Proceedings of the ASME 2019 38th International
Conference on Ocean, Offshore and Arctic Engineering
OMAE2019
June 9-14, 2019, Glasgow, Scotland, UK

OMAE2019-95651

NON-LINEARLY RESTORING PERFORMANCE AND ITS HYSTERESIS BEHAVIOR OF DYNAMIC CATENARY

Yilun Li
School of Aeronautics Sciences and Engineering, Beijing University of Aeronautics and Astronautics
Beijing, China

Shuangxi Guo
Institute of Mechanics, Chinese Academy of Sciences
Beijing, China

Yue Kong
School of Aeronautics Sciences and Engineering, Beijing University of Aeronautics and Astronautics
Beijing, China

Min Li
School of Aeronautics Sciences and Engineering, Beijing University of Aeronautics and Astronautics
Beijing, China

Weimin Chen*
Institute of Mechanics, Chinese Academy of Sciences
School of Engineering Science, University of Chinese Academy of Sciences
Beijing, China

ABSTRACT

Catenary is increasingly used as mooring-line and riser system as the water depth gets larger due to its lower cost and easier installment. Its dynamic response and restoring performance become more complicated, as the length of the mooring-line become larger, and the structural and fluid dynamics the mooring-line become consequently more obvious. Compared to the quasi-static method where the static restoring force is mainly involved, the dynamic behaviors and its hysteresis of the catenary mooring-line are considered here so as to comprehensively examine the non-linearly restoring performance of mooring-lines. Based on the 3d dynamic vector equations along with the modified FEM simulations, the hysteresis character of the restoring stiffness and the influences of the catenary dynamics on its restoring performance are presented and discussed.

It is found that, principally owing to the damping and inertial effect coming from the fluid and structural dynamics, the restoring force of the mooring-line depends on both the structural displacement and velocity. Moreover, the dynamic stiffness behaves as a hysteresis loop, instead of a curve. Our numerical results show that the energy consumption during one period rises nonlinearly with the increase of the body frequency ω_d *and amplitude* A_0*. And, the influence of nonlinear restoring stiffness on the structural response along with the slack-taut phenomenon caused by structural /hydrodynamic inertia and damping is discussed.*

* Contact author: wmchen@imech.ac.cn

Keywords: dynamic response; catenary mooring-line; dynamic behavior; hysteresis loop

NOMENCLATURE

ω_d	angular frequency
A_0.	amplitude of body motion
T	concentration force
q	distributed force
ρ	density of catenary
A	cross section area
r	position vector
EI	bending stiffness
λ	effective tension
$\boldsymbol{\varepsilon}$	strain
f	hydrodynamic force
C_D	drag coefficient
C_A	added mass coefficient
D	structural diameter
u	structure displacement
V	fluid velocity
$\mathbf{M}$	mass matrix
$\bar{\mathbf{M}}$	added mass matrix
$\boldsymbol{U}$	displacement vector
$\mathbf{C}$	damping matrix
$\mathbf{K}$	stiffness matrix

F external load vector
θ rotation angle

INTRODUCTION

In recent years, more and more ocean energy, oil/gas and wind industries have been developing towards deeper ocean area. The supporting systems of the floating bodies, such as oil rig/production platform and wind turbine, includes catenary mooring-lines, tendon lines and vertical tension legs[1]. Among the three ones, the first one, i.e. catenary mooring-line, is increasingly used due to its lower economic cost and easier installment implementation in deeper water. And, as the floating platform are developed toward deeper water depth, the length of the mooring-line become larger and consequently the dynamics such as the structural inertia and hydrodynamic damping of the mooring-line become more obvious. Moreover, the translating and rotational motions of the floating body should be no less the threshold value, and the economic cost of mooring-lines is supposed to be as low as possible under the condition of required structure strength. Therefore, more reasonable analysis and results of mooring-lines' restoring performance, along with its dynamic behaviors is crucial to safety design of mooring lines along with floating bodies.

There have been fruitful researches of the mooring-line, which are mostly on restoring force (top tension) and floating body stability[2-4]. Quasi-static method is one of the most popular approaches to calculate the static restoring force of catenary. For examples, Qiao[2] presented the static restoring force of a catenary, made up of three wire ropes and chains, with different buoyancy unit weights and structural properties based on classic static catenary theory and piecewise extrapolation. Van Den Boom[3] found that the nonlinearities coming from the geometry, elastic deformation and acting loads can significantly enlarge top tension. Fan[4] studied the mooring line damping for the design of a truncated mooring system using an improved quasi-static method.

However, as water depth and structural length increase, the dynamic characteristics of mooring-line become more significant. Chen[5] calculated the dynamic response of a system including a spar and its mooring-lines based on a linear coupling approach. He pointed out that if the inertial and damping forces of the mooring-line are involved during dynamic response, the top tension would get larger. By now, there have been two kinds of methods, i.e. lumped-mass method and flexible-bar method[6,7], were used to involve the dynamic behavior of catenary. In the lumped-mass method, the mooring-line is modeled by a series of concentrated-mass points connected by linear springs, the inertial force and the added mass coming from the fluid is calculated by the motion of the concentrated-mass points. For the flexible-bar model, the nonlinear dynamic equations of curved flexible bar are build and to be numerically solved so as to model nonlinear structural stiffness and to get large displacement of catenary. Given an ideal assumption of the two ends of catenary being at same level, Zhang[8] studied the nonlinear dynamic response of a catenary. Still, the dynamic behaviors of mooring line need further research to model a practical catenary with less assumptions, and few results of the extreme situations such as slack-taut based on the current approaches are seen.

In this study, the non-linearly restoring performance of catenary mooring-lines, under consideration of its dynamic behaviors, is comprehensively examine based on our 3d dynamic vector equations along with the modified FEM simulations model. Particularly, the hysteresis character of the restoring stiffness and the influences of the catenary dynamics on the structure's performance are presented and discussed. It is found that the restoring force of the mooring-line depends on both the structural displacement and velocity due to the fluid and structural dynamics, and the top-end response can be significantly decreased.

1 The Governing Equations and FEM Simulation of a Dynamic Catenary

The static restoring force and tension distribution of the mooring-line can be calculated according to the classical catenary theory[2], where only the static force of the mooring system is considered. Here, to consider the catenary dynamics along with nonlinear geometry/structural and the fluid dynamics, the dynamic equations based on 3d curved flexible beam approach is employed. The governing equations of a 3d catenary (see Fig.1) in terms of vectors[9] can be written as:

$$-(EIr'')''+(\lambda r')'+q=\rho A r' \tag{1}$$

where λ is a scalar variable represents the effective tension, r is the position vector of the catenary, q is the distribute force, ρ and A are structural mass density and area respectively, EI is the bending stiffness. And the deformation equation is:

$$r'\cdot r'=(1+\varepsilon)^2 \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the strain of the catenary. If the value of the bending moment in Eq.(1) is zero, we will have the dynamic equation of a catenary of which the external loads include the gravity, buoyancy and hydrodynamic forces.

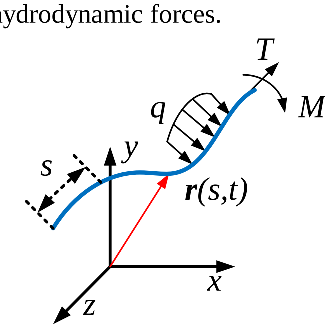


FIGURE 1: CATENARY MOORING-LINE MODELS

Generally, the hydrodynamic force acted on per unit structure length can be expressed by the Morison formula[10] as:

$$f=\frac{1}{2}C_D\rho D\left|V-\dot{u}\right|(V-\dot{u})+C_A\frac{\pi D^2}{4}\rho(\dot{V}-\ddot{u})+\frac{\pi D^2}{4}\rho\dot{V} \tag{3}$$

where D and u are the structural diameter and displacement respectively. V is the fluid velocity. Combing Eqs.(1), (2) and (3), we have a group of nonlinear equations of the dynamic catenary, and a numerical simulation based on FEM is used to solve the dynamic equations.

The catenary is uniformly divided into ***N*** two-node Euler beam elements. For representativeness and simplicity, only the translation displacement in ***x-y*** plane [u_i,v_i] and one rotation around ***z*** axis θ_i, of per node, are considered. Then the governing equation of the catenary with many degrees of freedom can be written as follow:

$$(\mathbf{M}+\bar{\mathbf{M}})\ddot{\mathbf{U}}+\mathbf{C}\dot{\mathbf{U}}+\mathbf{K}\mathbf{U}=\mathbf{F} \qquad (4)$$

where **M** and $\bar{\mathbf{M}}$ are respectively the structure mass matrix and the added mass matrix. **C** is the structure damping matrix. **K** is the stiffness matrix. **F** is the hydrodynamic force and gravity acted on the catenary. **U** is the displacement vector. In order to model simultaneously its original catenary shape and the large rotation/translation flexibilities the rotation motion between two neighboring beam elements is released in our model, it means that the rotation angles of the two beam elements are no longer consistent with each other at same grid. Subsequently, the system rotational degrees of the freedom $\boldsymbol{\theta}$ would double as $\boldsymbol{\theta},\boldsymbol{\theta}'$ because of the additional rotation angle. The displacement vector of beam element changes from the original form:

$$\mathbf{U}_i=[u_i,v_i,w_i,\theta_i,u_{i+1},v_{i+1},w_{i+1},\theta_{i+1}]^T \quad i=1,\cdots,N \qquad (5)$$

as

$$\begin{aligned}\mathbf{U}_i&=[u_i,v_i,w_i,\theta_i,u_{i+1},v_{i+1},w_{i+1},\theta_{i+1}]^T & i&=1,N\\ \mathbf{U}'_i&=[u_i,v_i,w_i,\theta_i,\theta'_i,u_{i+1},v_{i+1},w_{i+1},\theta_{i+1},\theta'_{i+1}]^T & i&=2,\cdots,N-1\end{aligned} \qquad (6)$$

Then, given the statically indeterminate characteristics along with stronger stiffness singularity of the system, here the original shape and top tension based on traditional static catenary theory is used as the definite conditions so as to eliminate the singularity of the stiffness matrix. To run the dynamic response analysis, the Newmark method is employed here so as to adjust the distribution of the structural acceleration and the nonlinearity of the catenary during the integration range by properly changing the integration parameters.

2 Dynamic Response and Restoring Performance of the Catenary Mooring-lines

The structural and geometrical parameters of the catenary are listed in Table 1. The dynamic response of the catenary under surge motion of the top-end body is analyzed based on our numerical simulations, i.e. the displacement, velocity and tension of the catenary along structural span are calculated. Then the restoring performance and its hysteresis are discussed.

TABLE 1: THE GEOMETRICAL AND MATERIAL PARAMETERS OF THE CATENARY

Geometrical	Value
Length	800m
Initial horizontal projection	706m
Initial vertical projection	350m
Diameter	0.19m
Young's modulus	210GPa
Density	2513kg/m3
Poisson's ratio	0.3

2.1 Top Tension and Displacement along Catenary

The typical dynamic response of the catenary for case of a regular motion, i.e. 0.05Hz frequency and 5m surge amplitude, of the floating wind turbine (FWT) is shown in Fig. 2a. In Fig.2a, the top tension of the quasi-static method is also plotted as a comparison. It is seen that either the value of the peaks or the trough get extremer than, the maximum tension increases by 30% and the gap value between the peak and trough is about 3 times of the quasi-static ones owing to the inertial and damping effects, while the period of dynamic tension is consistent with that of the top-end surge.

As the motion of the top-end gets larger, the top tension response under the condition of the top movement with 0.1Hz frequency and 5m amplitude is give in Fig.2b. There is an abrupt increase of the dynamic tension while the minimum tension almost reaches zero value, which means slack-taut happens. In this case, the maximum tension is around three times of the static method and even the peak-trough value is about ten times of the static method.

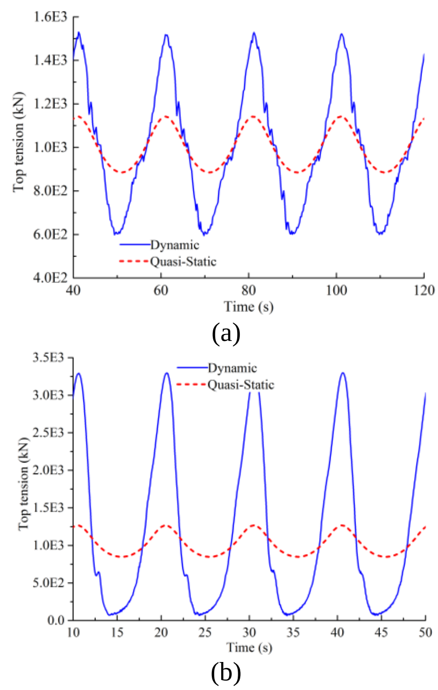
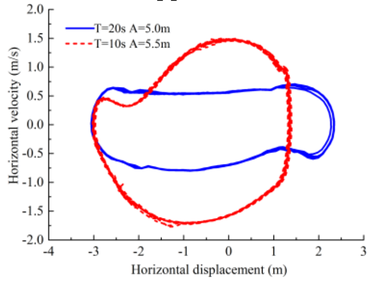
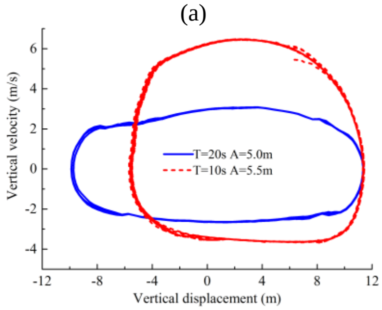


FIGURE 2: TOP TENSION RESPONSE OF THE CATENARY (a) TOP-END SURGE AMPLITUDE=4m, PERIOD=20s (b) TOP-END SURGE AMPLITUDE=5.5m, PERIOD=10s

The displacements and velocity change significantly as the motion of the top-end gets larger, see Fig.3 where the phase track of the catenary middle point. It is noted that the center of the blue curve, corresponding to smaller motion of the top-end (T=20s, A=5.0m) is close to the zero position, that indicates the dynamic balance position deviates from the static one. While the center of the red curve, i.e. the top-end motion of T=10s, A=5.5m, deviate from the zero position, and the shape of the track phase looks no longer like elliptical one. Moreover, in Fig.3b, the vertical velocity almost keeps being a constant value, i.e. the minimum velocity, as the displacement change from its positive peak to negative trough. That means owing to the additional dynamic behavior of the catenary, the structural initial/damping forces and fluid drag force can balance the structural gravity which is supposed to principally cause the structural tension, and consequently, slack would happen if no tension is caused.

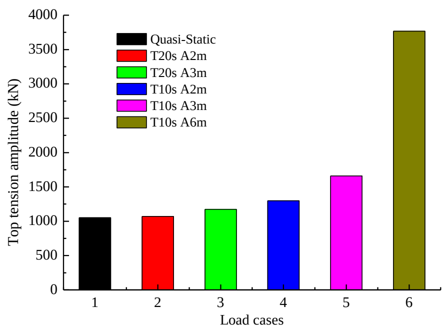


(a)

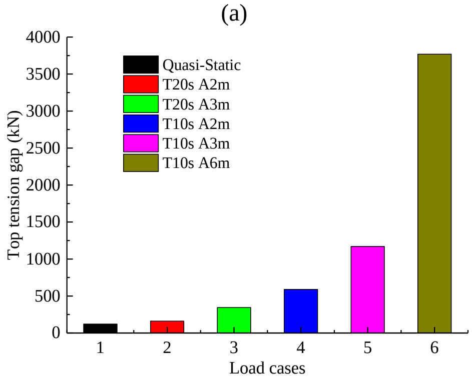


(b)

FIGURE 3: PHASE TRACK OF THE VELOCITY VERSUS DISPLACEMENT OF THE MIDDLE POINT (a) HORIZONTAL MOTION (b) VERTICAL MOTION



(a)



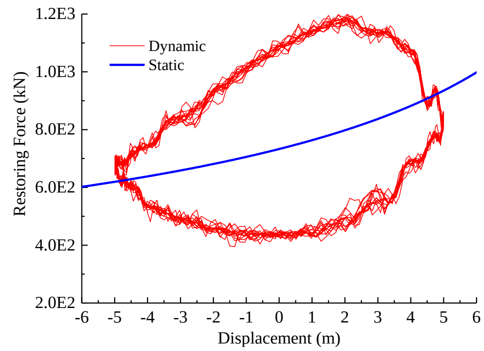
(b)

FIGURE 4: TOP TENSION RESPONSES OF THE CATENARY MOORING-LINE UNDER DIFFERENT CONDITIONS (a) TENSION AMPLITUDE OF THE CATENARY (b) TENSION GAP OF THE CATENARY

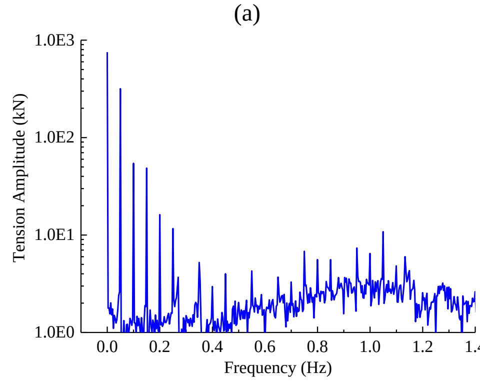
The maximum top tension and tension gap at different amplitudes and frequencies of the top-end motions are given in Fig.4. It shows that the values of the maximum top tension (and the tension gap) gets larger, e.g. up to 3.5 times of the static value particularly for case of snap, as the amplitude and/or frequency of the top end get larger.

2.2 The Non-Linear Restoring Stiffness and Its Hysteretic Behaviors

As presented above, the mooring-line dynamics could produce an increase of top tension along with tension amplitude difference. In fact, because of involvement of catenary dynamics, i.e. the inertial and damping effects, the restoring stiffness may change too. The horizontal restoring stiffness will be examined here, which are calculated under conditions of different top-end amplitudes, i.e. A=7, 5, 4 and 3m, and different periods, i.e. T=5, 10, 20 and 40s. The selected results are presented in Fig.5 and 6.



(a)



(b)

FIGURE 5: RESTORING CHARACTERISTICS OF SINGLE MOORING-LINE (a) RESTORING STIFFNESS OF THE MOORING LINE (b) SPECTRUM OF THE RESTORING FORCE

Fig.5a shows the horizontal restoring stiffness curve of the catenary at 20s period and 5m amplitude, and the static stiffness is also plotted as a comparison. Interestingly, the dynamic stiffness curve shows that the top tension is no longer linearly related to only the top-end displacement as it does for case of quasi-static scenario, but, notably, it depends on both the top-end displacement and velocity approximately in a way of approximately ellipse loop, which is called hysteresis loop. And differently from the static stiffness, the dynamic restoring force does not get its maximum value at the maximum displacement but at a smaller displacement, that produces a smaller dynamic stiffness. The hysteresis character of the dynamic stiffness is mainly due to the damping effect coming from the structure and fluid of the mooring-line. The spectrum plot, shown in Fig.5b, indicates that the peak values at frequencies of odd times of the excitation frequency is much larger than others.

It is found that the energy consumption during a period get larger as the amplitude (and/or the frequency) increases. In other words, the hysteresis effect of the dynamic stiffness gets more obvious as the amplitude and/or frequency increase, as shown in Fig.6. The energy consumption and mooring-line top tension under different load cases are presented in Fig.7.

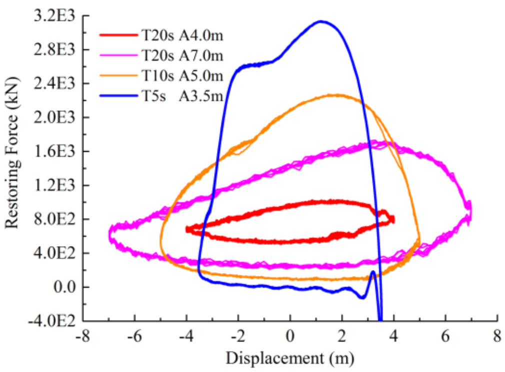
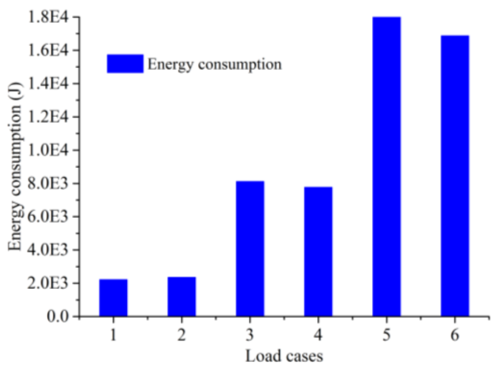
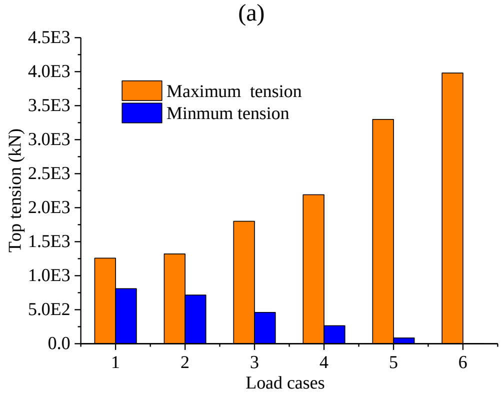


FIGURE 6: RESTORING LOOPS AT DIFFERENT FREQUENCY AND AMPLITUDE



(a)



(b)

FIGURE 7: ENERGY CONSUMPTION AND TOP TENSION RESPONSES UNDER DIFFERENT TOP-END AMPLITUDES AND FREQUENCIES (a) ENERGY CONSUMPTION (b) TOP TENSION

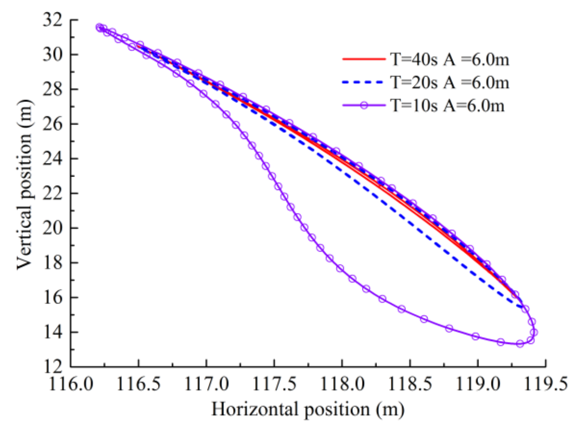
The load cases of the top-end motion are list in Table 2. It can be seen in Fig. 7b that, the value of energy consumption from Case1 to Case 6 gradually increases, except for Case 1 and Case 6, where the minimum top tension approaches to zero, or slack happens. Then we may say that, the energy consumed by hysteretic damping in one cycle may get smaller as the mooring-line becomes slack.

TABLE 2: THE LOAD CASES OF THE TOP-END MOTION

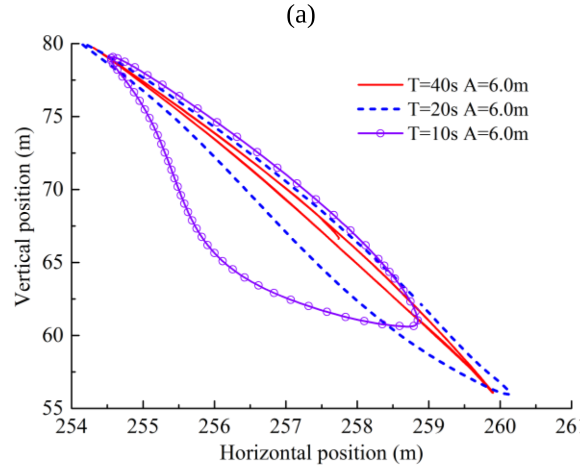
Load cases	1	2	3	4	5	6
Period/s	40	20	20	10	10	5
Amplitude/m	6	4	6	4	6	4

2.3 The Influence of Hysteresis Characteristics on the Catenary Response

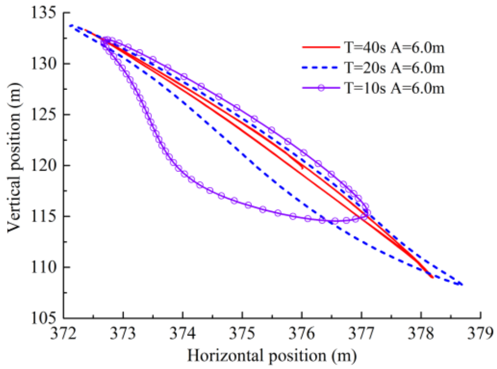
Fig.8 shows the trajectories of different positions along the mooring-line while the motion frequency changes (the amplitude of the top-end is 6m). It shows that the trajectory loop becomes more obvious and the vertical/horizontal displacements become smaller owing to the hysteresis characteristics of the mooring-line, as the motion of the top-end gets faster (or the period gets smaller). The trajectory loop becomes more obvious as the motion of catenary points, e.g. point 2 and point 3, get larger, because of the larger damping effect coming from larger catenary motion.



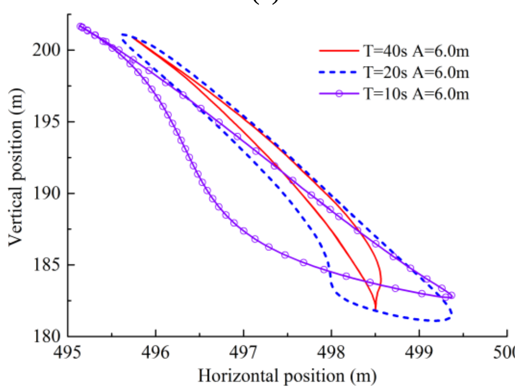
(a)



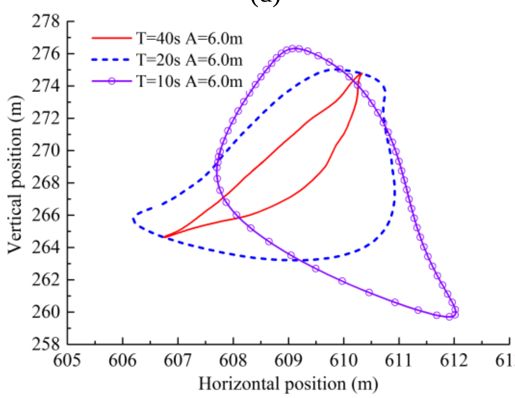
(b)



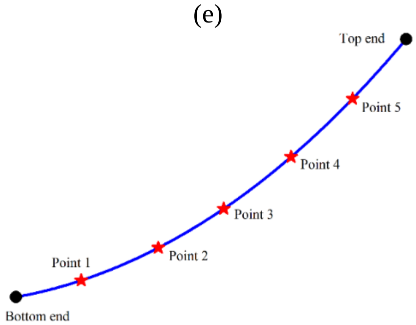
(c)



(d)



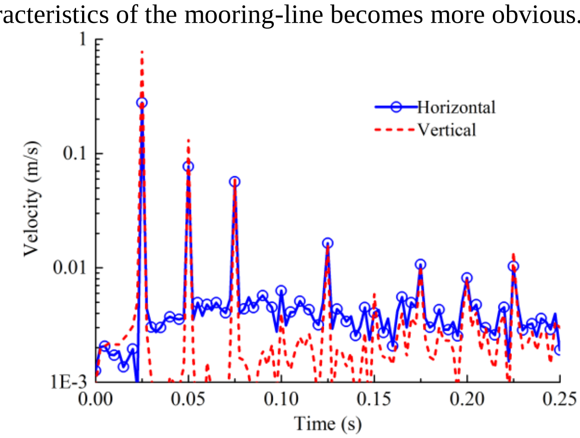
(e)



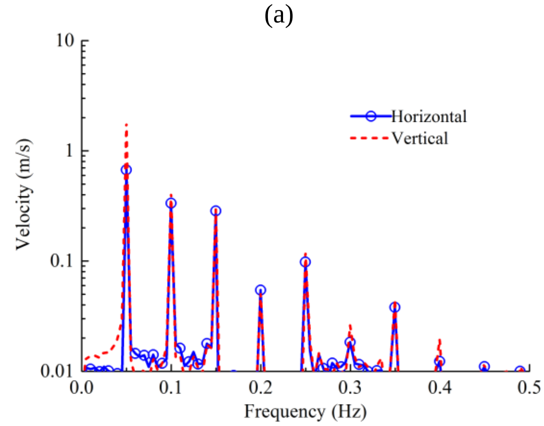
(f)

FIGURE 8: TRAJECTORY OF THE CATENARY (a) TRAJECTORY OF POINT 1 (b) TRAJECTORY OF POINT 2 (c) TRAJECTORY OF POINT 3 (d) TRAJECTORY OF POINT 4 (e) TRAJECTORY OF POINT 5 (f) AXIAL LOCATION OF POINT 1-5

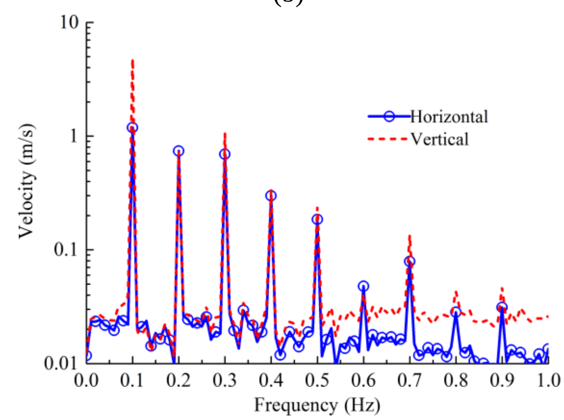
Particularly, the hysteresis of point 5 appears to be the most obvious, see Fig. 8e. And, Fig.9 shows the velocity spectrum of Point 5. It can be seen that, as the motion period decreases, the moving speed of Point 5 gradually increases. The peak values at higher frequencies also gradually increases. If comparing the peak values at double and triple excitation frequencies, we can see that peak value at triple excitation frequency gets larger, or even larger than that of double excitation frequency as the period of top-end motion decreases. That also means that the hysteresis characteristics of the mooring-line becomes more obvious.



(a)



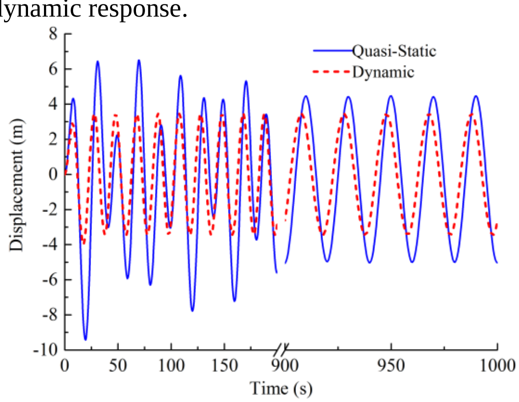
(b)



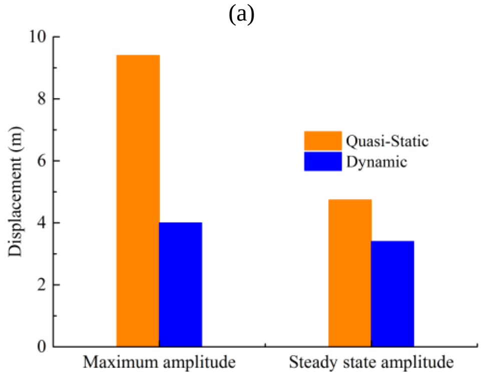
(c)

FIGURE 9: VELOCITY SPECTRUM OF THE CATENARY POINT 5 (a) TOP-END PERIOD 40s (b) TOP-END PERIOD 20s (c) TOP-END PERIOD 10s

In order to examine the influence of the mooring-line hysteresis on the motion the top-end, the dynamic response of the top-end is calculated and compared with the quasi-static method. The period of the excitation force is 20s, and the first-order natural period of the system is 30s. The displacement of the top-end is shown in Fig.10. It is noted that the displacement significantly decreases due to the hysteresis of the mooring-line. For examples, the maximum amplitude decreases from 9.4m to 4.0m, or by more than 50%, and the amplitude decreases from 4.7m to 3.4m, or by approximately 28% during the steady phase of the dynamic response.



(a)



(b)

FIGURE 10: COMPARISON OF THE TOP-END RESPONSES BETWEEN THE DYANAMIC AND STATIC RESULTS (a) TIME HISTORY OF TOP-END DISPLACEMENT (b) THE MAXIMUM AND STEADY STATE AMPLITUDE

3 Conclusion

The non-linearly restoring performance of catenary mooring-lines under consideration of its dynamic effects is comprehensively examined based on our modified flexible-beam model combining with the FEM simulations. By our numerical simulations, the hysteresis characteristics of the restoring stiffness and the influences of the catenary dynamics on the catenary and top-end responses are presented and discussed. Based our numerical results we draw the following conclusion:

Owing to the damping and inertial effect coming from the fluid and structural dynamics, the restoring force of the mooring-line depends on both the structural displacement and velocity. The dynamic stiffness behaves as a hysteresis loop, instead of a static line. The energy consumption during one period get larger as the amplitude (and/or the frequency) increases. The displacement significantly decreases due to the hysteresis of the mooring-line. The maximum amplitude decreases by around 50%, and the amplitude decreases by 28% during the steady phase of the dynamic response.

ACKNOWLEDGEMENTS

The authors of this paper would like to thank the financial supports provided by the Strategic Priority Research Programme of CAS (Grant No. XDB100203 and XDA100201) and the National Natural Sciences Foundation (Grant No. 11372320).

REFERENCES

[1] Musial, Walt, Sandy Butterfield, and Andrew Boone. "Feasibility of floating platform systems for wind turbines." 23rd ASME Wind Energy Symposium, Reno, NV. 2004.
[2] Qiao Dongsheng, Ou Jinping. Static analysis of a deepwater catenary mooring system, Ship&Ocean Engineering, 2009, Vol.38(2):120-124.
[3] Van Den Boom, H.J.J. Dynamic behaviour of mooring lines. Proceedings of Behaviour of Offshore Structures, 1985:359-368.
[4] Fan T, Qiao D, Yan J, et al. An improved quasi-static model for mooring-induced damping estimation using in the truncation design of mooring system[J]. Ocean Engineering, 2017, 136:322-329.
[5] Chen X, Zhang J, Ma W. On dynamic coupling effects between a spar and its mooring lines [J]. Ocean Engineering. 2001, 28(7): 863-887.
[6] Vassalos D, Huang S. Dynamics of small-sagged slack-taut marine cables [J]. Computers & structures, 1996, 58(3): 557-562.
[7] Huang S, Vassalos D. A numerical method for predicting snap loading of marine cables [J]. Applied Ocean Research, 1993, 15(4): 235-242.
[8] Zhang S, Tang Y, Liu X. Analysis of Nonlinear Dynamic Response of Mooring Lines in Deepwater[C]. ASME 2010 29th International Conference on Ocean, Offshore and Arctic Engineering. American Society of Mechanical Engineers, 2010: 489-494.
[9] Guo S, Li Y, Chen W, et al. Non-Linearly Restoring Performance of Catenary Mooring-Line Under Consideration of Its Dynamic Behaviors[C], ASME 2017, International Conference on Ocean, Offshore and Arctic Engineering. 2017:V07AT06A021.
[10] Mavrakos S A, Chatjigeorgiou J. Dynamic behaviour of deep water mooring lines with submerged buoys [J]. Computers & structures, 1997, 64(1): 819-835.