

THE MECHANISM OF THREE-DIMENSION STEADY SHOCK WAVE INTERACTION

Chun WANG*

- 1) State Key Laboratory of High-temperature Gasdynamics, Institute of Mechanics of Chinese Academy of Sciences, Beijing, China
- 2) School of Engineering Science, University of Chinese Academy of Sciences, Beijing, China

Ruixin YANG

- 1) State Key Laboratory of High-temperature Gasdynamics, Institute of Mechanics of Chinese Academy of Sciences, Beijing, China
- 2) School of Engineering Science, University of Chinese Academy of Sciences, Beijing, China

Zonglin JIANG

- 1) State Key Laboratory of High-temperature Gasdynamics, Institute of Mechanics of Chinese Academy of Sciences, Beijing, China
- 2) School of Engineering Science, University of Chinese Academy of Sciences, Beijing, China

ABSTRACT

The problem of three-dimensional steady shock wave interaction is a key issue for supersonic and hypersonic corner flow. Due to the complexity of shock configurations, there is no analytical theory to such problem and the mechanism of three-dimensional shock waves and boundary layer interaction has not been clearly known. In this paper, an analytical approach to the problem of three-dimensional steady shock wave interaction was exhibited to analytically interpret the mechanism of three-dimensional interaction of two oblique planar shock waves. The results showed that the problem of three-dimensional steady shock wave interaction could be transformed to that of two moving shock wave interaction in two-dimensional plane, and there are various interaction configurations such as regular interaction, Mach interaction and weak interaction. The mechanism of three-dimensional shock wave interaction is helpful to understand the complex flow mechanism induced by three-dimensional shock wave and boundary layer in hypersonic flow. The interaction of three-dimensional shock waves and boundary layer plays important role in the complex flow feature in hypersonic rudder region. The contact surface induced by three-dimensional shock waves represents a local jet. When the flow jet impinges on the boundary layer of wall surface, the jet makes the boundary layer thinner and will inevitably cause local heat flux peak. The interaction configurations of three-

dimensional shock wave play important role in the gasdynamic heating mechanisms of hypersonic complex flow.

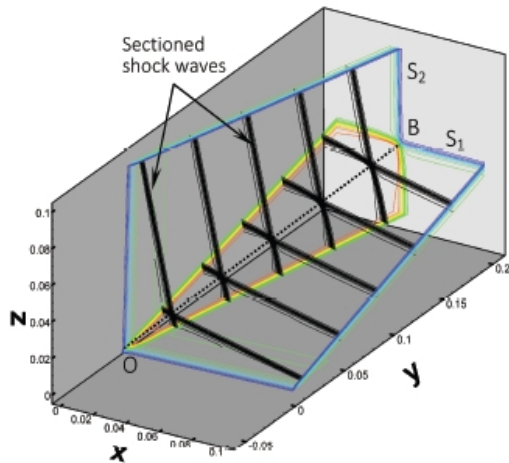
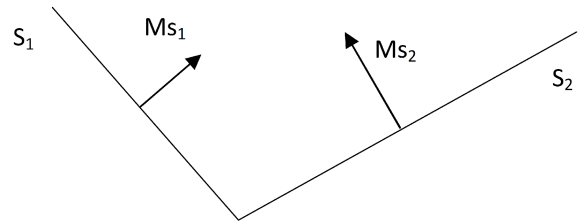
1. INTRODUCTION

Hypersonic heating is a very critical issue for hypersonic aviation. The stagnation of hypersonic flow on the surface of aircraft results in the high-temperature boundary layer which heats the surface of the aircraft and may cause the failure of structures. Also, the mechanism of hypersonic heating is a fundamental problem in the field of hypersonic gasdynamics, the correlation theory of hypersonic heating is of importance for the practical design of aircraft. It is noticed that the correlation of hypersonic heating to surface pressure has been widely used in the prediction of large-area gasdynamic heating. On the other hand, the correlation of hypersonic heating in the complex interaction region is difficult and there is no correlation theory, especially in the interaction regions of body and wing, body and rudder. Until now, there is plenty of work on this topic due to its importance in engineering, such as the experimental work of Michael K. Smart et al. in the University of Queensland^[1]. The research work on the mechanism of hypersonic three-dimensional shock wave interaction will help to understand the complex flow phenomena of shock wave and boundary layer interaction, which inevitably dominates the mechanism of hypersonic aerodynamic heating.

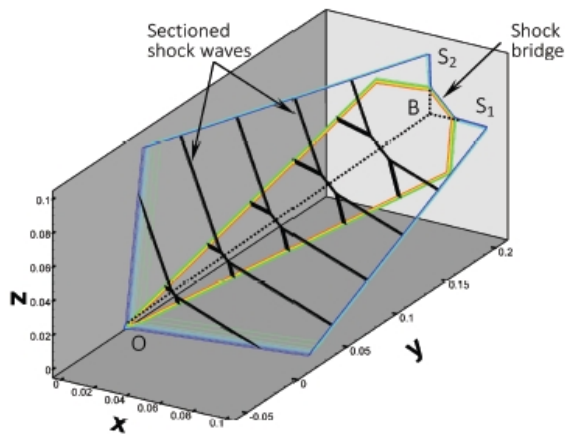
2. THE METHODS OF THEORETICAL ANALYSIS AND NUMERICAL SIMULATION

For the mechanism of three-dimensional shock wave interaction in the hypersonic interaction region, we built a theoretical approach to study the shock configuration. By transforming one spatial dimension to temporal dimension, the steady interaction of three-dimensional planar shock waves can be regarded as the interaction of two moving shock waves in a two-dimensional plane. By the shock-polar theory and shock dynamics theory, the shock configuration of three-dimensional shock waves interaction in hypersonic flow can be analyzed. There are several basic shock configurations exist in the hypersonic interaction region, such as Regular Interaction (RI), Mach interaction (MI) and Weak Shock Interaction(WSI). With different shock configurations, different flow mechanisms exist and influence the local flow feature in hypersonic interaction region.

Similarly, the interaction of three-dimensional steady shock waves shown in Fig.1 can be regarded as the interaction of two moving planar shock waves, these two moving planar shock waves propagates in their normal direction respectively. So, the problem of three-dimensional steady shock wave interaction can be theoretically transformed into that of two moving shock wave interaction in a two-dimensional plane, as shown in Fig.2^[2].



(a) Three-dimensional Regular Interaction



(b) Three-dimensional Mach Interaction

Fig.1 The shock configurations of three-dimensional steady interaction of two oblique shock waves

It is known that two-dimensional oblique shock wave can be regarded as a moving shock wave propagating in quiescent

media. Similarly, the interaction of three-dimensional steady shock waves shown in Fig.1 can be regarded as the interaction of two moving planar shock waves, these two moving planar shock waves propagates in their normal direction respectively. So, the problem of three-dimensional steady shock wave interaction can be theoretically transformed into that of two moving shock wave interaction in a two-dimensional plane, as shown in Fig.2^[2].

There are various shock configurations for the interaction of two moving shock waves. For theoretical analysis, regular interaction is first assumed. In the case of regular interaction, such a problem can be transformed into a full two-dimensional steady problem, so the shock-polar theory can be used. As shown in the Fig.3, the polar curves of two reflected shock waves R_1 and R_2 intersect at point $(3^W, 4^W)$ and point $(3^S, 4^S)$, the solution responding to point $(3^W, 4^W)$ is the most possible solution to the regular interaction of two shock waves.

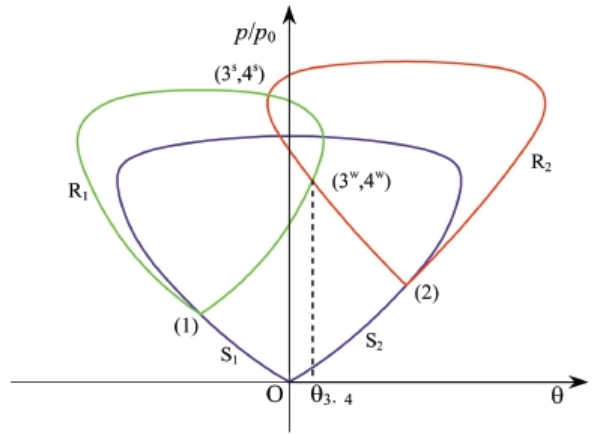


Fig.3 Shock-polar theory for shock regular interaction

If the polar curves of two reflected shock waves R_1 and R_2 do not intersect with each other, as shown in Fig.3, Mach interaction of two moving shock waves will take place. In the case of Mach interaction, the three-dimensional shock configuration can be illustrated in Fig.1b. It must be noticed that if Mach interaction of two planar shock waves takes place, the problem cannot be transformed into a steady problem due to the Mach stem will grow up infinitely.

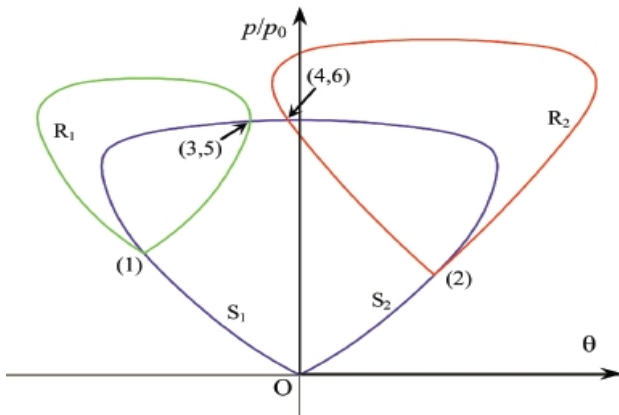


Fig.4 Shock-polar theory for shock Mach interaction

For the theoretic analysis to the Mach interaction of two moving shock waves, the shock dynamics method is valid. As shown in Fig.5, if a virtual wall is introduced, the problem of two moving shock waves interaction can be transformed into that of the reflections of two moving shock waves on the virtual wall. The parameters of the Mach stem including both the moving Mach number and the incline angles of the virtual wall, the problem can be solved with the following equations^[3,4].

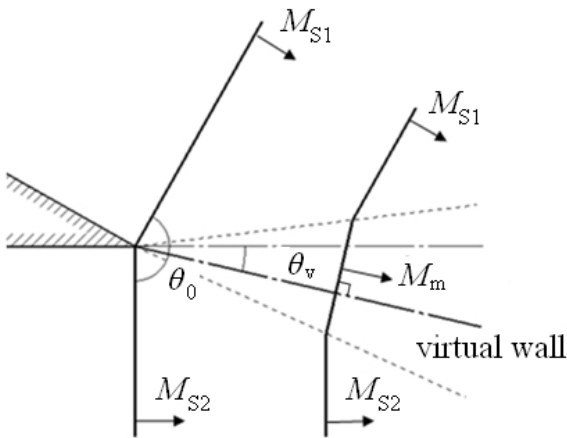


Fig.5 Shock dynamics approach to the shock Mach interaction

$$\tan \theta_v = \left(\frac{M_m}{M_{S2}} \right) \frac{\left[1 - \left(\frac{M_{S2}}{M_m} \right)^2 \right]^{\frac{1}{2}} \left\{ 1 - \left[\frac{f(M_m)}{f(M_{S2})} \right]^2 \right\}^{\frac{1}{2}}}{1 + \frac{f(M_m) M_m}{f(M_{S2}) M_{S2}}}$$

$$\tan(\pi - \theta_0 - \theta_v) = \left(\frac{M_m}{M_{S1}} \right) \frac{\left[1 - \left(\frac{M_{S1}}{M_m} \right)^2 \right]^{\frac{1}{2}} \left\{ 1 - \left[\frac{f(M_m)}{f(M_{S1})} \right]^2 \right\}^{\frac{1}{2}}}{1 + \frac{f(M_m) M_m}{f(M_{S1}) M_{S1}}}$$

After determining the parameters of the moving Mach stem, the reflected shock waves can be calculated further. A validation of Mach interaction of three-dimensional shock waves to the analytical method is shown in Fig.6, the experimental data was reported by West in 1972^[5]. Our results of theoretical analysis and computational fluid dynamics (CFD) simulation agree well with the experimental data in the case of three-dimensional Mach interaction of shock waves.

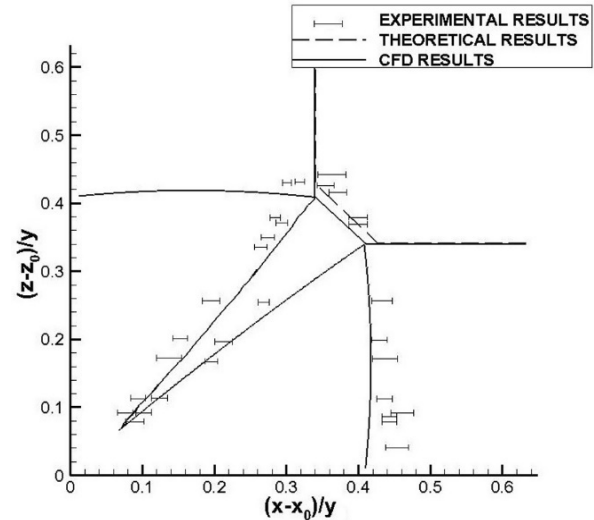


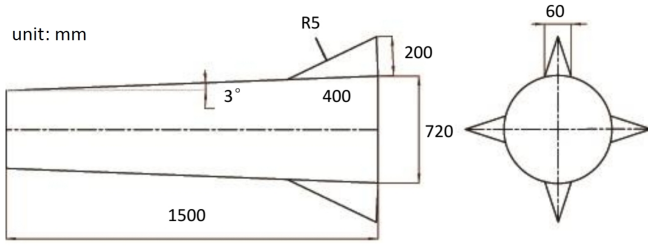
Fig.6 The validation of theoretical results to experimental and CFD results

With the theoretical analysis to the mechanism of three-dimensional shock waves interaction, various shock configurations exist in the supersonic or hypersonic corner flow, such as regular interaction, Mach interaction and complex shock interaction, which have been reported in ref.[2].

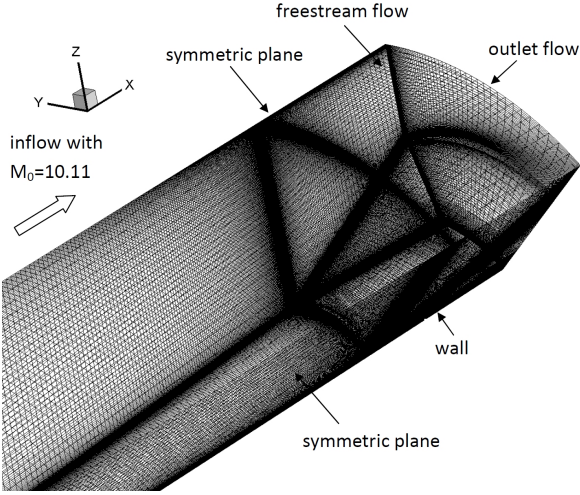
3. THE FLOW MECHANISM IN THE HYPERSONIC INTERACTION REGION

In most practical cases, the interaction of shock waves involves the boundary layer of hypersonic aircraft and the shock wave shapes are not planar. The analysis to such complex flow is difficult but the knowledge of three-dimensional shock waves interaction helps to understand the basic flow structures far from the boundary layer of corner flow.

For discussion of the role of three-dimensional shock wave interaction on hypersonic gasdynamic heating, a numerical simulation is conducted for studying the flow feature of a hypersonic interaction region similar to the experimental model of University of Queensland^[1]. The sketch of the hypersonic rudder model for simulation is shown in Fig.7a. Only 1/8 flowfield with half of the upper rudder is simulated for investigating the flow mechanism induced by shock waves and boundary layer interaction. The cases of 0° and -20° attack angles are concerned. The three-dimensional structured meshes are shown in Fig.7b. The total meshes are about 2.2 million and the minimal mesh size at the bottom of boundary layer is about 0.5mm.



(a) The aircraft model with four rudders



(b) The local meshes for simulation

Fig.6 The rudder model and meshes for numerical simulation

We solve the reactive Navier-Stocks equations as follows:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} + \frac{\partial \mathbf{G}}{\partial z} = \frac{\partial \mathbf{E}_v}{\partial x} + \frac{\partial \mathbf{F}_v}{\partial y} + \frac{\partial \mathbf{G}_v}{\partial z} + \mathbf{S}$$

where the primitive and conservative vectors are expressed as:

$$\mathbf{Q} = [\rho, \rho u, \rho v, \rho w, \rho E, \rho f_1, \dots, \rho f_{ns}, \rho k, \rho \varepsilon]^T$$

$$\mathbf{E} = [\rho u, \rho u^2 + p, \rho uv, \rho uw, \rho uE, \rho u f_1, \dots, \rho u f_{ns}, \rho uk, \rho u \varepsilon]^T$$

$$\mathbf{F} = [\rho v, \rho uv, \rho v^2 + p, \rho vw, \rho vE, \rho v f_1, \dots, \rho v f_{ns}, \rho vk, \rho v \varepsilon]^T$$

$$\mathbf{G} = [\rho w, \rho uw, \rho vw, \rho w^2 + p, \rho wE, \rho w f_1, \dots, \rho w f_{ns}, \rho wk, \rho w \varepsilon]^T$$

and the viscous vectors are expressed as:

$$\mathbf{E}_v = [0, \tau_{xx}, \tau_{xy}, \tau_{xz}, u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + q_x,$$

$$\rho D_1 f_{1x}, \dots, \rho D_{ns} f_{nsx}, (\mu + \frac{\mu_T}{\sigma_k}) k_x, (\mu + \frac{\mu_T}{\sigma_k}) \varepsilon_x]^T$$

$$\mathbf{F}_v = [0, \tau_{xy}, \tau_{yy}, \tau_{yz}, u\tau_{xy} + v\tau_{yy} + w\tau_{yz} + q_y,$$

$$\rho D_1 f_{1y}, \dots, \rho D_{ns} f_{nsy}, (\mu + \frac{\mu_T}{\sigma_k}) k_y, (\mu + \frac{\mu_T}{\sigma_k}) \varepsilon_y]^T$$

$$\mathbf{G}_v = [0, \tau_{xz}, \tau_{yz}, \tau_{zz}, u\tau_{xz} + v\tau_{yz} + w\tau_{zz} + q_z,$$

$$\rho D_1 f_{1z}, \dots, \rho D_{ns} f_{nsz}, (\mu + \frac{\mu_T}{\sigma_k}) k_z, (\mu + \frac{\mu_T}{\sigma_k}) \varepsilon_z]^T$$

and chemical reaction source vectors are expressed as:

$$\mathbf{S} = [0, 0, 0, 0, 0, \omega_1, \dots, \omega_{ns}, S_k, S_\varepsilon]^T$$

In above equations, ρ , p , E are the density, pressure and total energy of the fluid. u , v , w are the velocity components in x , y and z directions. f is the mass fraction of species. k and ε are energy and dissipation rate of turbulence. τ and q are stress tensor and heat flux. μ and μ_T are laminar and turbulent viscous coefficients. σ_k and σ_ε are constants of turbulence model.

High-temperature chemical reactions are concerned and 7-species and 6-step air chemical reactions are involved. The species includes N_2 , O_2 , N , O , NO , NO^+ and e ., as shown in Table.1. The above air reaction model can be referred to [6].

Table.1 High-temperature reaction model of Air

$N_2 + M = 2N + M$
$O_2 + M = 2O + M$
$NO + M = N + O + M$
$NO + O = O_2 + N$
$N_2 + O = NO + N$
$N + O = NO^+ + e$

The vibrational nonequilibrium effect is considered by solving the following dynamic equation[7]:

$$\frac{de_{vib}}{dt} = \frac{1}{\tau_{vib}} (E_{vib}^* - e_{vib})$$

where e_{vib} is the vibrational energy and E_{vib} is the vibrational energy in equilibrium state, τ_{vib} is the vibrational relaxation time.

In the numerical simulations, reactive Navier-Stocks equations are solved by the 2nd TVD scheme, the k - ε two-equation turbulence model is adopted and turbulence is forced to transition in the upstream of rudder. The boundary conditions for simulation are selected as: 1) the inflow condition is given hypersonic freestream condition with Mach number $M_0=10.11$, static pressure $p_0=8400\text{Pa}$ and static temperature $T_0=215\text{K}$; 2) the symmetric conditions are selected at the symmetric planes; 3) the outer boundary adopts the freestream flow condition as its distance to the wall of aircraft is rather far for hypersonic flow simulation; 4) the supersonic outlet flow condition is adopted at the exit of flowfield and the downstream condition will not affect the upstream flow.

The flow structures of hypersonic interaction region of body and rudder are numerically simulated in the conditions of 0 and -20° attack angle. The flow structures in the interaction region in both conditions above show the interaction feature of three-dimensional shock waves and boundary layer. Basically, three-dimensional shock wave interaction determines the flowfield far from the wall surfaces and the three-dimensional shock wave interaction has an important impact on the boundary layer and causes the complex flow in the interaction region of body and rudder.

Under the condition of zero attack angle, there is no interaction of body shock wave and rudder shock wave, three-dimensional shock wave is mainly determined by the configuration of the rudder, as shown in the Fig.8. In the cross section of the interaction region, λ -shaped shock configuration forms and the contact surface inclines to the wall surface of the rudder and makes the boundary layer thinner.

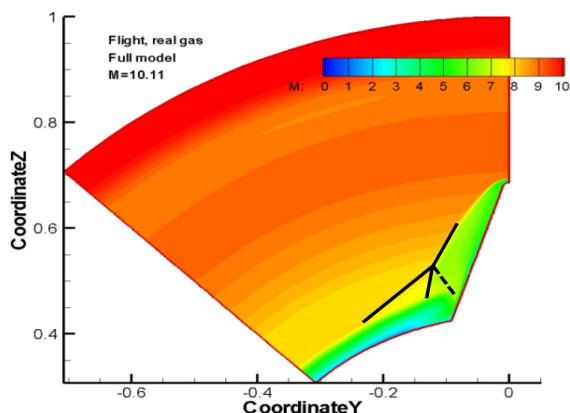


Fig.8 Sectioned flowfield of the interaction region of hypersonic rudder with 0° attack angle.

Under the condition of -20° attack angle, the body shock wave impinges on the rudder, which interacts with the rudder shock wave and forms complex three-dimensional shock configuration. According to the numerical result, in the region of ladder leading edge, regular three-dimensional shock wave interaction takes place, there is a contact surface of fluid inclines to the surface of rudder.

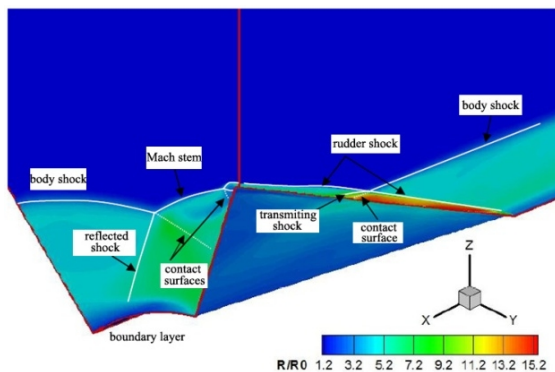


Fig.9 Three-dimensional flow feature of hypersonic rudder with -20° attack angle

In physics, the contact surface represents an impinging jet to the wall surface, which makes the boundary layer thinner and raises the local heat flux. In the lateral region of the rudder,

Mach interaction of three-dimensional shock waves takes place and there are two contact surfaces between the two tri-points. In physics, there is a high-speed impinging jet between the two above contact surfaces, which further makes the boundary layer thinner and raises the local heat flux greatly.

It is obvious that the configuration of three-dimensional shock wave interaction plays important role in the local heat flux peak of hypersonic aircraft. The research of mechanism of three-dimensional steady shock wave interaction is of great importance in the field of hypersonic gasdynamic heating.

4. SUMMARY

In the hypersonic interaction region, complex shock interaction configurations exist with the different flight conditions. The interaction of three-dimensional shock waves and boundary layer plays important role in the complex flow of hypersonic rudder region. The contact surfaces induced by three-dimensional shock waves represent a local flow jet. When the flow jet bounded by contact surfaces impinges on the boundary layer of wall surface, it makes the boundary layer thinner and will inevitably cause a local heat flux peak. The interaction configuration of three-dimensional shock waves plays important role in the gasdynamic heating mechanism in hypersonic complex flow.

REFERENCES

- [1] Michael K.S., Experimental Study of hypersonic wing/fin root heating at Mach 8, AOARD-104124, 2012.
- [2] G. Xiang, C. Wang, H. Teng, Z. Jiang, Three-dimensional shock wave configurations induced by two asymmetrical intersecting wedges in supersonic flow, Shock Waves, DOI:10.1007/s00193-017-0757-1.
- [3] Yang, Y., Wang, C., Jiang, Z.L.: Analytical and numerical investigations of the reflection of asymmetric nonstationary shock waves. Shock Waves, 2012(22):435–449.
- [4] Han ZY, Yin XZ. Shock Dynamics. Dordrecht: Kluwer Academic Publishers and Science Press, 1993. 22-67.
- [5] West J, Korkegi R. Supersonic interaction in the corner of intersecting wedges at high Reynolds numbers. 1972, 10(3): 652-656.
- [6] Ghislain Tchuen and Yves Burtschell (2011). Physico - Chemical Modelling in Nonequilibrium Hypersonic Flow Around Blunt Bodies, Aeronautics and Astronautics, Prof. Max Mulder (Ed.), InTech, DOI: 10.5772/18941.
- [7] John, D. Anderson, Jr., Hypersonic and High-temperature Gas Dynamics, published by the AIAA Inc., 2006.