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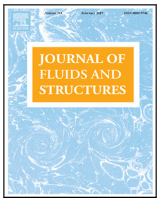

An sequential optimization and aeroelastic constraint transformation method for strength-aeroelastic comprehensive design

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ABSTRACT

With the development of modern aircraft technology, aeroelasticity plays a more and more crucial role in aircraft structural design. However, low efficiency of present aeroelastic analysis and optimization methods makes it difficult to apply in engineering practice. This paper presents a sequential optimization and aeroelastic constraint transformation method (SOACTM) for comprehensive design of airplane wings with strength and aeroelastic constraints. Optimization with structural strength constraint and aeroelastic constraint is transformed into a serial of cycles of decoupled structural strength sub-optimizations and aeroelastic sub-optimizations based on sequential optimization strategy. In structural strength sub-optimization, structural strength constraint is translated along its normal direction to make optimal design point satisfying aeroelastic constraint. And the goal of aeroelastic sub-optimization is to find the translational distance of structural strength constraint. Aeroelastic constraint is transformed to equivalent structural strength constraint via above approach. In this way, number of aeroelastic analyses in SOACTM is less than that in traditional optimization method and total computational time decreases. SOACTM is verified based on two examples. Traditional optimization method is applied for the sake of validation. The results demonstrated the accuracy and efficiency of SOACTM for wing comprehensive optimization considering both structural strength and aeroelastic constraints.

1. Introduction

Nowadays, aeroelasticity is a significant factor in the field of aeronautics and astronautics (Yurkovich, 2003; Dowell et al., 2003). Aeroelastic analysis and simulation are necessary in design process of airplane wings, especially long-span wings with low stiffness (Frulla, 2004) and high flight speed (Mcnamara and Friedmann, 2011). Aeroelasticity is a subject about the coupling relationship between aerodynamic loads and elastic deformation of wings. Aeroelastic response is related to combined action of elastic force, inertial force and aerodynamic force. Collar (1946) classified aeroelastic problems and describes different kinds of aeroelastic problems based on this relationship. Aeroelasticity can be classified into static aeroelasticity and dynamic aeroelasticity. Static aeroelasticity aims at analyzing static deformation of wings burdening aerodynamic loads, while structural dynamic response is paid attention to dynamic aeroelasticity. Aeroelastic demand always attracts much attention in wing design. However, modern aircrafts have not get rid of problems caused by

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Nomenclature

d_k	Translational distance of optimal design point in the kth cycle, m
D_k	Total correction of structural strength constraint after kth cycle, Pa
h	Difference between maximum stress in structure and allowable stress of material, Pa
M	Total mass of airplane wing, kg
$\mathbf{n}$	Unit normal vector of $h(\mathbf{X})$
Q	Aerodynamic load matrix, N
V_{cr}	Critical speed of airplane wing, m/s
V_{cr_0}	Minimum allowable critical speed in aeroelastic demand, m/s
x_1	The first design variable, m
x_2	The second design variable, m
x_m	The mth design variable, m
$\mathbf{X}$	Design point
$\mathbf{X}^L$	Lower bound of $\mathbf{X}$
$\mathbf{X}^U$	Upper bound of $\mathbf{X}$
$\mathbf{X}_k$	Optimal design point after structural strength sub-optimization in kth cycle
$\mathbf{X}_k^*$	Translated optimal design point after aeroelastic sub-optimization in kth cycle
ΔD_k	Correction of structural strength constraint in the kth cycle, Pa
Δx	Small perturbation quantity of variables, m
ε	Convergence threshold
$\sigma_{\max}$	Maximum stress in structure, Pa
$[\sigma]$	Allowable stress of material, Pa

aeroelasticity (Livne, 2003). These problems threat the safety of aircrafts and sometimes lead to serious accidents (Farrick and Reed, 1981; Wang et al., 2014, 2019). Therefore, aeroelastic constraint is a necessary part for wing design of modern aircraft.

On the one hand, static deformation or vibration of airplane wing occurs under the influence of aerodynamic loads; on the other hand, static deformation and vibration change flow distribution, thus changing aerodynamic loads conversely (Zheng and Qiu, 2018b). Aeroelastic analysis is a complex work which refers to this kind of interaction between aerodynamic load and structural deformation in both static and dynamic aeroelastic problems (Zheng and Qiu, 2018a; Geuzaine et al., 2003). Therefore, aeroelastic analysis requires large computational cost, especially when CFD/CSD coupling technology is applied in it. Bartels and Sayma (2007) and Bartels (2015) indicated that although the aeroelastic and multidisciplinary optimization of designs are already making use of higher level methods of computational aeroelasticity, these methods are still too expensive for routine application.

The former design method is to perform aerodynamic optimization and structural optimization sequentially. This method which ignores the influences made by aeroelasticity cannot satisfy present design demand. Nowadays, aeroelasticity becomes a notable factor in aircraft design. Maute et al. (2001) and Maute et al. (2003) presented a complete optimization methodology which includes a staggered algorithm for the computation of the aeroelastic steady state. Nikbay et al. (2009) presents a practical methodology for static aeroelastic analysis and optimization via coupling of high-fidelity commercial codes. At present, airplane wing design is usually regarded as an optimization considering both structural strength constraints and aeroelastic constraints. Optimization result can be acquired after multiple aeroelastic analyses which cost much computational time. Low efficiency of aeroelastic analysis and optimization is a remarkable disadvantage which limits comprehensive optimization methods in engineering practice (Kamakoti and Wei, 2004).

In order to improve efficiency of airplane wing comprehensive design, researchers proposed feasible approaches from many aspects. Some researchers make efforts to improve the efficiency of aeroelastic analysis using approximate calculation method of aerodynamic force (Zhang et al., 2012; Yan and Wan, 2018). Although these methods improve efficiency of aeroelastic analysis, they sacrifice accuracy more or less. Many researchers have made efforts to improve efficiency of comprehensive optimization methods for airplane structures (Wang et al., 2018d). Some people pay attention to reduced-order models(ROMs) (Lucia et al., 2004) and surrogate models (Wan et al., 2016). Zhang et al. (2015) applied unsteady aerodynamic ROM of an aeroelastic wing using arbitrary mode shapes in aeroelastic design optimization and the computational cost is reduced obviously. Wu et al. (2018) studied on robust aerodynamic optimization which is computationally expensive due to expensive CFD costs, and successfully improved efficiency of it by introducing surrogate model technology. However, application of these methods are limited because accuracy of reduced-order models and surrogate models have great influences on optimization results. Part of researchers concentrate on optimization algorithm (Manan et al., 2010). Efficiency improvement of these methods is not satisfying in many circumstances. However, few people pay attention to optimization strategies. Many researches indicate that an appropriate optimization strategy can promote optimization efficiency obviously (Dababneh et al., 2018). Moreover, optimization efficiency can be further improved by combining optimization strategy and above-mentioned means.

Sequential optimization strategy is often applied in reliability optimization and has been proved efficient. Reliability optimization consists of deterministic optimization and reliability estimation which makes finite element analysis consuming much time (Wang et al., 2018a,b). In order to solve this problem, Du and Chen (2004) firstly proposed sequential optimization and reliability analysis(SORA) method. This method is widely applied in reliability optimization problems subsequently. Cho and Lee (2011) combined convex approximations for probabilistic constraint with SORA method, and the proposed method requires much less function evaluations of probabilistic constraint in the deterministic optimization. Yin and Chen (2006) proposed enhanced SORA method considering varying design variance in probabilistic optimization. Wang et al. (2018c) developed an single-loop strategy for reliability-based multidisciplinary design under non-probabilistic set theory and illustrated the effectiveness and efficiency of the proposed method. In recent years, some people make attempts to introduce sequential optimization strategy into solution of multidisciplinary optimization problems. Lei et al. (2008) presented a sequential optimization method for design of electromagnetic device, the proposed method is composed of coarse optimization process and fine optimization process (Zhang and Huang, 2010) applied sequential optimization strategy in multidisciplinary design optimization under aleatory and epistemic uncertainties, and proved that sequential optimization method can reduce computation load effectively. Sequential optimization strategy has been proved to be effective in multidisciplinary optimization. However, there are still many problems need to be further studied. Although strength-aeroelastic comprehensive optimization problem is a typical multidisciplinary problem, studies about application of sequential optimization strategy in strength-aeroelastic comprehensive optimization of airplane wings are quite rare yet.

The main contribution of this paper is a novel comprehensive optimization method for airplane wings considering structural strength and aeroelastic constraints based on sequential optimization strategy. The main original feature of the proposed method is transforming an optimization with multiple constrains into a sequential optimization via introducing an approach to transforming aeroelastic constrain to equivalent structural strength constrain. Sequential optimization strategy is introduced to decouple structural strength analyses and aeroelastic analyses. An optimization scheme consists of a series of cycles includes structural strength sub-optimization and aeroelastic sub-optimization. In this way, frequency of aeroelastic analyses in optimization is reduced, and optimization efficiency is improved while not losing accuracy.

The remainder of this paper is organized as follows. Constraints and optimization model of comprehensive design for airplane wing is simply introduced in Section 2. Section 2 also presents the main content and limitation of traditional optimization method. Main idea of sequential optimization strategy and detailed procedures of SOACTM are described in Section 3, and this section specially explains the modifying approach of structural strength constraint. Two numerical examples are presented in Section 4 to demonstrate validity of SOACTM. At last, Section 5 provides a brief summary and conclusion.

2. Problem statement

Aeroelasticity has great influence on airplane wings, especially long-span wings. On the one hand, aeroelasticity influences deformation and stress distribution of airplane wings, which may lead to structural failure; on the other hand, aeroelastic problems such as static divergence and flutter occur and threat flight safety when flight speed is higher than critical speed. Therefore, comprehensive design considering structural strength and aeroelastic constraints is an unavoidable task in airplane wing design. Structural strength constraint limits the maximum value of stress in airplane wing to prevent structural failure, and it is written as follows:

$$\sigma_{\max} \le [\sigma] \tag{1}$$

where $\sigma_{\max}$ is the maximum stress in structure and $[\sigma]$ is the maximum allowable stress. Aeroelastic constraints limits the minimum value of critical speed of airplane wing to avoid aeroelastic problems and it is written as follows:

$$V_{cr} \ge V_{cr\_0} \tag{2}$$

where V_{cr} is the critical speed of structure and V_{cr_0} is minimum allowable critical speed in aeroelastic demand. Critical speed stands for divergence speed in static elastic problems and critical flutter speed in dynamic elastic problems.

In many circumstances, the essence of structural design in engineering is an optimization which consists of optimization variables, optimization objective and constraint conditions. Above all, the optimization model of this comprehensive design problems for airplane wings are written as follows:

$$\begin{cases} \text{find} & \mathbf{X} \\ \min & M(\mathbf{X}) \\ \text{s.t.} & \sigma_{\max}(\mathbf{X}) \le [\sigma] \\ & V_{cr}(\mathbf{X}) \ge V_{cr\_0} \\ & \mathbf{X}^L \le \mathbf{X} \le \mathbf{X}^U \end{cases} \tag{3}$$

in which $\mathbf{X}$ is the vector consisting of design variables and M stands for total mass of airplane wing.

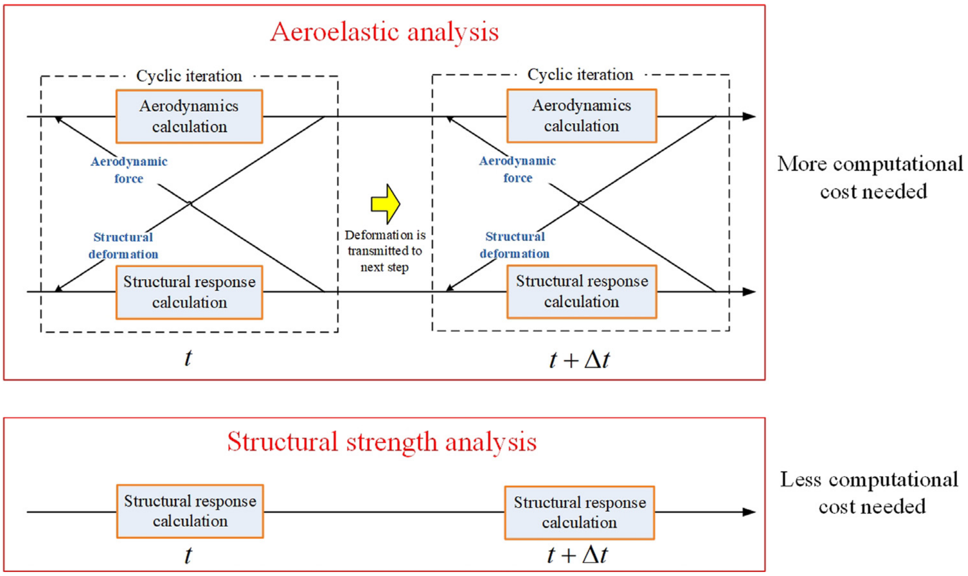

Fig. 1. Main procedure of a typical aeroelastic analysis process.

Comprehensive design problem of airplane wing is an optimization problem with several constraints which can be transformed into unconstrained optimization problem via penalty function method. Solving this kind of optimization problems via traditional optimization method is a cyclic iterative process with multiple structural strength analyses and aeroelastic analyses. Aeroelastic analysis is a complex coupling process which consists of cyclic iteration of structural analyses and aerodynamic analyses. A typical aeroelastic analysis process is shown in Fig. 1. Therefore, aeroelastic analysis needs much more computational cost than structural strength analysis.

As we can see, aeroelastic analyses consume most computational cost of whole optimization process. If we execute aeroelastic analyses for less times, we are able to solve comprehensive design problems more efficiently. Then, traditional optimization method for comprehensive design shows its limitation. For above optimization model of comprehensive design problem, structural strength analysis and aeroelastic analysis are both carried out to validate the constraints in every iteration. So aeroelastic analysis is carried out as the same times as structural strength analysis. Redundant aeroelastic analyses may be carried out when we apply above common optimization model to solve comprehensive design problems. Comprehensive design problem for airplane wing always concludes many optimization variables. The design space is high-dimensional and many iterations are needed to obtain converged optimization result. Therefore, number of aeroelastic analyses is large. Above all, reducing the number of aeroelastic analyses is a feasible way to improve efficiency of comprehensive design for airplane wings.

3. Sequential optimization and aeroelastic constraint transformation method

This part proposed an efficient comprehensive optimization method for airplane wings based on sequential optimization strategy. The main idea of SOACTM is transforming the optimization with structural strength constraint and aeroelastic constraint into a serial of cycles of decoupled structural strength sub-optimizations and aeroelastic sub-optimizations. In structural strength sub-optimization, we only consider the structural strength constraint and ignore the aeroelastic constraint. Aeroelastic sub-optimization is an unconstrained optimization with single variable of which the aeroelastic constraint is transformed into optimization objective. For every cycle, structural strength constraint is modified based on the result of aeroelastic sub-optimization in last cycle, and the structural strength sub-optimization supplies initial design point for aeroelastic sub-optimization in this cycle. Aeroelastic constraint is transformed into equivalent structural strength constraint through above approach. Aeroelastic sub-optimization is easy to get convergent in many circumstances, which reduces number of aeroelastic analyses.

3.1. Modifying approach of structural strength method

The key point of sequential optimization strategy is to transform aeroelastic constraint to equivalent structural strength constraint. In SOACTM, structural strength constraint is modified based on the optimization result of aeroelastic sub-optimization, and the modification is to translate optimal design point along normal direction of structural strength constraint as shown in Fig. 2. In many cases, optimal design point obtained from structural strength sub-optimization

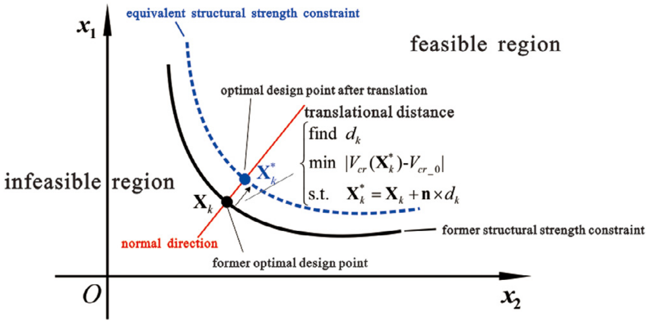

Fig. 2. Diagrammatic sketch about modification of structural strength constraint based on result of aeroelastic sub-optimization.

is out of feasible region because it does not satisfy the aeroelastic constraint. So this design point is translated to make the aeroelastic constraint satisfied. The minimum translation distance is obtained from aeroelastic sub-optimization. Then the structural strength constraint is modified, and the translated design point located on the boundary corresponding to equivalent structural strength constraint. Thus, optimal design point acquired form optimization considering equivalent structural strength constraint. Hence, determining the normal direction of structural strength constraint at optimal design point is of great significance for SOACTM.

The functional expression of structural strength constraint is written as follows:

$$\sigma_{\max}(\mathbf{X}) \le [\sigma] \tag{4}$$

Therefore, the corresponding boundary of feasible region is expressed as follows:

$$h\left(\mathbf{X}\right) = \sigma_{\max}(\mathbf{X}) - [\sigma] = 0 \tag{5}$$

The unit normal vector of $h\left(\mathbf{X}\right)$ is

$$\mathbf{n} = \frac{\partial h\left(\mathbf{X}\right)}{\partial \mathbf{X}} = \frac{\partial \sigma_{\max}\left(\mathbf{X}\right)}{\partial \mathbf{X}} = \left(\frac{\partial \sigma_{\max}\left(\mathbf{X}\right)}{\partial x_1}, \frac{\partial \sigma_{\max}\left(\mathbf{X}\right)}{\partial x_2}, \ldots, \frac{\partial \sigma_{\max}\left(\mathbf{X}\right)}{\partial x_m}\right)^T \tag{6}$$

This paper introduce difference method to calculate each component in $\frac{\partial h(\mathbf{X})}{\partial \mathbf{X}}$, and the detailed calculation formulas are shown as follows:

$$\begin{aligned}
&\frac{\partial \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\partial x_1} \approx \frac{\sigma_{\max}\left(x_1 + \Delta x, x_2, \ldots, x_m\right) - \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\Delta x} \\
&\frac{\partial \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\partial x_2} \approx \frac{\sigma_{\max}\left(x_1, x_2 + \Delta x, \ldots, x_m\right) - \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\Delta x} \\
&\ldots \\
&\frac{\partial \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\partial x_m} \approx \frac{\sigma_{\max}\left(x_1, x_2, \ldots, x_m + \Delta x\right) - \sigma_{\max}\left(x_1, x_2, \ldots, x_m\right)}{\Delta x}
\end{aligned} \tag{7}$$

where Δx is a small perturbation quantity of variables.

Then we can obtain translational distance d_k of optimal design point via aeroelastic sub-optimization as normal direction of structural strength constraint at optimal design point is known. Next, aeroelastic constraint is transformed into equivalent structural strength constraint through modifying structural strength constraint with translational distance d_k.

As we can see, above modifying approach is based on this hypothesis: normal direction of structural strength constraint is the same as that of aeroelastic constraint. In fact, this hypothesis is not strictly true in many circumstances. However, this error is acceptable for metal wings. If a part in metal wing is changed to promote the aeroelastic properties, its structural strength is improved at the same time. That is to say, structural strength constraint and aeroelastic constraint have inner relationship and their normal directions are much close for metal wings. Furthermore, structural strength sub-optimization and aeroelastic sub-optimization are carried out again sequentially after modifying structural strength constraint in SOACTM. The difference between normal direction of structural strength constraint and that of aeroelastic constraint will decrease and can be neglected after several iterations.

3.2. Main procedure of SOACTM

Fig. 3 presents detailed procedure of SOACTM method which is shown in following contents in this part:

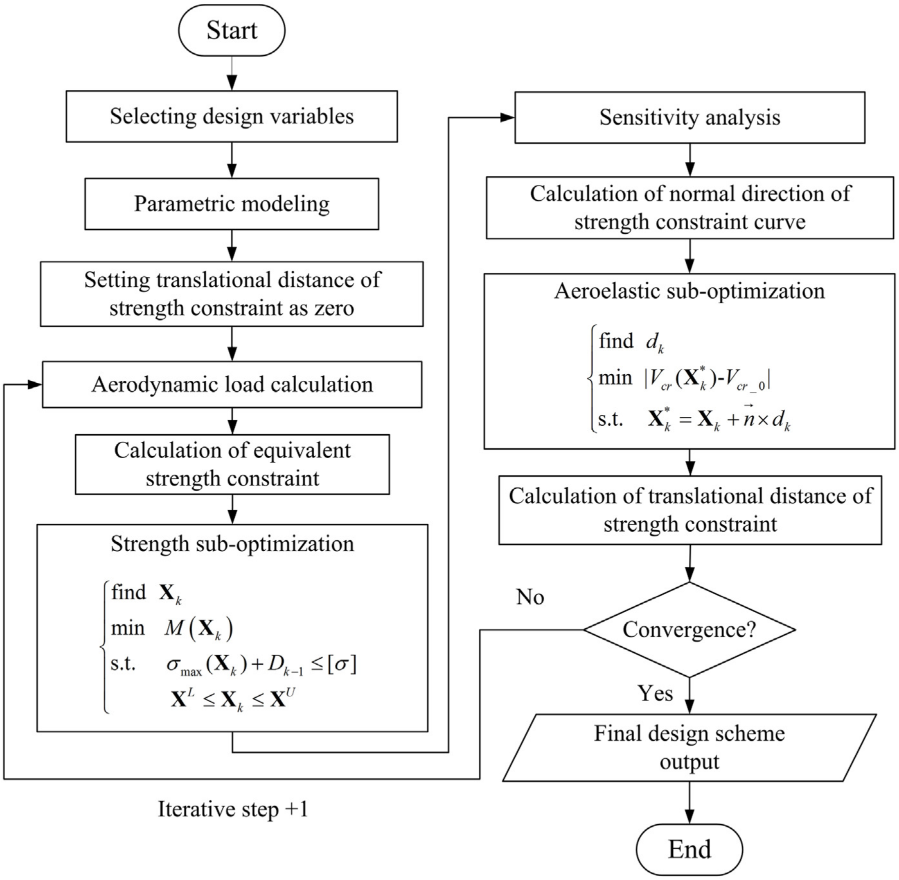

Fig. 3. Flow chart of SOACTM for comprehensive design of airplane wings.

(1) Aiming at airplane wing, selecting parameters which have large influence on structural weight and strength as design variables based on result of sensitivity analysis.

(2) Parametric modeling of airplane wing.

(3) Setting initial translational distance of structural strength constraint as zero.

(4) Calculating aerodynamic load Q considering the elastic deformation of airplane wing.

(5) Modifying structural strength constraint based on the total translational distance after the $(k-1)$th cycle, thus obtaining the equivalent structural strength constraint:

$$\sigma_{\max}(\mathbf{X}_k)+D_{k-1}\le[\sigma] \tag{8}$$

where D_{k-1} stands for the total correction of structural strength constraint in the $(k-1)$th cycle, $\mathbf{X}_k$ stands for the optimal design point acquiring from the structural strength sub-optimization in the kth cycle.

(6) Carrying out structural strength sub-optimization whose optimization model is shown as follows:

$$\begin{cases}\text{find} & \mathbf{X}_k\\ \text{min} & \mathrm{M}(\mathbf{X}_k)\\ \text{s.t.} & \sigma_{\max}(\mathbf{X}_k)+D_{k-1}\le[\sigma]\\ & \mathbf{X}^L\le\mathbf{X}_k\le\mathbf{X}^U\end{cases} \tag{9}$$

Structural strength sub-optimization is similar to general structural strength optimization. So one can regard decrease of structural mass of two adjacent iterative steps as convergence criterion in structural strength sub-optimization.

(7) Calculating the partial derivative of maximum stress on current design point $\mathbf{X}_k$ which is shown as follows:

$$\frac{\partial\sigma_{\max}(\mathbf{X}_k)}{\partial\mathbf{X}_k}=\left(\frac{\partial\sigma_{\max}(\mathbf{X}_k)}{\partial x_{1k}},\frac{\partial\sigma_{\max}(\mathbf{X}_k)}{\partial x_{2k}},\ldots,\frac{\partial\sigma_{\max}(\mathbf{X}_k)}{\partial x_{ik}},\ldots,\frac{\partial\sigma_{\max}(\mathbf{X}_k)}{\partial x_{mk}}\right)^T \tag{10}$$
$$i=1,2,\ldots,m$$

where x_{ik} is the ith component of $\mathbf{X}_k$ which stands for the value of the ith optimization variable after strength sub-optimization in the kth cycle.

(8) Calculating unit normal vector of strength constraint curve via following formula:

$$\mathbf{n}=\frac{\partial\sigma_{\max}\left(\mathbf{X}_k\right)}{\partial\mathbf{X}_k}\bigg/\left|\frac{\partial\sigma_{\max}\left(\mathbf{X}_k\right)}{\partial\mathbf{X}_k}\right| \tag{11}$$

(9) Carrying out aeroelastic sub-optimization of airplane wing aiming at finding the minimum translational distance of structural strength constraint to satisfy the aeroelastic constraint. Its optimization model is presented as follows:

$$\begin{cases}\text{find} & d_k\\ \min & \left|V_{cr}(\mathbf{X}_k^*)-V_{cr\_0}\right|\\ \text{s.t.} & \mathbf{X}_k^*=\mathbf{X}_k+\mathbf{n}\times d_k\end{cases} \tag{12}$$

where d_k strands for the translational distance of optimal design point $\mathbf{X}_k$ along the normal direction of the structural strength constraint at optimal design point in the kth cycle. $\mathbf{X}_k^*$ represents the optimal design point after translation which satisfies the aeroelastic demand:

$$V_{cr}(\mathbf{X}_k^*)\geq V_{cr\_0} \tag{13}$$

where V_{cr} is critical speed of airplane wing and V_{cr_0} is the minimum critical speed demand in aeroelastic constraint. Variance in translational distance d_k of two adjacent iterative steps is used as convergence criterion in this sub-optimization.

(10) Calculating correction of structural strength constraint in the kth cycle based on following formula:

$$\varDelta D_k=\sigma_{\max}\left(\mathbf{X}_k\right)-\sigma_{\max}\left(\mathbf{X}_k^*\right) \tag{14}$$

Then calculating the total correction of structural strength constraint:

$$D_k=D_{k-1}+\varDelta D_k \tag{15}$$

(11) Repeating step (4) to step (10) until following convergence condition is satisfied:

$$d_k\leq\varepsilon \tag{16}$$

where ε is a small threshold value used to judge that if the optimization converges or not. At last, outputting the design point $\mathbf{X}_k$ in last cycle as the final result.

The number of aeroelastic analyses is not equal to that of structural strength analyses in SOACTM. The number of aeroelastic analyses in SOACTM is less than that in traditional optimization method because aeroelastic sub-optimization is an optimization with single variable which is easy to get convergent in many circumstances. Hence, this method achieves higher efficiency than traditional optimization method in many circumstances.

4. Numerical examples

In this part, two numerical examples are given to prove that SOACTM has high efficiency and accuracy in comprehensive design for airplane wings.

4.1. Stiffened panel

Stiffened panel is a simple structure which is a common part of airplane wing. In this example, the stiffened panel shown in Fig. 4 is regarded as optimization object. The side length and thickness of square plate are 1000 mm and 5 mm respectively. Four stiffener are all 40 mm high and two of them are located along flow direction while others are perpendicular to flow direction. This stiffened panel is made from steel and simply supported on four sides of it. Detailed information of this stiffened panel is given in Table 1.

Thicknesses of stiffeners is a significant factor which has great influences on weight, strength, stiffness and static divergence speed of the stiffened panel, so we choose them as optimization variables. We might regard the thickness of two stiffeners along flow direction as the first optimization variable denoted as x_1, and regarded the thickness of two stiffeners perpendicular to flow direction as the second optimization variable denoted as x_2. Optimization objective is to make weight of stiffened panel minimum. Structural strength constraint is that maximum stress is less than 225 MPa while flow speed is 1750 m/s and the flight altitude is 0 km above sea-level. Aeroelastic constraint demands that static aeroelastic divergence speed is not less than 1745 m/s. The initial condition is $x_1 = x_2 = 0.03$ m and convergence condition is $d_k \leq 0.0001$ m. The convergence criterion for structural strength sub-optimization in example 1 is $|m_{k+1}-m_k|\leq 0.01$ kg, where m_k and m_{k+1} stands for the structural mass of the kth iterative step and $k+1$th iterative step respectively. The convergence criterion for aeroelastic sub-optimization in example 1 is $\left|V_{cr,k+1}-V_{cr,k}\right|\leq 0.1$ m/s, where $V_{cr,k}$ stands for the critical speed of the kth iterative step and $k+1$th iterative step respectively.

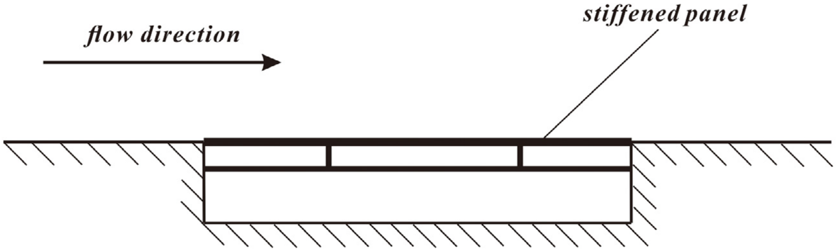

Fig. 4. Diagrammatic sketch of the stiffened panel and flow direction.

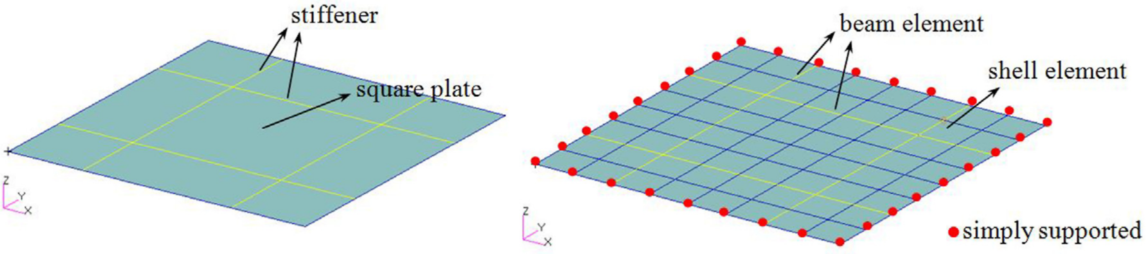

Fig. 5. Geometric and finite element model of the stiffened panel.

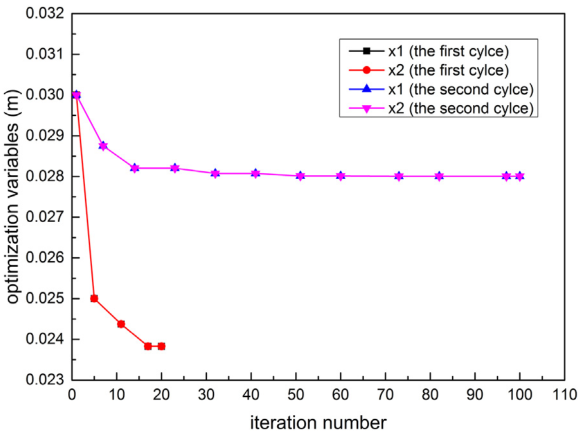

Fig. 6. Iteration history of optimization variables of structural strength sub-optimization in panel example.

Geometric and finite element model of stiffened panel shown in Fig. 5 is constructed. In this example, finite-element-based solver Nastran is used to acquire structural response while aerodynamic loads are calculated based on ZONA51 aerodynamic theory (Liu et al., 1991; Liu and Liu, 1985). Static aeroelastic divergence speed is obtained via $v-g$ method while reduced frequency is set to 0, and non-linear programming by quadratic Lagrangian (NLPQL) is applied in solution of both structural strength sub-optimizations and aeroelastic sub-optimizations.

Convergence condition is satisfied after two cycles in this example. Translational distances and corrections of structural strength constraint in each cycle are listed in Table 2. Iteration history of two optimization variables in structural strength sub-optimization are shown in Fig. 6. Iteration history of optimization variables in aeroelastic sub-optimization, i.e. translational distance, is shown in Fig. 7. Iteration history of structural strength sub-optimization objective, i.e. total mass of this stiffened panel, and aeroelastic sub-optimization objective are presented in Fig. 8. Only improved solutions are presented in these figures.

The processor of computer used for numerical calculation is Intel(R) Core(TM) i7-4790 CPU @3.60 GHz 3.60 GHz and RAM is 16.0 GB. Computation time of single structural strength analysis and aeroelastic analysis are 10.50 s and 27.20 s respectively.

Next, SOACTM is compared with traditional optimization method in order to prove its accuracy and efficiency. Comparison between optimization results of two optimization method are listed in Table 3. It is obvious that result of SOACTM is close to that of traditional optimization method. Therefore, SOACTM has satisfying accuracy. Table 4

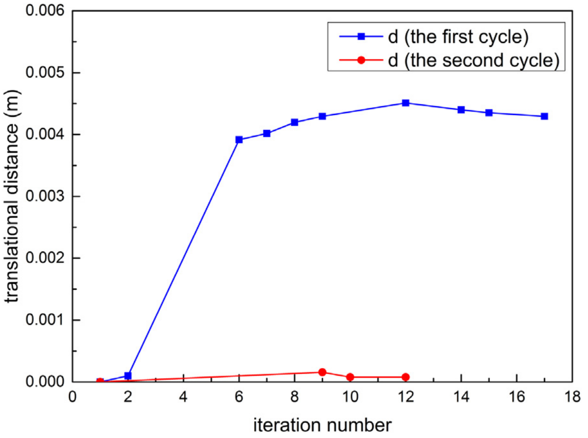

Fig. 7. Iteration history of optimization variable of aeroelastic sub-optimization in panel example.

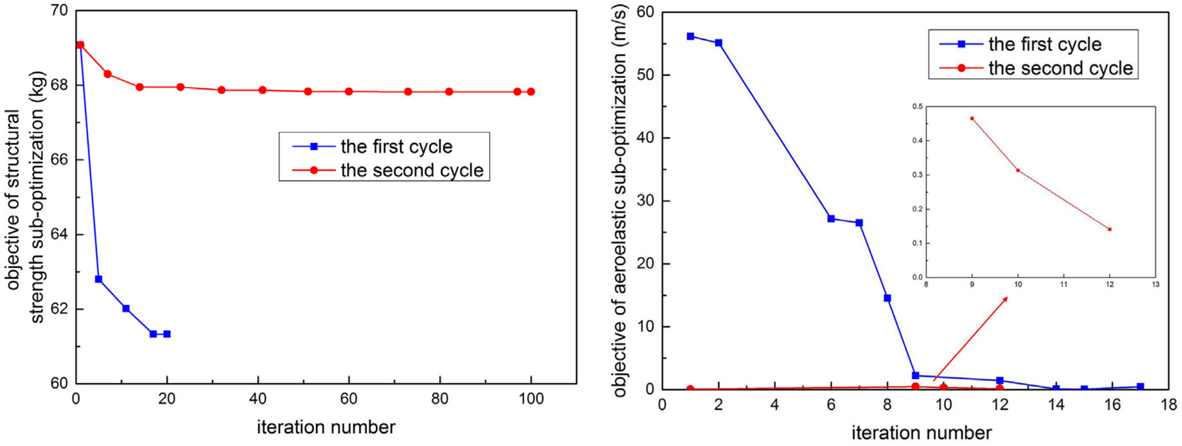

Fig. 8. Iteration history of objectives of sub-optimizations in panel example.

Table 1
Detailed information of stiffened panel.

Length		1000 mm
Width		1000 mm
Thickness		5 mm
Height of stiffeners		40 mm
Boundary restraint		Simply supported on four sides
Material properties	Elastic modulus	210 GPa
	Poisson ratio	0.3
	Density	7850 kg/m^3
Optimization variables		Thickness of stiffeners (along and perpendicular to flow direction)
Optimization objective		Minimum structural weight
Structural strength constraint		$\sigma_{\max} \leq 225$ MPa
Aeroelastic constraint		$V_{cr} \geq 1745$ m/s

Table 2
Translational distances and corrections of strength constraint in panel example.

Cycle number	d_k (mm)	ΔD_k (MPa)	D_k (MPa)
1	4.295	3.4361	3.4361
2	0.0781	0.0532	3.4893

presents number of structural strength and aeroelastic analyses and total time they cost in two method. Although SOACTM increases number of structural strength analyses, it reduces the number of aeroelastic analyses which need much more

Table 3
Comparison of optimization results of two methods in panel example.

Optimization variables	Optimization range/mm	Optimization results/mm		Relative difference
		SOACTM	Traditional optimization method	
x_1	[20, 50]	28.00404228	28.20312528	−0.706%
x_2	[20, 50]	28.00404228	28.20312528	−0.706%

Table 4
Comparison of number of analyses and computational time of two methods in panel example.

Optimization method		Computational time (number of analyses)		Total computational time
		Structural strength analysis	Aeroelastic analysis	
SOACTM	The first cycle	210 s (20)	462.4 s (17)	2048.8 s
	The second cycle	1050 s (100)	326.4 s (12)	
	Total	1260 s (120)	788.8 s (29)	
Traditional optimization method		819 s (78)	2121.6 (78)	2940.6 s

Table 5
Detailed information of ONERA M6 wing.

Span		2000 mm
Root chord		806 mm
Tip chord		216.5 mm
Leading sweepback		30 degrees
Attack angle		2 degrees
Boundary restraint		Root-fixed
Material		Duralumin
Material properties	Elastic modulus	71 GPa
	Poisson ratio	0.33
	Density	2850 kg/m^3
Optimization variables		Thicknesses of spars and ribs
Optimization objective		Minimum structural weight
Structural strength constraint		$\sigma_{\max} \leq 200$ MPa
Aeroelastic constraint		$V_{cr} \geq 300$ m/s

computational cost. From the perspective of total computational time, SOACTM costs 2048.8 s and traditional optimization costs 2940.6 s. Total computational time of SOACTM is 30.33% shorter than that of traditional optimization method. Above all, sequential optimization strategy can improve optimization efficiency while not losing much accuracy.

4.2. ONERA M6 wing

In this example, an ONERA M6 wing with two spars and twenty-one ribs is regarded as optimization object. Span of the wing is 2000 mm. Tip chord and root chord are 216.5 mm and 806 mm respectively. Leading sweepback is 30 degrees and attack angle is 2 degrees. The whole wing is made from duralumin and fixed at root. Detailed information of this wing is listed in Table 5.

Thickness of two spars is regarded as the first optimization variable which is denoted as x_1. Twenty-one ribs are divided into three groups from root to tip, and thicknesses of ribs in these three groups are regarded as three optimization variables which are denoted as x_2, x_3 and x_4. Structural strength constraint is that maximum stress cannot be more than 200 MPa under the condition that flow speed is 200 m/s at sea-level. Critical flutter speed cannot be less than 300 m/s. Initial values of optimization variables are $x_1 = 2$ mm, $x_2 = x_3 = x_4 = 1.5$ mm and convergence threshold is 0.0001 mm. The convergence criterion for structural strength sub-optimization is $|m_{k+1} - m_k| \leq 0.0001$ kg . The convergence criterion for aeroelastic sub-optimization is $|V_{cr,k+1} - V_{cr,k}| \leq 0.1$ m/s .

Geometry model of this wing is constructed in CATIA, and meshed in Patran. Surface and interior of Finite element model of the ONERA M6 wing is presented in Fig. 9. In this example, Nastran is applied for structural response calculation, aerodynamic loads are calculated based on doublet lattice method, critical flutter speed is obtained via $v - g$ method, and NLPQL is applied in solution of both structural strength sub-optimizations and aeroelastic sub-optimizations.

The optimization result is convergent after two cycles in this example. Table 6 lists translational distances and corrections of structural strength constraint in each cycle. Iteration history of optimization variables in structural strength sub-optimization and aeroelastic sub-optimization are shown in Figs. 10 and 11 respectively. Fig. 12 presents the iteration history of structural strength sub-optimization objective, i.e. total mass of this ONERA M6 wing, and aeroelastic sub-optimization objective. Only improved solutions are presented in these figures.

The processor of computer used for numerical calculation is Intel(R) Core(TM) i7-4790 CPU @3.60 GHz 3.60 GHz and RAM is 16.0 GB. Computation time of single structural strength analysis and aeroelastic analysis are 15.80 s and 39.79 s respectively.

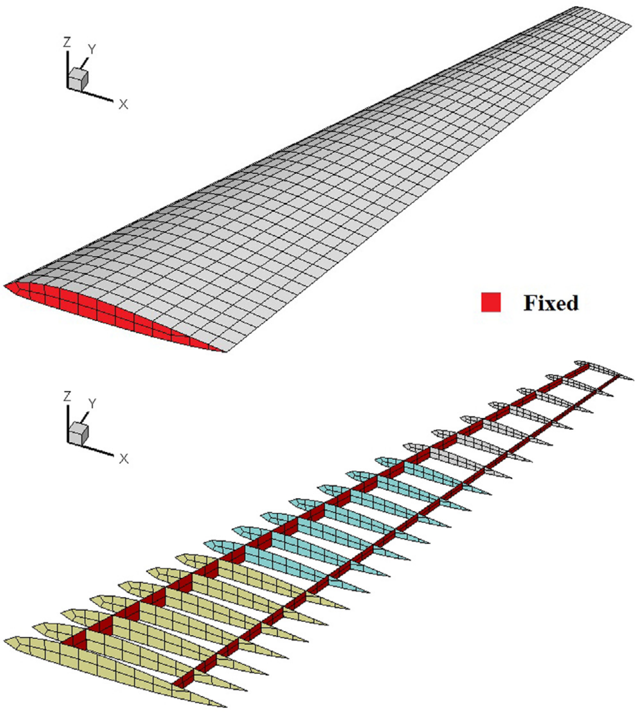

Fig. 9. Finite element model of the ONERA M6 wing.

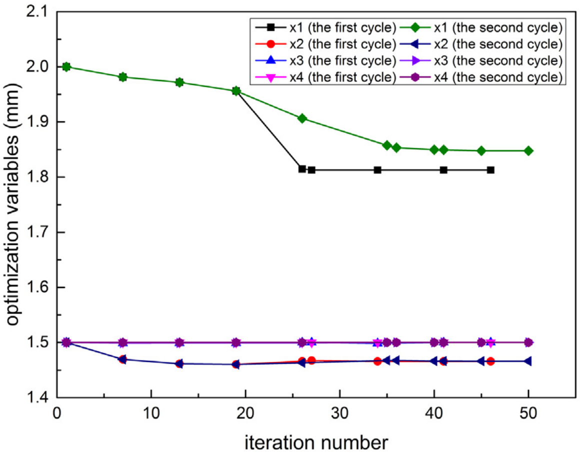

Fig. 10. Iteration history of optimization variables of structural strength sub-optimization in wing example.

Table 6
Translational distances and corrections of strength constraint in wing example.

Cycle number	d_k (mm)	ΔD_k (MPa)	D_k (MPa)
1	0.01555	3.6247	3.6247
2	0.0001	0.0328	3.6575

Then, we compare SOACTM with traditional optimization method to prove that SOACTM is valid in optimization problems of such engineering structures. Results obtained from two methods are listed in Table 7 and we can easily find that they are in accordance. Number of structural strength and aeroelastic analyses and total time they cost in two methods are presented in Table 8. It is obvious that sequential optimization strategy reduces number of both structural strength analyses and aeroelastic analyses. From the perspective of total computational time, SOACTM costs 4620.42 s

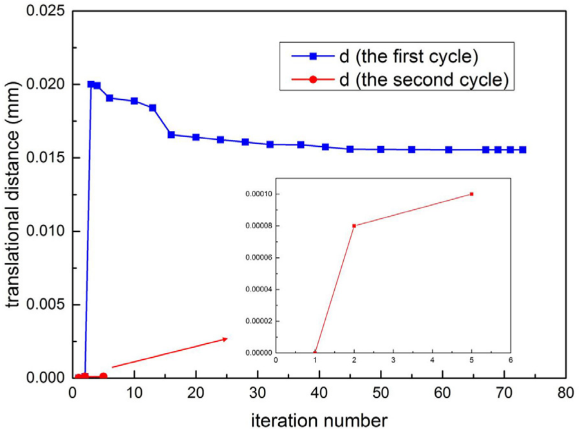

Fig. 11. Iteration history of optimization variable of aeroelastic sub-optimization in wing example.

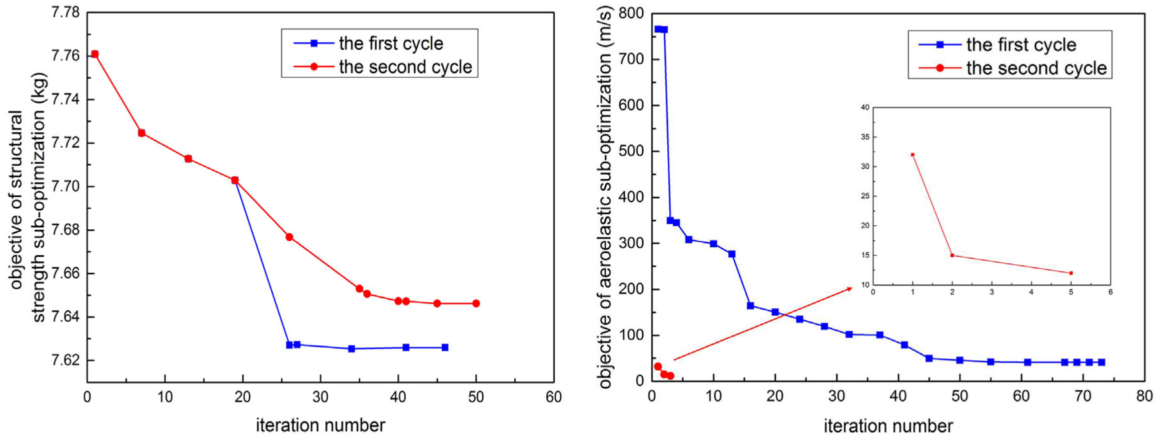

Fig. 12. Iteration history of objectives of sub-optimizations in wing example.

Table 7
Comparison of optimization results of two methods in wing example.

Optimization variables	Optimization range/m	Optimization results		Relative difference
		SOACTM	Traditional optimization method	
x_1	[1.0, 2.0]	1.847888007	1.813095886	1.919%
x_2	[1.0, 1.5]	1.466305535	1.465792190	0.035%
x_3	[1.0, 1.5]	1.5	1.5	0.000%
x_4	[1.0, 1.5]	1.5	1.5	0.000%

and traditional optimization costs 5447.82 s. Compared with traditional optimization method, SOACTM costs 15.20% less time. In conclusion, sequential optimization strategy is also feasible to reduce computational cost for relatively complex engineering structures.

5. Conclusions

This paper presents an efficient comprehensive design method for airplane wings based on sequential optimization strategy. The proposed method is named as SOACTM. SOACTM decouples structural strength analyses and aeroelastic analyses and transform aeroelastic constraint into equivalent structural strength constraint. This paper presents the main idea of detailed procedure of SOACTM. Then, several conclusions were reached based on two examples: one is stiffened panel, the other is ONERA M6 wing. First, we found that SOACTM reduces number of aeroelastic analyses which need much more computational cost than structural strength analyses, although SOACTM may increase the number of structural

Table 8
Comparison of number of analyses and computational time of two methods in wing example.

Optimization method		Computational time (number of analyses)		Total computational time
		Structural strength analysis	Aeroelastic analysis	
SOACTM	The first cycle	726.8 s (46)	2904.67 s (73)	4620.42 s
	The second cycle	790 s (50)	198.95 s (5)	
	Total	1516.8 s (96)	3103.62 s (78)	
Traditional optimization method		1548.4 s (98)	3899.42 s (98)	5447.82 s

strength analyses. We also found SOACTM yields close result while expending less time compared with traditional optimization method. SOACTM expends 30.33% and 15.20% less time than traditional optimization method under the same conditions in these two numerical examples respectively. Above all, SOACTM proposed in this paper yields accurate comprehensive optimization for airplane wings considering structural strength and aeroelastic constraints very efficiently.

Moreover, SOACTM can save much more computation time when applied in more complex cases or using more complex solution algorithms. For design of a large-scale engineering structure, even only once aeroelastic analysis is reduced, the total computational time saving is still considerable.

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