this context, gravel pile, as an effective foundation treatment method for replacing soil mass, is more and more used in engineering. On the one hand, gravel pile has high water permeability, which enhances the dissipation ability of excess pore water pressure; on the other hand, gravel pile has high rigidity and reduces the load distributed on the soil. These two mechanisms work together to effectively increase the bearing capacity of soil mass.

Scholars at home and abroad have done a lot of research work on gravel pile foundation reinforcement. Based on Biot's consolidation theory, Yoshikuni [1] and Onoue[2] have derived the most rigorous and complex analytical solutions for isostrain boundary conditions. The two analytical solutions consider both vertical and radial drainage in soil and well resistance effects, in which the application effect is also considered in Onoue's theoretical solution and the vertical strain is allowed to vary with radius and depth when the upper boundary displacements are equal. Based on Terzaghi's one-dimensional seepage consolidation theory, Barron [3] and Hansbo[4] obtained corresponding analytical



solutions under the assumptions of ignoring vertical soil flow, considering well resistance effect and smearing effect as isostrain analysis. Hansbo has been widely used because of its relatively simple analytical form and relatively reliable analytical results. Barksdale& Bachus[5] In the study, it is found that the aggregate excess pore water pressure is reduced by 40% by reducing the vertical loads around the soft soil; Han[6] gives an analytical solution of seepage consolidation considering both high rigidity and high water permeability of gravel piles based on the assumption of equal strain. Tan[7] Based on the assumption of equal strain, two simple plane strain transformation methods are proposed with equal drainage path and equal drainage capacity control respectively, and are compared and analyzed by finite element simulation. Gaoguangyun [8] put forward the transformation method under non-isostrain condition based on Tan and other drainage path transformation methods and verified by finite element analysis. Shigang [9] establishes consolidation equation for large deformation of sand well foundation by combining Hansbo seepage model and demonstrates it in comparison with existing research results.

Gravel piles are densely distributed on the foundation in engineering. In the two-dimensional numerical simulation, it is necessary to find a suitable method to convert them into equivalent plane strain model. Based on the existing research results, it is found that there is little research on equivalent plane strain transformation model of pile-soil unit on dam foundation of soft soil overburden in weak permeable layer in China. In this paper, three reasonable equivalent plane strain transformation models are proposed in combination with domestic and international research results, and the rationality of these three transformation models is compared by finite element analysis.

2. Equivalent plane strain transformation model

Based on the analysis method of the same area replacement rate and the genetic algorithm, this paper is based on Hansbo axisymmetric seepage theory, Han pile-soil seepage theory and Taishaji one-dimensional seepage consolidation theory respectively. Considering that the average consolidation degree before and after conversion remains unchanged when the axisymmetric problem is transformed into an equivalent plane strain problem, three equivalent plane strain transformation models for gravel pile reinforced foundation are proposed.

2.1 Description of problems

The thickness of a homogeneous saturated fully consolidated soft soil is assumed to be 1. The boundary condition of soil body is set as permeable at the top and impervious at the bottom. Gravel piles are set in the soil for replacement. The spacing of gravel piles is a and the effective radius is r_c . Equidistant plum blossom piles are used for arrangement. According to the principle of area equivalence before and after model transformation, the influence radius R of single gravel pile is $0.525a$.

2.2 Conversion of Axial Seepage Theory Based on Hansbo

Figure 1 shows the cross section of gravel pile unit. Figure 1(a) is a cross-sectional diagram of an axisymmetric model with the influence radius R of a single gravel pile derived from above. r_c is the radius of gravel pile.

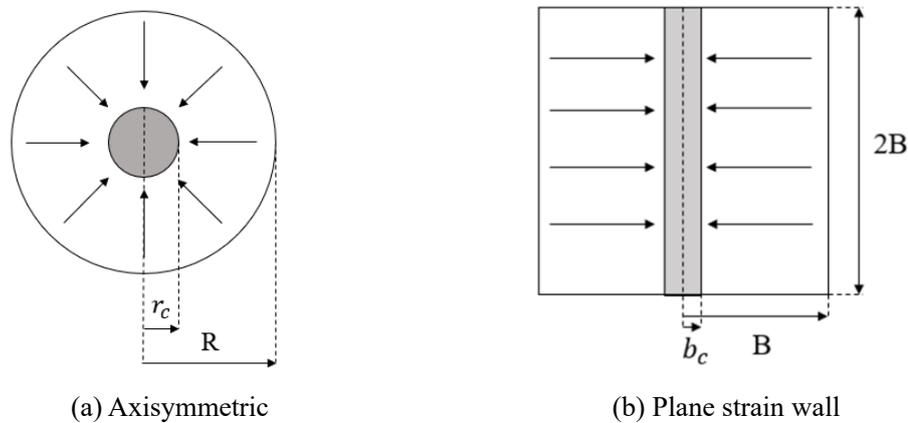


Fig. 1 Cross section diagram of stone column unit

Based on the analysis method of the same area replacement rate, the axisymmetric model is converted into an equivalent plane strain wall model as shown in Figure 1 (b). Among them, the size can be determined by the equal area of the two models: $B=0.885R$, $b_c = B \frac{r_c^2}{R^2}$.

Based on the equivalent transformation of the plane strain wall model, a general model of plane strain is obtained as shown in Figure 2, where l is the thickness of saturated soft clay layer.

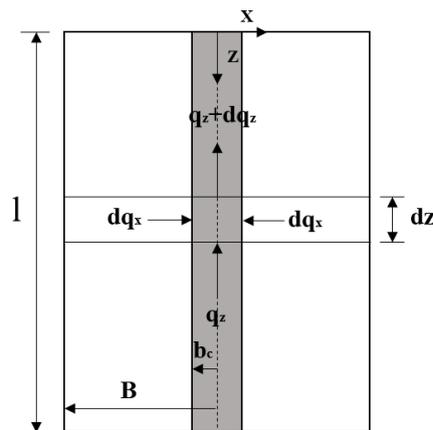


Fig. 2 Generalized model of plane strain

2.2.1 Establishment of Mathematical Model

This part is similar to the derivation of the equivalent plane strain model of vertical drainage well by indraratna et al[10]. The difference is that the water flow in gravel pile is expressed as:

$$dq_z = \frac{k_{z, pl} 2b_w}{\gamma_w} \frac{\partial^2 u}{\partial z^2} dz dt \quad (x \leq b_c) \tag{1}$$

And then the average degree of consolidation is:

$$U_{h, pl} = 1 - \frac{\tilde{u}}{\tilde{u}_0} = 1 - \exp\left(\frac{-8T_{h, pl}}{\mu_{pl}}\right) \tag{2}$$

Furthermore, the analytical solution of the average degree of consolidation for Hansbo axisymmetric seepage is

$$U_{h, ax} = 1 - \exp\left(\frac{-8T_{h, ax}}{\mu}\right) \tag{3}$$

The core idea is to keep the average degree of consolidation constant before and after conversion, i.e. $U_{h,ax} = U_{h,pl}$

It is obtained by equating Formula (2) with Formula (3)

$$\frac{T_{h,pl}}{T_{h,ax}} = \frac{k_{x,pl}R^2}{k_x B^2} = \frac{\mu_{pl}}{\mu} \tag{4}$$

It is obtained by transforming the form of equation (4)

$$k_{x,pl} = \frac{2\alpha}{\ln(n) - 0.75} \frac{B^2}{R^2} k_{x,ax} \tag{5}$$

In equation: $\alpha = \frac{1}{3} + \frac{b_c}{2B} - \frac{b_c}{6B} - \frac{b_c^2}{6B^2} + \frac{b_c^2}{2B^2} - \frac{b_c}{B}$.

2.3 Conversion of Axisymmetric Seepage Theory Based on Han Pile Soil

The plane strain model of this method is shown in Figure 2 and has the same size as the transformation model based on Hansbo's axisymmetric seepage theory.

2.3.1 Establishment of Mathematical Model

This part is similar to the derivation of Han's analytical solution of pile-soil consolidation settlement.

In the two-dimensional plane, the seepage consolidation equation of pile-soil element can be expressed as

$$c'_{x,pl} \left(\frac{\partial^2 u}{\partial x^2} \right) = \frac{\partial \bar{u}_x}{\partial t} \tag{6a}$$

$$c'_{z,pl} \left(\frac{\partial^2 u}{\partial z^2} \right) = \frac{\partial \bar{u}_z}{\partial t} \tag{6b}$$

In equation: $c'_{x,pl} = \frac{k_{x,pl}}{\gamma_w} \frac{m_{v,s} b_c + m_{v,c}(B - b_c)}{m_{v,s} m_{v,c}(B - b_c)}$, $c'_{z,pl} = \frac{k_{z,pl}}{\gamma_w} \frac{m_{v,s} b_c + m_{v,c}(B - b_c)}{m_{v,s} m_{v,c}(B - b_c)}$

Furthermore, the analytical solution of axial symmetrical consolidation seepage of pile-soil derived by Han is:

$$c'_{r,ax} \left(\frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \right) = \frac{\partial \bar{u}_r}{\partial t} \tag{7a}$$

$$c'_{z,ax} \left(\frac{\partial^2 u}{\partial z^2} \right) = \frac{\partial \bar{u}_z}{\partial t} \tag{7b}$$

In equation: $c'_{r,ax} = \frac{k_{r,pl}}{\gamma_w} \frac{m_{v,c}(1 - a_s) + m_{v,s} a_s}{m_{v,s} m_{v,c}(1 - a_s)}$, $c'_{z,ax} = \frac{k_{z,pl}}{\gamma_w} \frac{m_{v,c}(1 - a_s) + m_{v,s} a_s}{m_{v,s} m_{v,c}(1 - a_s)}$.

The core idea is also to keep the average degree of consolidation constant before and after conversion. For vertical seepage, easy to get $k_{z,ax} = k_{z,pl}$ at the same replacement rate. For axisymmetric seepage, analytical solution of consolidation by Han, average degree of consolidation U_r is

$$U_r = 1 - \exp\left(\frac{-8T_r}{f(n)}\right) \tag{8}$$

In equation: $T_r = \frac{c'_{r,ax} t}{R^2}$, $f(n) = \frac{n^2}{n^2 - 1} \ln(n) - \frac{3n^2 - 1}{4n^2}$.

The average degree of consolidation of horizontal seepage under two-dimensional seepage can be expressed as:

$$U_x = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v) \tag{9}$$

Let $U_x = U_r$ so that the average degree of consolidation is equal, the result is:

$$\sum_{m=1}^{\infty} \frac{2}{M^2} \exp(-M^2 T_v) = \exp\left(\frac{-8T_r}{f(n)}\right) \tag{10}$$

Formula (10) cannot be directly solved by analytic solution. The genetic algorithm principle is used to solve the problem numerically, thus the ratio of the horizontal permeability coefficient of soil before and after conversion can be obtained.

2.4 Conversion of one-dimensional seepage consolidation theory based on Terzaki

Based on Terzaghi's one-dimensional seepage consolidation theory, the equivalent plane strain model is derived directly from the equivalent horizontal and vertical drainage

2.4.1 Establishment of Mathematical Model

When the plane problem is equivalent, the equivalent drainage horizontal permeability coefficient $k_{x,pl}$ for the whole area is:

$$k_{x,pl} = \frac{(R - r_0)k_{x,ax}}{R} / \ln\left(\frac{R}{r_0}\right) \tag{11}$$

Using the same average degree of consolidation before and after the model transformation of the whole region, the result is obtained.

$$T_{v,pl} = \frac{E_s k_{x,pl} \cdot t}{\gamma_w H_{pl}^2} = \frac{E_s k_{x,ax} \cdot t}{\gamma_w H_{ax}^2} = T_{v,ax} \tag{12}$$

From Eq. (12), the drainage distance H_{pl} of the equivalent plane strain model is:

$$H_{pl} = \sqrt{R(R - r_0) \ln\left(\frac{R}{r_0}\right)} \tag{13}$$

As the replacement ratio of product η before and after model transformation remains unchanged, the pile width D is:

$$D = 2\eta H_{pl} / (1 - \eta) \tag{14}$$

3. Finite element simulation verification

3.1 Model Size

Based on the analysis and calculation of an engineering example, the gravel pile is arranged with plum-blossom pile, with a distance of 4 m and an effective diameter of 1 m.

For axisymmetric model and three-dimensional model, the dimension remains unchanged, the diameter of gravel pile is 1m, and the influence radius of single gravel pile is $R=2.1m$. For the three planar strain models that need to be established, the model dimensions are calculated according to the above formulas.

3.2 Model parameters

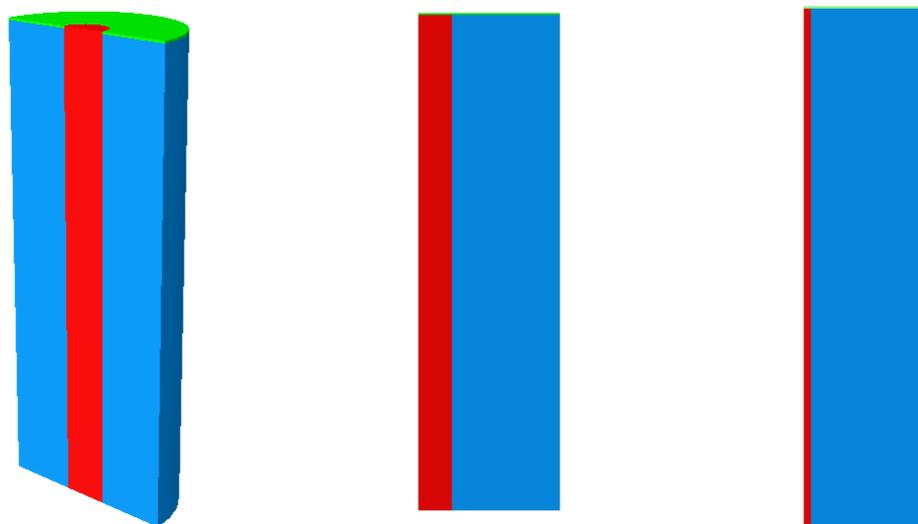
The basic material parameters of the model are shown in Table 1. The vertical permeability coefficient is set at 1/3 of the horizontal permeability coefficient. In the calculation simulation, the gravel pile is idealized as a homogeneous drainage material.

Table 1 Material parameters of element model

Model	$\gamma_s = \gamma_c$ kN·m ⁻³	ν'	E_s MPa	E_c MPa	k_h m·s ⁻¹	k_v m·s ⁻¹	c_s' kPa	φ_s' deg	c_c' kPa	φ_c' deg
Axisymmetric/three-dimensional model	15	0.3	3	30	$3.00 \cdot 10^{-9}$	$1 \cdot 10^{-9}$	0.1	22	1	40
Plane strain model 1	15	0.3	3	30	$2.04 \cdot 10^{-9}$	$1 \cdot 10^{-9}$	0.1	22	1	40
Plane strain model 2	15	0.3	3	30	$1.55 \cdot 10^{-9}$	$1 \cdot 10^{-9}$	0.1	22	1	40
Plane strain model 3	15	0.3	3	30	$3.00 \cdot 10^{-9}$	$1 \cdot 10^{-9}$	0.1	22	1	40

3.3 Finite element modeling

The specific finite element modeling process is as follows: firstly, material parameters and boundary conditions are set to restrain lateral and bottom displacements of the model side; secondly, the analysis section is set up: to establish in-situ stress balance; to activate the instantaneous 100kPa vertical load on the top surface of the model; to conduct settlement and consolidation analysis. The finite element model of the basic unit of pile-soil is shown in Figure 3, in which the different models of plane strain only need to modify the model size:



(a) Three-dimensional model (b) axisymmetric model (c) Plane strain model

Fig. 3 Finite element model of column soil element

3.4 Result analysis

The results of calculation simulation are expressed in the form of surface settlement and excess pore water pressure in Figures 4 and 5 respectively.

From the settlement map in Figure 4, it can be seen that the trend of several settlement curves is basically the same, and the settlement curves of plane strain model 1 and plane strain model 3 basically coincide, which may be due to the fact that both are based on the transformation of ignoring the size of rigid body of gravel pile itself. In the early to middle stage of consolidation, if the settlement curves of axisymmetric model and three-dimensional model are taken as a settlement interval, the settlement curves of plane strain 2 model basically lie in this settlement interval, while those of plane strain model 1 and 3 deviate significantly from this interval. From the end of the curve where Settlement Consolidation tends to be stable, the deviation values of plane strain model 1, 2, 3 and axisymmetric model settlement are 18.2%, 13.9% and 16.8% respectively, and the settlement values of plane strain model 2 are lower and closer to the settlement values of axisymmetric model.

Figure 5 shows the trend of excess pore water pressure in different models, and the initial excess pore water pressure of 100 kPa generated by instantaneous loads in each model is adequately reflected in the

result diagram. It can be seen in the diagram that the change trend of over pore pressure of each model is similar. Combined with data analysis, the over-pore pressure deviation values of axisymmetric model and plane strain model 1, 2 and 3 are 69.4%, 8.9% and 60.3% 100 days after consolidation, only the over-pore pressure values of plane strain model 2 and axisymmetric model are in one order of magnitude. From the dimension of consolidation time, the corresponding over pore pressure of the axisymmetric model at 100 days after consolidation is the value of plane strain model 1 and 3 at 70 days after consolidation. The plane strain 2 model is in good agreement with the axisymmetric model.

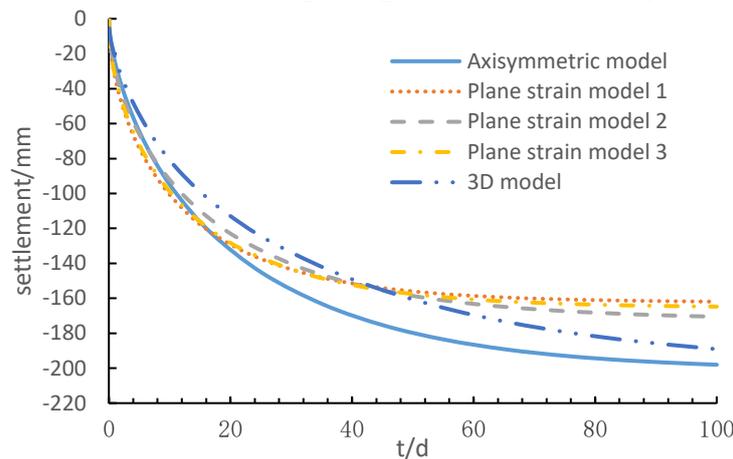


Fig. 4 Comparison of surface settlement simulation of unit

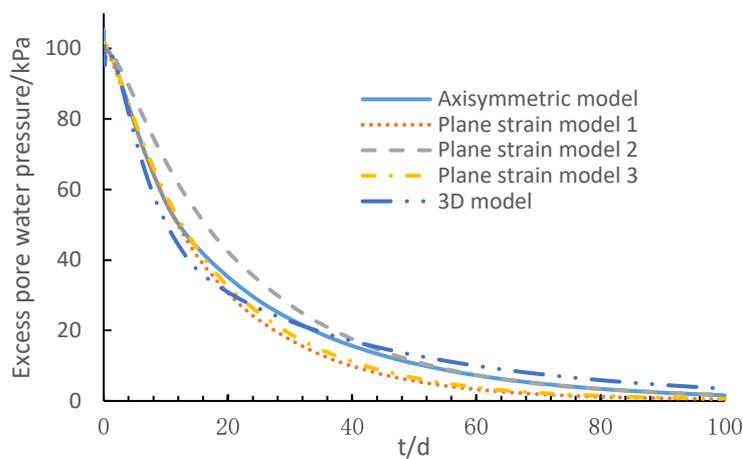


Fig. 5 Comparison of excess pore pressure simulation of cell model

4. Conclusion

Based on the analysis method of the same area replacement rate and the genetic algorithm, three methods are proposed to simulate and analyze the gravel pile plane strain transformation model. From the simulation results, the conclusions are as follows:

(1) The settlement and excess pore pressure changes of the Hansbo axial seepage theory conversion method are basically the same as those of the direct one-dimensional seepage consolidation theory conversion method based on Taishaki, because both are essentially transformed based on ignoring the rigidity of gravel piles. The difference is the conversion of Hansbo's axial seepage theory to change the permeability coefficient of soil layer. The one-dimensional seepage consolidation theory conversion method based on Taishaki is the conversion to change the size of soil between piles.

(2) Han-based transformation method of seepage solution of pile-soil considering both high rigidity of gravel pile itself and high permeability has better adaptability than other two methods. When the

settlement tends to be stable 100 days after consolidation, the settlement stability values of the other two methods are relatively small, and only the over-pore pressure values of the conversion method based on Han's seepage solution of pile-soil and the simulation values of axisymmetric model are in one order of magnitude. In conclusion, the Han-based transformation model of seepage solution of pile-soil is more consistent with the simulation results of axisymmetric model.

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