

PORO-ELASTOPLASTIC MODELLING OF UPLIFT RESISTANCE OF SHALLOWLY-BURIED PIPELINES

Wen-gang Qi

Key Laboratory for Mechanics in Fluid Solid
Coupling Systems, Institute of Mechanics,
Chinese Academy of Sciences,
Beijing, China

School of Engineering Science, University of
Chinese Academy of Sciences, Beijing, China

Yu-min Shi

Key Laboratory for Mechanics in Fluid Solid
Coupling Systems, Institute of Mechanics,
Chinese Academy of Sciences,
Beijing, China

School of Engineering Science, University of
Chinese Academy of Sciences, Beijing, China

Fu-ping Gao

Key Laboratory for Mechanics in Fluid Solid Coupling Systems, Institute of
Mechanics, Chinese Academy of Sciences, Beijing, China
School of Engineering Science, University of Chinese Academy of Sciences
Beijing, China

ABSTRACT

During operational cycles of heating and cooling of submarine pipelines, variations of temperature and internal pressure may induce excessive axial compressive force along the pipeline and lead to global buckling of the pipeline. Reliable design against upheaval buckling of a buried pipeline requires the uplift response to be reasonably predicted. Under wave loading, the effective stress of soil could be reduced significantly in the seabed under wave troughs. To investigate the effects of wave-induced pore-pressure on the soil resistance to an uplifted buried pipeline, a poro-elastoplastic model is proposed, which is capable of simulating the wave-induced pore-pressure response in a porous seabed and the development of plastic zones while uplifting a shallowly-buried pipeline. The uplift force on the buried pipeline under wave troughs can be generated by the pore-pressure nonuniformly distributed along the pipe periphery. Numerical results show that the value of uplift force generally increases linearly with the wave-induced mudline pressure under troughs. Parametric study indicates that the peak soil resistance (under wave troughs) decreases with increasing wave height and wave period, respectively. The ratio of peak soil resistance under wave action to that without waves is mainly dependent on the normalized wave-induced mudline pressure, but influenced slightly by the internal friction angle of soil.

INTRODUCTION

The buckling of pipelines could occur due to the axial compressive forces caused by the constrained expansions set up by thermal and internal pressure actions during operational cycles of heating and cooling of subsea pipelines. Such compressive forces can lead to either lateral buckling in the plane of the seabed or upheaval buckling in a vertical plane, of which vertical buckling is of particular interest with respect to buried pipelines [1]. Moreover, when the pipelines approach the onshore terminal, the water depth gradually gets shallow and waves become the prevailing hydrodynamic load. Wave-induced pore-pressure in the seabed would affect the responses of uplifted pipelines. On one hand, the wave-induced pore-pressure induces an uplift force on the pipe under wave troughs. Meanwhile, the uplift soil resistance provided by the overburden soil to the pipe is reduced with decreasing effective stress due to the excess pore-pressure under wave troughs.

As for the wave-induced pore-pressure distribution around and uplift force on the pipeline, many studies have been conducted [2-5]. For evaluating the soil resistance to the pipeline inclining to move upward, several prediction formulas have been proposed. According to the ASCE Guideline [6], the vertical peak soil resistance to the pipeline per unit length (F_{rp})

was expressed as $F_{rp} = \gamma' H_c N_{qc} D$, where γ' is the submerged unit weight of soil, H_c is the burial depth to the centre of the pipeline, N_{qc} is a coefficient determined by soil internal friction angle and embedment ratio H_c/D , and D is the diameter of the pipeline. White et al. [7] reviewed the previous prediction models for vertical soil resistance. They classified the models into two categories, i.e. limit equilibrium and plasticity models, and proposed a limit equilibrium solution for predicting the vertical soil resistance of pipes and plate anchors in sands. Cheuk et al. [8] summarized several prediction formulae for peak vertical soil resistance and corresponding assumed mechanism. It was indicated that the simplified mechanism known as the vertical slip model provided a somewhat conservative but the most convenient result.

The aforementioned studies focus on either the wave-induced pore-pressure around or the uplift soil resistance to a buried pipe. In a more practical scenario, these two issues should be considered concurrently. In the present study, a poro-elastoplastic model for simulating wave-seabed-pipeline interaction is established. The effects of wave-induced pore-pressure on the responses of uplifted pipelines are further investigated for various wave and soil parameters.

PORO-ELASTOPLASTIC MODEL OF PIPE-SOIL INTERACTION

Finite element mesh and boundary conditions

The interaction between a pipeline and the surrounding soil under the action of waves can be considered as a plane strain problem. A plane strain finite element model is proposed in ABAQUS to investigate the effect of wave-induced pore-pressure response on the uplifting process of a buried pipeline. A fully saturated sandy seabed is considered in this study. For the fully saturated sandy seabed, the shear modulus of soil is very small compared to the true modulus of elasticity of pore-water, and pore-pressure response in the seabed is independent of the permeability of the soil [9, 10]. Quasi-static analysis is adopted in the present study to capture the pore-pressure response in the surrounding soil of the pipe. The user subroutines are used to load the wave pressure and the pore-pressure: wave pressure acting on the mudline can be loaded by the subroutine DLOAD, whereas pore-pressure at the mudline can be loaded to seabed surface by subroutine DISP. This approach has been indicated to be feasible in the existing numerical studies (e.g., [4]). A contact-pair algorithm is adopted to characterize the interfacial constitutive relationship between pipe exterior surfaces and surrounding soil. The pipe-soil interface frictional coefficient (μ) is calculated with the following formula proposed by Randolph & Wroth [11]

$$\mu = \tan \left[\sin \phi \times \cos \phi / (1 + \sin^2 \phi) \right] \quad (1)$$

Fig. 1 illustrates the geometry of the finite element model, which is mainly consisting of the pipeline and the surrounding soil. The pipe is composed with 4-node bilinear plane strain reduced-integration elements (CPE4R). And element CPE4RP (4-node plane strain, bilinear displacement, bilinear pore-pressure, reduced-integration) is chosen for the soil mesh. In this study, attentions are primarily focused on distribution of pore pressure response in the seabed, especially in vicinity of pipeline zone. Therefore, the computational grids get denser in the closer proximity to the pipe, as shown in Fig. 1. The grid size of the present model has been proved fine enough for ensuring the accuracy of the results, by comparing results between models with different grid sizes. The width of the numerical model is set as 250 m and the depth as 20 m, and the pipeline is located at 0.5m depth (measured to the upper point of pipeline). The extent of the soil domain was found to be sufficient to acceptably limit boundary effects.

The boundary conditions are as follow. On two vertical sides, the normal component of displacement is fixed, and no flow of pore fluid through the walls is permitted. At the bottom of the model, the translational degrees of freedom in two directions are fixed. The top surface of the soil allows perfect drainage so that the excess pore pressure is always zero on this surface.

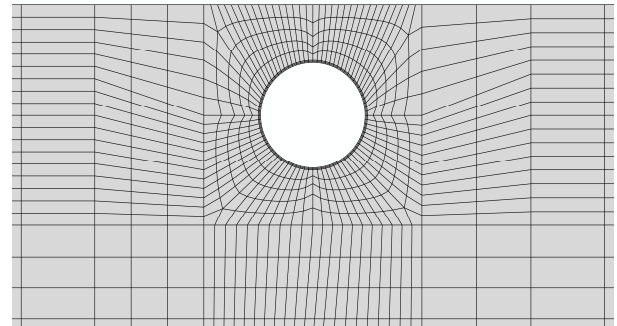


FIGURE 1. MESHES OF BURIED PIPELINE AND SURROUNDING SOIL

Constitutive models and properties of materials

The Mohr-Coulomb constitutive model (M-C model) is utilized to simulate the elasto-plastic behavior of the soil for the present drained conditions. The yield function of the M-C model is written as (see [12])

$$J - \left(\frac{c}{\tan \phi} + p' \right) g(\theta) = 0 \quad (2)$$

in which, J is the deviatoric stress invariant; c is the cohesion strength; p' is the mean effective stress;

$$g(\theta) = \frac{\sin \phi}{\cos \theta + \sin \theta \sin \phi / \sqrt{3}} \quad (3)$$

where θ is the Lode's angle and ϕ is the internal friction angle of the soil.

A very high Young's modulus of $E_p=210$ GPa is adopted for the steel pipe, although it was treated as a rigid body using a feature of ABAQUS that forces all nodes on the body to move according to displacements and rotations specified at a load reference point (LRP). The LRP is taken at the center of the pipe section. The input data of soil parameters, wave conditions, and pipe properties for the investigations are listed in Table 1. The values of the calculating parameters are taken from Table 1 unless otherwise stated hereinafter.

TABLE 1. INPUT DATA FOR SEABED, WAVE AND PIPELINE

| | Parameter | Value |
|----------|---|---------------|
| Seabed | Internal friction angle (ϕ) | 30° (various) |
| | Young's modulus [E_s (MPa)] | 50 (various) |
| | Porosity of soil (n) | 0.45 |
| | Poisson ratio of soil [ν] | 0.3 |
| | Submerged unit weight of soil [γ' (kN/m ³)] | 7.84 |
| | Cohesion strength [c (kPa)] | 0 |
| | Seabed thickness [d_o (m)] | 20 |
| Wave | Water depth [h (m)] | 12 (various) |
| | Wave period [T (s)] | 7.0 (various) |
| | Wave height [H (m)] | 2.0 (various) |
| Pipeline | Young's modulus [E_p (GPa)] | 210 |
| | Poisson ratio [ν] | 0.19 |
| | Diameter [D (m)] | 1.0 |
| | Embedment depth [d (m)] | 0.5 |

Loading procedures

Three load steps are utilized to perform the analysis. In the first step, the initial geostress is generated by exerting a uniform body force on the soil. In this step, a technique named equilibrium of geostress is adopted to eliminate the displacement of soil, which shouldn't occur in reality. In the second step, the wave loading is applied on the soil surface to mimic the pore-pressure distribution in the soil. Note that the wave loading doesn't propagate with increasing time, i.e., the pipelines will be always under wave trough in the following analysis. After the background stress field, which considers the wave-induced pore-pressure, has been set up, an upward concentrated force is exerted on the pipe at LRP in the last step. And the vertical interaction between pipe and soil can be executed and analyzed. Note that in the present model, the pore-pressure surrounding the pipe cannot be exerted on the exterior surface of the pipe, limited by the software. Therefore, the uplift responses of pipelines obtained from the numerical

simulations are only dependent on the externally exerted concentrated force, but irrelevant to the pore-pressure-induced uplift force.

VERIFICATION OF NUMERICAL MODEL

The present model covers two scenarios. One is the vertical interaction between pipe and soil. The other is the wave-induced pore-pressure response in the seabed. The similar model has been widely verified and utilized for simulating the pipe-soil interaction in many existing studies (e.g. [13, 14]). In this study, we focus on the validation of wave-induced pore-pressure response in the seabed.

Fig. 2 presents a comparison of normalized pore-pressure distributions between the present model and the previous analytical solution without a pipeline in saturated sand [10]. As shown in the figure, the present numerical model agrees well with the previous analytical solution for relatively small wave periods. With increasing wave period, the discrepancy gradually becomes significant in the lower part of the soil while good consistency is still observed in the upper part. The discrepancy in the lower part of the soil can be attributed to the assumption of infinite thickness of seabed in the analytical solution (refer to [15]). In this study, we focus on the effects of pore-pressure on the pipe-soil interaction, which happens in the upper zone of the seabed. Therefore, the numerical results of pore-pressure response are considered to be precise enough for this investigation.

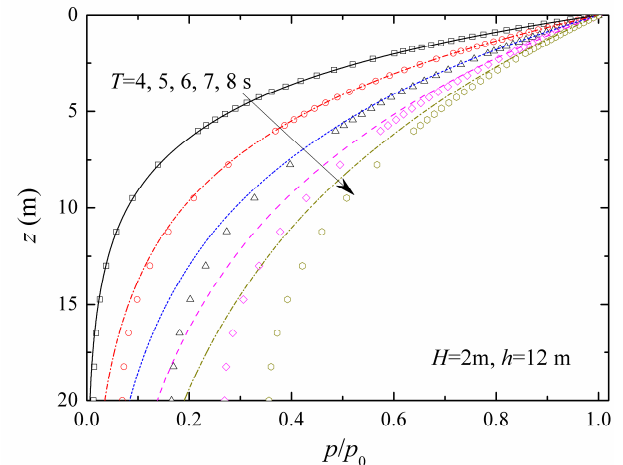


FIGURE 2. COMPARISON OF NORMALIZED WAVE-INDUCED FAR-FIELD PORE-PRESSURE (p/p_0) ALONG SOIL DEPTH BETWEEN THE PRESENT MODEL AND THE PREVIOUS ANALYTICAL SOLUTION (LINES REPRESENT ANALYTICAL SOLUTION AND SCATTERED POINTS REPRESENT NUMERICAL RESULTS)

RESULTS AND DISCUSSIONS

Pore-pressure response and uplift force

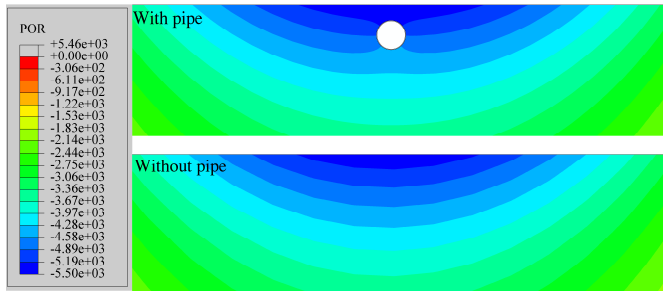


FIGURE 3. COMPARISON OF WAVE-INDUCED PORE-PRESSURE AT WAVE TROUGH BETWEEN THE CASES WITH AND WITHOUT A PIPELINE

Under wave troughs, the wave-induced pore-pressure response along the periphery of pipeline will result in an uplift force F_w acting on the buried pipeline. In this section, the pore-pressure response and uplift force are analyzed. Since we only consider linear wave loading, wave heights are included in the reference wave pressure at the seabed surface (p_0), and wavelength is determined by wave dispersion relation with a given wave period and water depth. Thus, we only examine the influence of wave period and water depth.

Fig. 3 shows a contour of the wave-induced pore-pressure in the seabed. The pipeline locates under the center of a wave trough. It can be seen that the existence of pipeline alters the pore-pressure distribution obviously in its vicinity.

The distributions of linear wave-induced pore-pressure ($|p|/p_0$) along the soil depth in far-field and through the center of the pipeline for various water depths and wave periods are compared in Figs. 4(a) and (b), respectively. Fig. 4 indicates that the pipe has a sheltering effect on the pore-pressure transmission. Specifically, the magnitude of excess pore-pressure is smaller at the pipe top and larger at the pipe bottom than the same depth of far field zone under wave troughs. The perturbation effect of the pipeline on the original pore-pressure distribution in the soil will render the uplift force significantly larger than the non-perturbed one.

For the fully saturated sandy seabed in the present study, the vertical pore-pressure profile normalized with mudline pressure ($|p|/p_0$) is practically controlled by the wave number λ ($\lambda = 2\pi/L$, where L is wave length; see [10]). Both water depth and wave period can affect the profile of $|p|/p_0$ by influencing the value of wave length. By comparing Fig. 4(a) with (b), it can be seen that both increasing water depth and wave period are beneficial to the transmission of pore-pressure, and the wave period has a greater effect on the normalized vertical pore-pressure profile. It should be noted that both water depth and wave period also have significant effects on the value of mudline pressure p_0 . Specifically, increasing water depth results in a smaller mudline pressure while increasing wave period induces a larger one. While evaluating the effects of water depth and wave period on the uplifted pipe response, the specific values of pore-pressure in the seabed are the primary

concern and thus the effects of water depth and wave period on both wave length and mudline pressure will be included.

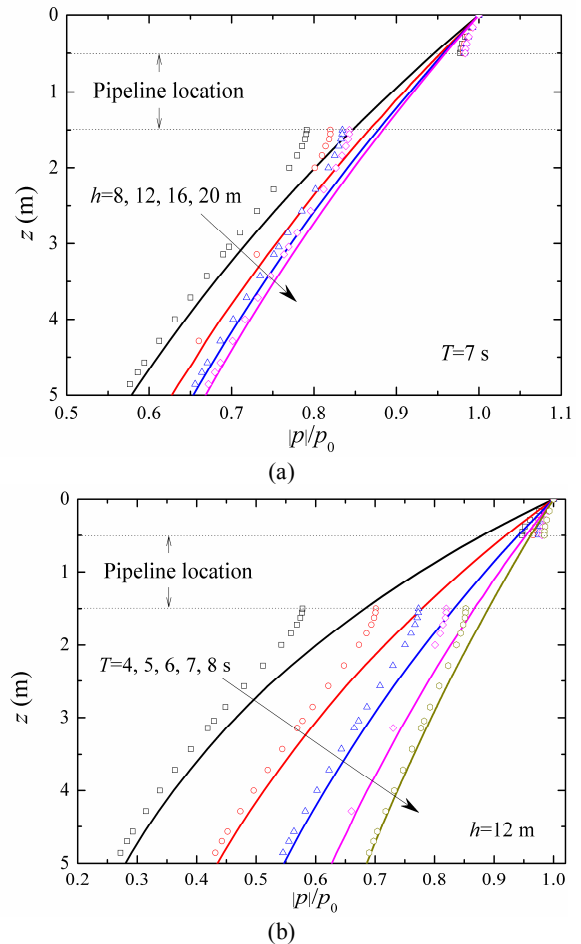


FIGURE 4. COMPARISON OF THE NORMALIZED WAVE-INDUCED PORE-PRESSURE ($|p|/p_0$) ALONG THE SOIL DEPTH IN FAR-FIELD AND THROUGH THE CENTER OF THE PIPELINE FOR VARIOUS (a) WATER DEPTHS; AND (b) WAVE PERIODS

The pore-pressure distribution along the periphery of the pipeline are extracted and integrated to obtain the uplift force under wave troughs F_w . The variations of F_w with water depth and wave period are given in Figs. 5(a) and (b), respectively. The value of F_w decreases with increasing water depth. With increasing wave period, the value of F_w increases first, then reaches a maxima and slightly decreases. For the examined conditions, although the maximum magnitude of F_w is only approximately 13% of the displaced water weight of pipeline, the uplift force could significantly affect the vertical on-bottom stability of pipelines. This is because that the soil resistance would also be small and thus comparable to F_w for the conditions under which the largest F_w occurs.

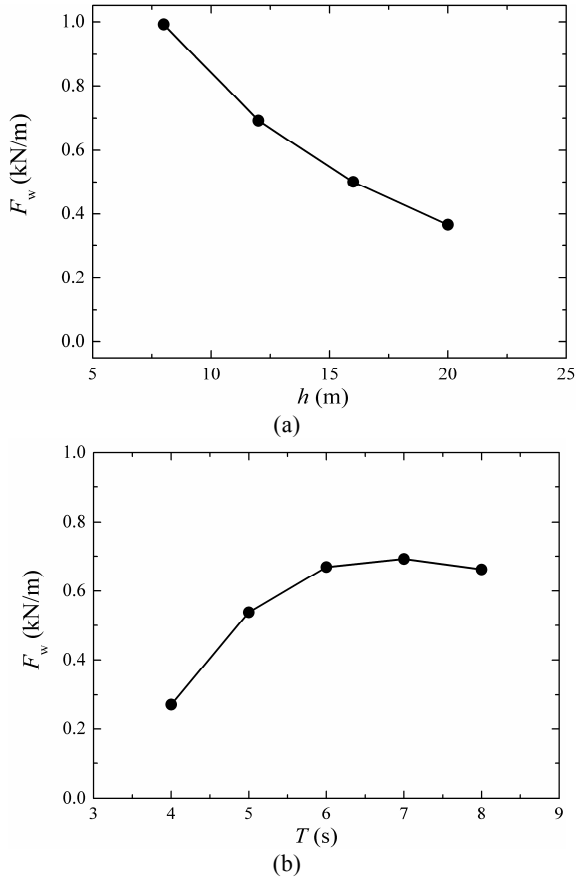


FIGURE 5. VARIATION OF WAVE-INDUCED UPLIFT FORCE F_w WITH (a) WATER DEPTH; AND (b) WAVE PERIOD

For a fixed value of pipe embedment, the uplift force is mainly dependent on the mudline pressure p_0 and the wave length L . The variation of $F_w/\gamma'D^2$ with $p_0/\gamma'D$ is shown in Fig. 6. The value of $F_w/\gamma'D^2$ generally increases linearly with $p_0/\gamma'D$, implying that the mudline pressure rather than the wave length plays a key role in determining F_w . For the other values of embedment, this conclusion needs to be further verified.

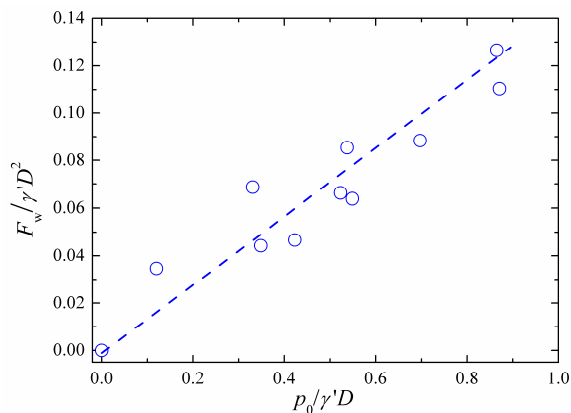


FIGURE 6. VARIATION OF $F_w/\gamma'D^2$ WITH $p_0/\gamma'D$

Parametric study on the response of uplifted pipelines

The effects of several influential parameters on the response of uplifted pipelines are investigated. Fig. 7(a) shows the force-displacement curves under various wave heights. It is shown that with increasing wave height, the curves gradually shifted downwards. The peak soil resistance under wave troughs F_{tp} are gathered from the plateau of the force-displacement curves and shown in Fig. 7(b). The value of F_{tp} decreases with increasing wave height linearly. F_{tp} is approximately halved by a wave of $H=1.5$ m. The plastic strain contours under various wave heights are compared in Fig. 8. It's clearly observed that as wave height increases, the zone of relatively large plastic strain shrinks. This implies that the wave-induced pore-pressure response in the seabed has significant effects on the interaction between an uplifted pipeline and surrounding soil.

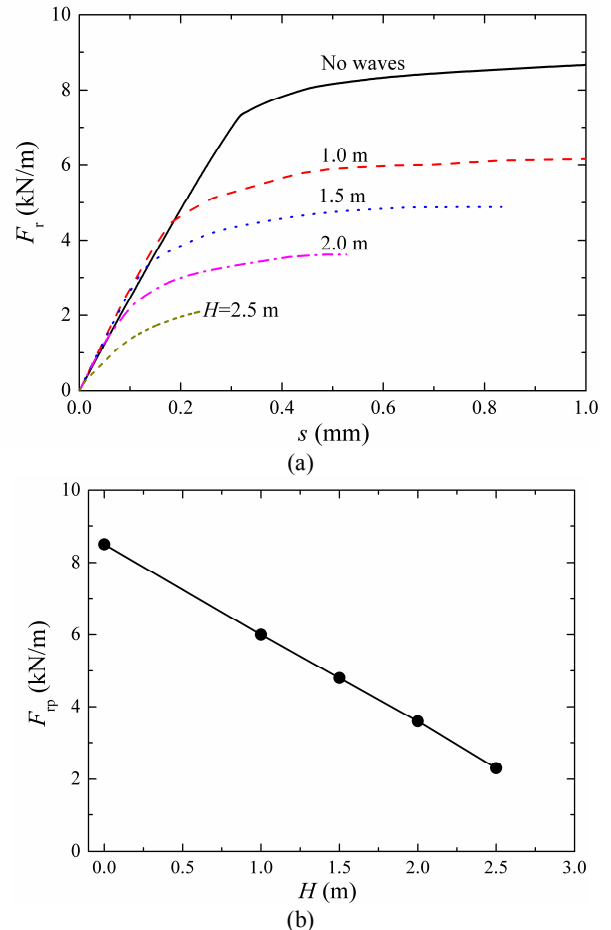


FIGURE 7 EFFECTS OF WAVE HEIGHT ON (a) FORCE-DISPLACEMENT CURVES OF UPLIFTED PIPELINES; AND (b) PEAK SOIL RESISTANCE OF UPLIFTED PIPELINES

The effects of water depth and wave period on the response of uplifted pipelines are shown in Fig. 9 and 10, respectively. As shown in the figures, the plateau period of force-displacement curves emerges earlier with decreasing water depth and increasing wave period. As aforementioned, the effects of decreasing water depth on both wave length and mudline pressure are beneficial to the occurrence of larger excess pore-pressure in the shallow zone of seabed, which would result in more reduction of peak soil resistance for shallowly-embedded pipelines. In contrast, for increasing wave period, only the induced larger mudline pressure is beneficial to the reduction of peak soil resistance. The significant decrease of F_{rp} with T shown in Fig. 10(b) indicates that the effects of enlarged mudline pressure are in control rather than the increasing wave length.

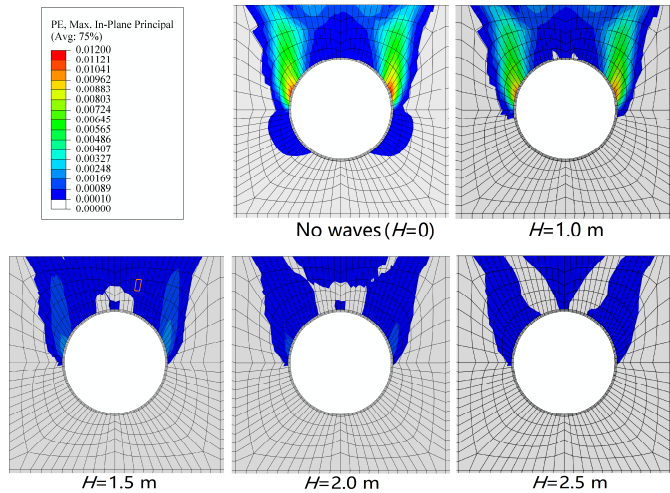


FIGURE 8. COMPARISON OF PLASTIC STRAIN CONTOURS UNDER VARIOUS WAVE HEIGHTS

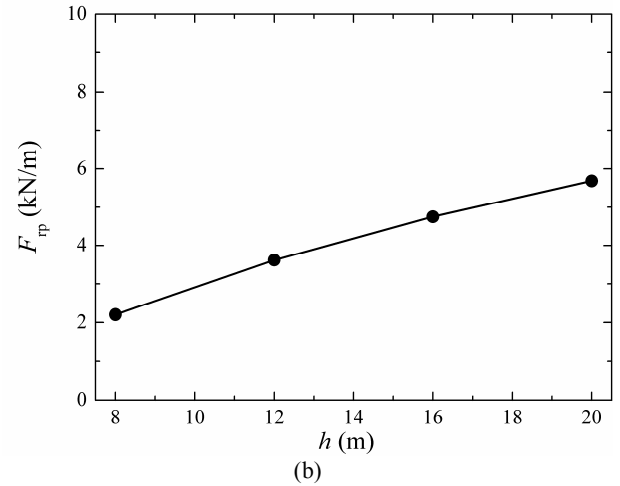
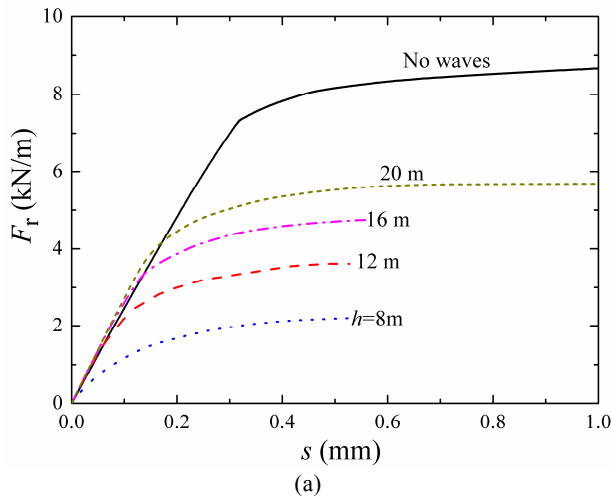


FIGURE 9. EFFECTS OF WATER DEPTH ON (a) FORCE-DISPLACEMENT CURVES OF UPLIFTED PIPELINES; AND (b) PEAK SOIL RESISTANCE OF UPLIFTED PIPELINES

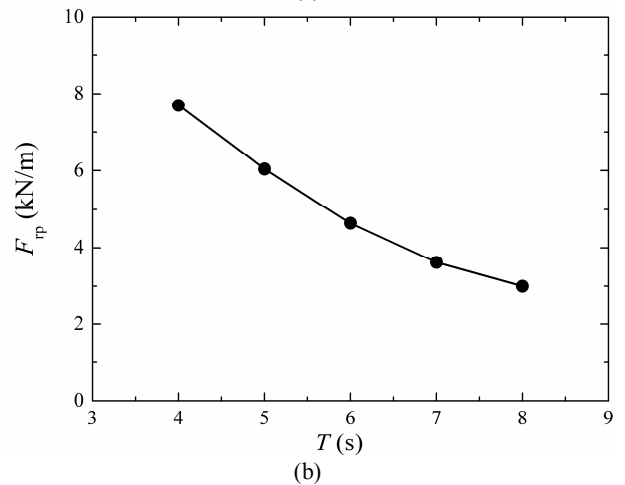
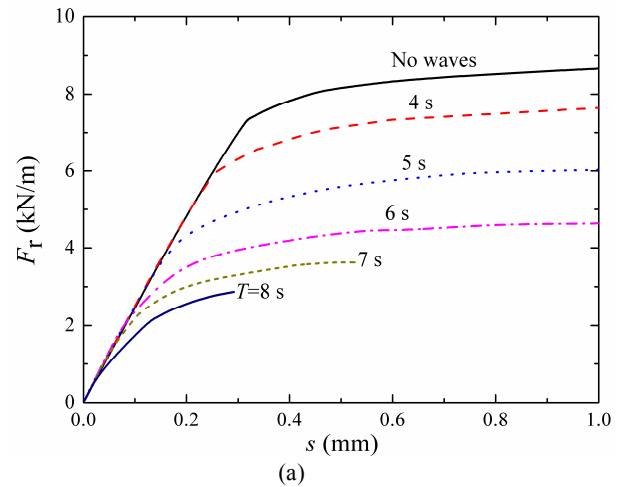


FIGURE 10. EFFECTS OF WAVE PERIOD ON (a) FORCE-DISPLACEMENT CURVES OF UPLIFTED PIPELINES; AND (b) PEAK SOIL RESISTANCE OF UPLIFTED PIPELINES

The effects of soil parameters on the wave-induced reduction of peak soil resistance are examined. Fig. 11 shows the force-displacement curves and corresponding peak soil resistance for two different internal friction angles. It's indicated that the initial stiffness of the force-displacement curves is not affected by the internal friction angle. The peak soil resistance is increased by a larger internal friction angle. Nevertheless, Fig. 11(b) indicates that the relative reduction of peak soil resistance induced by wave-induced pore-pressure keeps unchanged for different internal friction angles.

Fig. 12 gives the comparison of force-displacement curves under different elastic modulus of soil. In contrast to the effects of internal friction angle, the elastic modulus of soil only affects the initial stiffness of the curves while the peak soil resistance generally keeps constant. By conducting the parametric study, it can be concluded that while evaluating the relative reduction of peak soil resistance resulted from wave-induced pore-pressure, the effects of soil parameters can be neglected and only the wave parameters should be included (refer to Fig. 13).

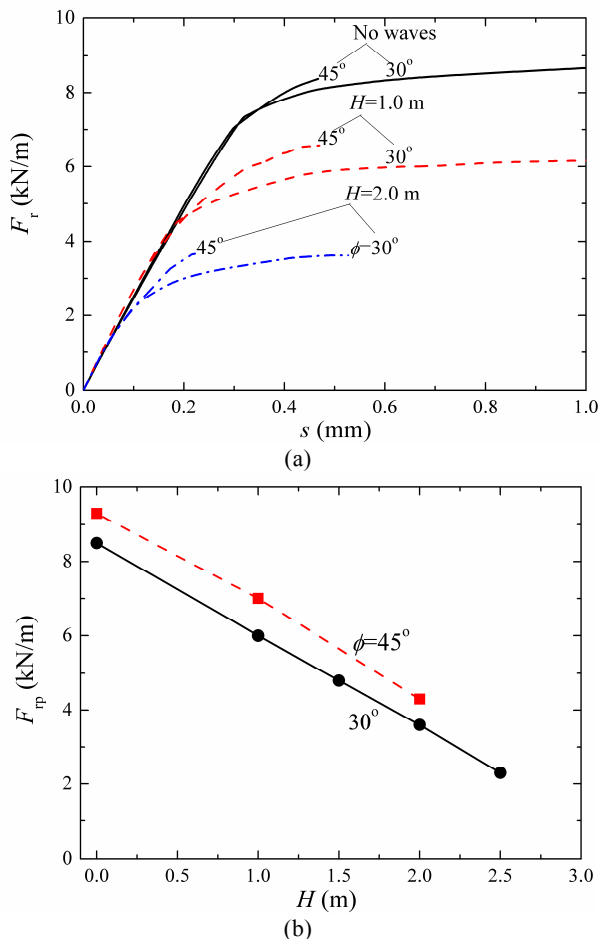


FIGURE 11. EFFECTS OF INTERNAL FRICTION ANGLE ON (a) FORCE-DISPLACEMENT CURVES OF UPLIFTED PIPELINES; AND (b) PEAK SOIL RESISTANCE OF UPLIFTED PIPELINES

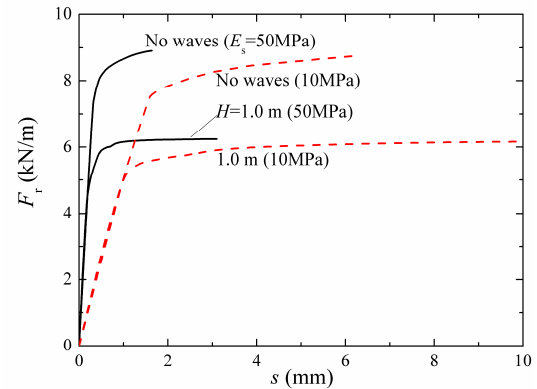


FIGURE 12. EFFECTS OF ELASTIC MODULUS OF SOIL ON FORCE-DISPLACEMENT CURVES OF UPLIFTED PIPELINES

Effect of wave-induced pore-pressure on peak uplift resistance

To qualitatively evaluate the effect of wave-induced pore-pressure on the peak uplift resistance, the ratio of peak soil resistance under wave action to that without waves (F_{rp}/F_{rp0}) is calculated. The variation of F_{rp}/F_{rp0} with $p_0/\gamma'D$ is given in Fig. 13. In spite of various wave parameters (water depth, wave period, and wave height), all the data generally lies around one single curve. By modifying the peak soil resistance according to the curve in Fig. 13, the effect of wave-induced pore-pressure can be taken into consideration. For different embedments of pipelines, the curve of F_{rp}/F_{rp0} vs. $p_0/\gamma'D$ could change. This effect will be examined in the further investigation.

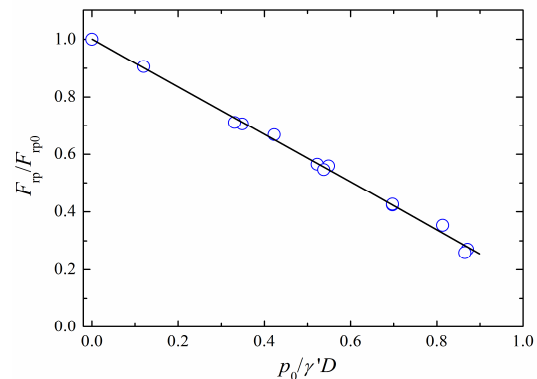


FIGURE 13. VARIATION OF F_{rp}/F_{rp0} WITH $p_0/\gamma'D$

CONCLUSIONS

Under wave loading, the effective stress of soil can be reduced in the seabed and the uplift soil resistance would be compromised significantly under wave troughs. A poroelastoplastic model is proposed to investigate the effects of wave-induced pore-pressure on the response of an uplifted buried pipeline. The proposed numerical model is capable of simulating the wave-induced pore-pressure response in a porous seabed and the development of plastic zones while

uplifting a shallowly-buried pipeline. Following conclusion can be drawn:

(1) Under wave troughs, uplift force can be induced by the nonuniformly distributed pore-pressure around the pipe periphery. The value of uplift force is mainly dependent on the mudline pressure for the examined pipeline embedment.

(2) Parametric study indicated that the peak soil resistance decreases with increasing wave height and wave period, respectively. The elastic modulus of soil only affects the initial stiffness of the force-displacement curves while the peak soil resistance generally keeps constant.

(3) The ratio of peak soil resistance under wave action to that without waves is mainly dependent on the normalized wave-induced mudline pressure, but influenced slightly by the internal friction angle of soil.

NOMENCLATURE

| | |
|-----------|---|
| c | Cohesion strength |
| D | Diameter of pipeline |
| d | Embedment of pipeline (measured to the upper point of pipeline) |
| d_0 | Seabed thickness |
| E_p | Young's modulus of pipeline |
| E_s | Young's modulus of soil |
| F_{Tp} | Vertical peak soil resistance under wave troughs acting on the pipeline per unit length |
| F_w | Wave-induced uplift force acting on pipeline under wave troughs |
| H_c | Burial depth to the centre of pipeline |
| H | Wave height |
| h | Water depth |
| J | Deviatoric stress invariant |
| L | Wave length |
| n | Porosity of soil |
| N_{qc} | A dimensionless coefficient in the expression of vertical peak soil resistance by Nyman [6] |
| p' | Mean effective stress |
| p_0 | Wave pressure at the seabed surface |
| p | Wave-induced pore-pressure in the soil |
| T | Wave period |
| ϕ | Internal friction angle of soil |
| γ' | Submerged unit weight of soil |
| λ | Wave number, $\lambda = 2\pi/L$ |
| μ | Pipe-soil interface frictional coefficient |
| ν | Poisson ratio of soil or pipeline |
| θ | Lode's angle |

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11232012, 11372319, 11602273) and the Strategic Priority Research Program (Type-B) of CAS (Grant No. XDB22030000).

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