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HIGHER-ORDER ANALYSIS OF NEAR-TIP FIELDS AROUND AN INTERFACIAL CRACK BETWEEN TWO DISSIMILAR POWER LAW HARDENING MATERIALS*

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ABSTRACT: By means of an asymptotic expansion method of a regular series, an exact higher-order analysis has been carried out for the near-tip fields of an interfacial crack between two different elastic-plastic materials. The condition of plane strain is invoked. Two group of solutions have been obtained for the crack surface conditions: (1) traction free and (2) frictionless contact, respectively. It is found that along the interface ahead of crack tip the stress fields are co-order continuous while the displacement fields are cross-order continuous. The zone of dominance of the asymptotic solutions has been estimated.

KEY WORDS: interfacial crack, elastic-plastic, near-tip fields, higher-order asymptotic analysis

I. INTRODUCTION

Up to now, the elastic problems in interface fracture mechanics have been widely investigated. A detailed review of its advances can be found in Ruhle et al.[1]. By comparison, less work has been done for the elastic-plastic problems.

Recently Shih and Asaro[2−5] have made some numerical finite element analyses of the near-tip fields for elastic-plastic interfacial crack, and found that the asymptotic behaviors of the near-tip fields directly depend on the lower hardening material. As r approaches zero, the behaviors of interfacial crack are much more similar to those of the crack lying along the interface between a plastic solid and rigid substrate. They finally obtained a nearly separable form of near-tip fields of the HRR type.

Wang[6] presented an exact asymptotic analysis for a crack lying on the interface between an elastic-plastic material and an elastic material. A separable singular stress field of the HRR type has been found. These solutions only correspond to the certain mixity parameter M^p. Some other advances in this field can be found in Gao and Lou[7].

Based on the work of Wang[6], Xia and Wang[8] not only extensively analyzed the interfacial crack problem with the traction vanishing on crack surfaces and obtained the full-continuous, separable form of solutions of the HRR type, but also obtained the similar solutions with the frictionless contact of crack surfaces. Furthermore, for any given mixity parameter value M^p, the solution of the HRR type has been found with the weak discontinuity of the third-order derivative.

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As for the interfacial crack problem with the frictionless contact of the crack surfaces, Xia and Wang[9] have also made a high-order asymptotic analysis for its near tip fields. Aravas and Sharma[10] also got some results in this aspect.

In this paper an asymptotic expansion method of regular series was introduced to implement the high-order analysis of the near-tip fields for the interfacial crack in elastic-plastic power-law hardening bimaterials. Two different solutions in separable form have been obtained. Among them, one corresponds to the traction free condition of the crack surfaces, and another one corresponds to the frictionless contact condition of the crack surfaces. Our results indicated that along the interface ahead of crack tip, the stress fields are co-order continuous while the displacement fields are cross-order continuous.

Using the high-order asymptotic solutions obtained here, the rational zone has been estimated in which the solutions in separable form of asymptotic series dominate.

II. BASIC EQUATIONS

Fig.1 shows an interfacial crack between two materials. These two materials are all elastic-plastic power-law hardening materials but with different hardening properties. Under the plane strain condition their constitutive relations can be written as

$$\varepsilon_{\beta\gamma} = \frac{1+\nu}{E}\sigma_{\beta\gamma} + \delta_{\beta\gamma}\frac{\Gamma}{E}\sigma_{\rho\rho} + \frac{3}{2}\bar{\alpha}\left(\frac{\sigma_e}{\sigma_o}\right)^{n-1}\frac{P_{\beta\gamma}}{E} \quad (2.1)$$

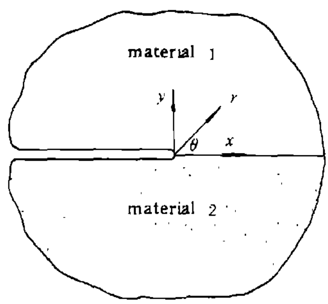


Fig.1 Interface crack tip region

where $\sigma_o = \min\{\sigma_{o1},\ \sigma_{o2}\}$ is the smaller one of the yield stresses of two materials. E and ν are the Young's modulus and Poisson's ratio, respectively. n is hardening exponent; $\bar{\alpha}$ can be expressed by the hardening coefficient α, i.e. $\bar{\alpha}_1 = \alpha_1(\sigma_o/\sigma_{o1})^{n_1-1}$ for upper part, and $\bar{\alpha}_2 = \alpha_2(\sigma_o/\sigma_{o2})^{n_2-1}$ for lower part. All properties mentioned abvove will take different values for upper material 1 and lower material 2, respectively. i.e. $(E_1, \nu_1, n_1, \bar{\alpha}_1)$ and $(E_2, \nu_2, n_2, \bar{\alpha}_2)$.

In (2.1),

$$\sigma_{\rho\rho} = \sigma_r + \sigma_\theta \qquad P_{\beta\gamma} = \sigma_{\beta\gamma} - \frac{1}{2}\sigma_{\rho\rho}\delta_{\beta\gamma}$$

where $\Gamma = -(1+\nu)\nu + (\frac{1}{2} - \nu)^2$ in the asymptotic sense; σ_e is the effective stress. For our high-order analysis it is accurate enough to write the effective stress in the following form

$$\sigma_e = \sqrt{\frac{3}{4}(\sigma_r - \sigma_\theta)^2 + 3\tau_{r\theta}^2} \quad (2.2)$$

Note that Greek letters are used for subscript indices running over 1, 2.

The stresses can be expressed in terms of the stress functions

$$\left.\begin{aligned} \sigma_r &= \frac{1}{r}\left(\frac{\partial\phi}{\partial r} + \frac{1}{r}\frac{\partial^2\phi}{\partial\theta^2}\right) \\ \sigma_\theta &= \frac{\partial^2\phi}{\partial r^2} \\ \tau_{r\theta} &= -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\theta}\right) \end{aligned}\right\} \quad (2.3)$$

where $\phi = \bar{\phi}/L^2$ is the nondimensional stress function, $r = \bar{r}/L$ is the nondimensional radial coordinate. And L is the characteristic length of crack.

Let

$$\phi = \sigma_o K r^2 \psi(\rho, \theta) \tag{2.4}$$

we have

$$\left.\begin{aligned} \sigma_r &= \sigma_o K(\ddot{\psi} + 2\psi + \psi'\Omega) \\ \sigma_\theta &= \sigma_o K(2\psi + 3\psi'\Omega + \psi''\Omega^2 + \psi'\Omega\Omega') \\ \tau_{r\theta} &= \sigma_o K[-(\dot{\psi} + \dot{\psi}'\Omega)] \end{aligned}\right\} \tag{2.5}$$

where $(') = \dfrac{\partial}{\partial\rho}(\ )$, $(\dot{\ }) = \dfrac{\partial}{\partial\theta}(\ )$.

$$\Omega = r\frac{\mathrm{d}\rho}{\mathrm{d}r} \tag{2.6}$$

Suppose

$$\Omega = \sum_{k=1}^{\infty} c_k \rho^k = c_1\rho + c_2\rho^2 + c_3\rho^3 + \ldots \tag{2.7}$$

and

$$\psi = \frac{1}{\rho}F_*(\theta) + F_0(\theta) + \rho F_1(\theta) + \rho^2 F_2(\theta) + \ldots \tag{2.8}$$

Therefore, stresses can be written as

$$\sigma_{\beta\gamma} = \sigma_o K\{\frac{1}{\rho}\tilde{\sigma}_{\beta\gamma_*} + \tilde{\sigma}_{\beta\gamma_0} + \tilde{\sigma}_{\beta\gamma_1} + \ldots\} \tag{2.9}$$

where

$$\left.\begin{aligned} \tilde{\sigma}_{r_0} &= \ddot{F}_* + (2 - c_1)F_* \\ \tilde{\sigma}_{\theta_*} &= (2 - c_1)(1 - c_1)F_* \\ \tilde{\tau}_{r\theta_*} &= -(1 - c_1)\dot{F}_* \end{aligned}\right\} \tag{2.10a}$$

$$\left.\begin{aligned} \tilde{\sigma}_{r_0} &= \ddot{F}_* + 2F_0 - c_2F_* \\ \tilde{\sigma}_{\theta_0} &= 2F_0 + (c_1 - 3)c_2F_* \\ \tilde{\tau}_{r\theta_0} &= -\dot{F}_0 + c_2\dot{F}_* \end{aligned}\right\} \tag{2.10b}$$

$$\left.\begin{aligned} \tilde{\sigma}_{r_1} &= \ddot{F}_1 + (2 + c_1)F_1 - c_3F_* \\ \tilde{\sigma}_{\theta_1} &= (2 + c_1)(1 + c_1)F_1 - 3c_3F_* \\ \tilde{\tau}_{r\theta_1} &= -(1 + c_1)\dot{F}_1 + c_3\dot{F}_* \end{aligned}\right\} \tag{2.10c}$$

Substituting (2.9) into (2.2) yields an expression for the effective stress

$$\left(\frac{\sigma_e}{\sigma_0}\right)^2 = K^2\frac{1}{\rho^2}[\tilde{\sigma}_{e_*}^2 + (\rho\tilde{\sigma}_{e_0} + \rho^2\tilde{\sigma}_{e_1} + \rho^3\tilde{\sigma}_{e_2} + \ldots)] \tag{2.11}$$

where

$$\left.\begin{aligned}
\tilde{\sigma}_{e_*}^2 &= \frac{3}{4}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*})^2 + 3\tilde{\tau}_{r\theta_*}^2 \\
\tilde{\sigma}_{e_0} &= 2[\frac{3}{4}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*})(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0}) + 3\tilde{\tau}_{r\theta_*}\tilde{\tau}_{r\theta_0}] \\
\tilde{\sigma}_{e_1} &= \frac{3}{4}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0})^2 + 3\tilde{\tau}_{r\theta_0}^2 + 2[\frac{3}{4}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*})(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + 3\tilde{\tau}_{r\theta_*}\tilde{\tau}_{r\theta_1}] \\
\tilde{\sigma}_{e_2} &= 2[\frac{3}{4}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*})(\tilde{\sigma}_{r_2} - \tilde{\sigma}_{\theta_2}) + 3\tilde{\tau}_{r\theta_*}\tilde{\tau}_{r\theta_2} \\
&\quad + \frac{3}{4}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0})(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + 3\tilde{\tau}_{r\theta_0}\tilde{\tau}_{r\theta_1}]
\end{aligned}\right\} \tag{2.12}$$

As $r \to 0$, the quantity in parentheses is much smaller than the first term in (2.11), so that

$$\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1} = \frac{K^{n-1}}{\rho^{n-1}}(\Lambda_* + \Lambda_0\rho + \Lambda_1\rho^2 + ...) \tag{2.13}$$

where

$$\left.\begin{aligned}
\Lambda_* &= (\tilde{\sigma}_{e_*}^2)^{(n-1)/2} \\
\Lambda_0 &= \frac{n-1}{2}(\tilde{\sigma}_{e_*}^2)^{(n-3)/2}\tilde{\sigma}_{e_0} \\
\Lambda_1 &= \frac{(n-1)(n-3)}{8}(\tilde{\sigma}_{e_*}^2)^{(n-5)/2}\tilde{\sigma}_{e_0}^2 + \frac{n-1}{2}(\tilde{\sigma}_{e_*}^2)^{(n-3)/2}\tilde{\sigma}_{e_1}
\end{aligned}\right\} \tag{2.14}$$

From (2.1), we can get the strains

$$\varepsilon_{\beta\gamma} = \frac{\sigma_0 K}{E\rho}(\tilde{\varepsilon}_{\beta\gamma_*}^e + \rho\tilde{\varepsilon}_{\beta\gamma_0}^e + \rho^2\tilde{\varepsilon}_{\beta\gamma_1}^e + ...) + \frac{\bar{\alpha}\sigma_0 K^n}{E\rho^n}(\tilde{\varepsilon}_{\beta\gamma_*}^p + \rho\tilde{\varepsilon}_{\beta\gamma_0}^p + \rho^2\tilde{\varepsilon}_{\beta\gamma_1}^p + ...) \tag{2.15}$$

where

$$\tilde{\varepsilon}_{\beta\gamma i}^e = (1+\nu)(\tilde{\sigma}_{\beta\gamma i} - \delta_{\beta\gamma}\nu\tilde{\sigma}_{\rho\rho i}) \qquad (i = *, 0, 1) \tag{2.16}$$

$$\left.\begin{aligned}
\tilde{\varepsilon}_{r_*}^p &= -\tilde{\varepsilon}_{\theta_*}^p = \frac{3^{\frac{n+1}{2}}}{4}\tilde{g}^{\frac{n-1}{2}}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*}) \\
\tilde{\varepsilon}_{r\theta_*}^p &= \frac{3^{\frac{n+1}{2}}}{4}\tilde{g}^{\frac{n-1}{2}}2\tilde{\tau}_{r\theta_*}
\end{aligned}\right\} \tag{2.17a}$$

$$\left.\begin{aligned}
\tilde{\varepsilon}_{r_0}^p &= -\tilde{\varepsilon}_{\theta_0}^p = A_1(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0}) + A_2\tilde{\tau}_{r\theta_0} \\
\tilde{\varepsilon}_{r\theta_0}^p &= \frac{A_2}{2}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0}) + A_3\tilde{\tau}_{r\theta_0}
\end{aligned}\right\} \tag{2.17b}$$

$$\left.\begin{aligned}
\tilde{\varepsilon}_{r1}^p &= -\tilde{\varepsilon}_{\theta_1}^p = A_1(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + A_2\tilde{\tau}_{r\theta_1} + B_1 \\
\tilde{\varepsilon}_{r\theta_1}^p &= \frac{A_2}{2}(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + A_3\tilde{\tau}_{r\theta_1} + B_2
\end{aligned}\right\} \tag{2.17c}$$

where $\tilde{g}$ and coefficients A_1, A_2, A_3, B_1, B_2 are given in Appendix A.

The strain compatibility equation is

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\varepsilon_\theta) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\varepsilon_r - \frac{1}{r}\frac{\partial}{\partial r}\varepsilon_r - \frac{2}{r^2}\frac{\partial^2}{\partial r\partial\theta}(r\varepsilon_{r\theta}) = 0 \tag{2.18}$$

Substituting (2.15) into (2.18), we can obtain the governing equations of all orders

$$(1)\ \frac{1}{\rho^n}:\qquad \ddot{\tilde{\varepsilon}}^p_{r_*} - nc_1(nc_1-2)\tilde{\varepsilon}^p_{r_*} - 2(1-nc_1)\dot{\tilde{\varepsilon}}^p_{r\theta_*} = 0 \tag{2.19}$$

$$(2)\ \frac{1}{\rho^{n-1}}:\qquad \ddot{\tilde{\varepsilon}}^p_{r0} - (n-1)c_1[(n-1)c_1-2]\tilde{\varepsilon}^p_{r0} - 2[1-(n-1)c_1]\dot{\tilde{\varepsilon}}^p_{r\theta 0}$$
$$= n[(2n-1)c_1c_2 - 2c_2]\tilde{\varepsilon}^p_{r_*} - 2nc_2\dot{\tilde{\varepsilon}}^p_{r\theta_*} \tag{2.20}$$

$$(3)\ \frac{1}{\rho^{n-2}}:\qquad \ddot{\tilde{\varepsilon}}^p_{r1} - (n-2)c_1[(n-2)c_1-2]\tilde{\varepsilon}^p_{r1} - 2[1-(n-2)c_1]\dot{\tilde{\varepsilon}}^p_{r\theta 1}$$
$$= \{2nc_3[(n-1)c_1-1] + nc_2^2(n-1)\}\tilde{\varepsilon}^p_{r_*}$$
$$+(n-1)[c_1(2n-3)-2]c_2\tilde{\varepsilon}^p_{r0} - 2nc_3\dot{\tilde{\varepsilon}}^p_{r\theta_*} - 2(n-1)c_2\dot{\tilde{\varepsilon}}^p_{r\theta 0} \tag{2.21}$$

In addition, there is a term which is related with the elasticity of the material:

$$(4)\ \frac{1}{\rho}:\qquad \ddot{\tilde{\varepsilon}}^e_{r_*} + c_1\tilde{\varepsilon}^e_{r_*} - c_1(1-c_1)\tilde{\varepsilon}^e_{\theta_*} - 2(1-c_1)\dot{\tilde{\varepsilon}}^e_{r\theta_*} \tag{2.22}$$

(2.22) will be added to the right side of the governing equation of the order $\frac{1}{\rho}$ after divided by $\tilde{\alpha}K^{n-1}$.

The boundary conditions have two different cases:

(1) Traction free condition on the crack face

$$\sigma_\theta\big|_{\theta=\pm\pi} = 0 \qquad \tau_{r\theta}\big|_{\theta=\pm\pi} = 0 \tag{2.23}$$

(2) Frictionless contact condition on the crack face

$$\left.\begin{aligned} &u\big|_{\theta=\pi} - u_\theta\big|_{\theta=-\pi} = 0 \\ &\tau_{r\theta}\big|_{\theta=\pi} = \tau_{r\theta}\big|_{\theta=-\pi} = 0 \\ &\sigma_\theta\big|_{\theta=\pi} = \sigma_\theta\big|_{\theta=-\pi} \le 0 \end{aligned}\right\} \tag{2.24}$$

The traction continuity on interface requires

$$\sigma_\theta^+ - \sigma_\theta^- = 0 \qquad \tau_{r\theta}^+ - \tau_{r\theta}^- = 0 \qquad \theta = 0 \tag{2.25}$$

And the displacement continuity on interface requires

$$\left.\begin{aligned} &\varepsilon_r\big|_{\theta=+0} - \varepsilon_r\big|_{\theta=-0} = 0 \\ &(\dot{\varepsilon}_r - 2\varepsilon_{r\theta} - 2\Omega\varepsilon'_{r\theta})\big|_{\theta=+0} - (\dot{\varepsilon}_r - 2\varepsilon_{r\theta} - 2\Omega\varepsilon'_{r\theta})\big|_{\theta=-0} = 0 \end{aligned}\right\} \tag{2.26}$$

Eqs.(2.19)–(2.26) comprise the governing equations for the asymptotic expansion.

III. SOLUTION OF GOVERNING EQUATIONS

It is noted that, our analysing method is only suitable to the cases where both n_1 and n_2 take integer values. In what follows a interfacial crack problem with $n_1 = 5, n_2 = 3$ will be investigated in detail.

For the first-order near-tip field, its governing equation is (2.19), in which $n = n_1$ within $0 \le \theta \le \pi$ while $n = n_2$ in $0 \le \theta \le \pi$. The corresponding boundary condition on crack faces can be expressed by the stress function as

(1) Traction free

$$F_*(\pm\pi) = \dot{F}_*(\pm\pi) = 0 \tag{3.1a}$$

(2) Frictionless contact

$$\dot{F}_*(\pm\pi) = \ddot{F}_*(\pi) = 0 \qquad F_*(\pi) = F_*(-\pi) \leq 0 \tag{3.1b}$$

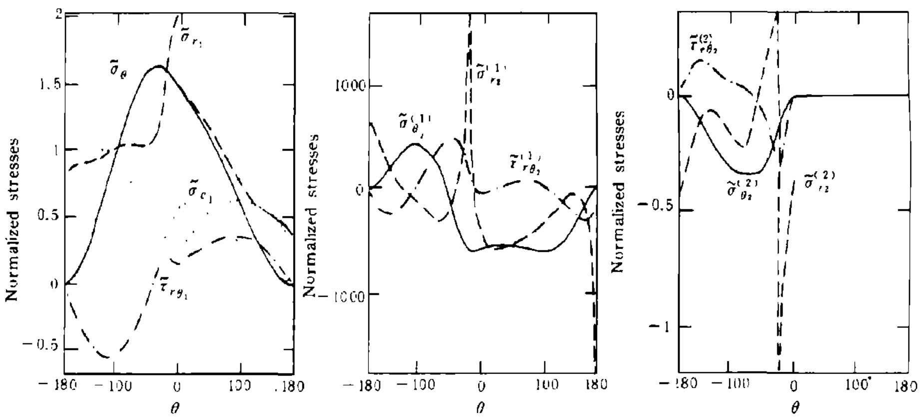
The traction continuity on the interface

$$F_*^+(0) = F_*^-(0) \qquad \dot{F}_*^+(0) = \dot{F}_*^-(0) \tag{3.2}$$

The displacement continuity on the interface

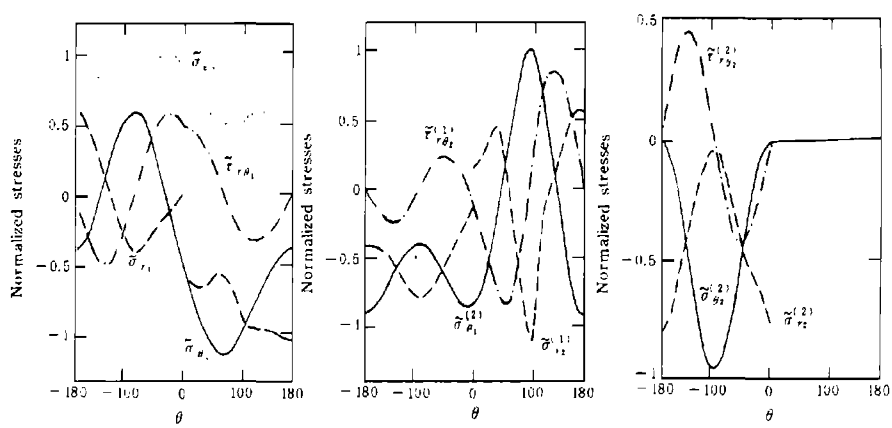
$$\tilde{\varepsilon}^p_{r_*}\big|_{\theta=+0} = 0 \qquad [\dot{\tilde{\varepsilon}}^p_{r_*} - 2(1-n_1c_1)\tilde{\varepsilon}^p_{r\theta_*}]\big|_{\theta=+0} = 0 \tag{3.3}$$

The solution of the first-order near-tip fields has been obtained by Xia and Wang[8] using the normalization $\max\{\tilde{\sigma}_{e_*}\} = 1$, shown in Fig.2(a) and Fig.3(a). The results proved that the singular exponent c_1 is determined by the lower hardening material (i.e. the material with larger n), i.e. $c_1 = 1/(n_1+1)$.



(a) The first-order stress field (b) The second-order stress field(1) (c) The second-order stress field (2)

Fig.2 Traction free condition on the crack faces $n_1 = 5, n_2 = 3, M^p = 0.937284$



(a) The first-order stress field(1) (b) The second-order stress field(1) (c) The second-order stress field(2)

Fig.3 Frictionless contact condition on the crack faces $n_1 = 5, n_2 = 3, M^p = -0.515412$

Fig.2(a) and Fig.3(a) correspond to the traction free condition and frictionless contact condition on the crack faces, respectively. Their mixity parameters are $M^p_{\text{open}} = 0.937284$ and $M^p_{\text{contact}} = -0.515412$. (The mixity parameter is defined by Shih[13]: $M^p = \frac{2}{\pi}\text{tg}^{-1}\left(\frac{\tilde{\sigma}_\theta(0)}{\tilde{\tau}_{r\theta}(0)}\right)$.

The analogous analysis can be carried out for the second-order near-tip fields. Its governing equation is (2.20). Here $n = n_1$ in the upper part and $n = n_2$ in the lower part.

The boundary conditions on the crack faces and the traction continuity conditions on the interface are the same as (3.1a), (3.1b) and (3.2) except that F_* is replaced by F_0.

The second-order displacement continuity on the interface is

$$\left.\begin{array}{l} \tilde{\varepsilon}^p_{r_0}\big|_{\theta=+0} = 0 \\ \{\dot{\tilde{\varepsilon}}_{r_0} - 2[1-(n_1-1)c_1\tilde{\varepsilon}^p_{r\theta_0} + 2n_1c_2\tilde{\varepsilon}_{r\theta_*}\}\big|_{\theta=+0} = 0 \end{array}\right\} \tag{3.4}$$

Through a simple analysis, we find that, the solutions of the second-order fields are $c_2(n_1+1)$ times as large as those of the first order fields, i.e.

$$\tilde{\sigma}_{\beta\gamma_*} = c_2(n_1+1)\tilde{\sigma}_{\beta\gamma_*} \tag{3.5}$$

However, the situation becomes relatively complicated for the third-order near-tip fields. Using (2.21) and (2.22) directly yields the governing equation

$$\begin{array}{ll} D_1\ddot{\ddot{F}}_1 + D_2\dddot{F}_1 + D_3\ddot{F}_1 + D_4\dot{F}_1 + D_5F_1 = c_3D_a & 0 \le \theta \le \pi \\ D_1\ddot{\ddot{F}}_1 + D_2\dddot{F}_1 + D_3\ddot{F}_1 + D_4\dot{F}_1 + D_5F_1 = c_3D_a - \dfrac{1}{\bar{\alpha}_2K^{n_2-1}}D_b & \pi \le \theta \le 0 \end{array} \tag{3.6}$$

where the coefficients D_1 to D_5, D_a and D_b are the functions of F_* and its corresponding derivatives, given in Appendix A.

The boundary conditions of the third-order fields are

(1) Traction free

$$\left.\begin{array}{l} F_1(\pm\pi) = 0 \\ \dot{F}_1(\pm\pi) = 0 \end{array}\right\} \tag{3.7}$$

(2) Frictionless contact

$$\left.\begin{array}{l} \dot{F}_1(\pm\pi) = 0 \\ F_1(\pi) = F_1(-\pi) \le 0 \\ \dfrac{\bar{\alpha}_1K^{n_1}}{E_1}\left\{\dot{\tilde{\varepsilon}}^p_{r_1} - 2[1-(n_1-2)c_1]\tilde{\varepsilon}^p_{r\theta_1} + 2(n_1-1)c_2\tilde{\varepsilon}^p_{r\theta_0} + 2n_1c_3\tilde{\varepsilon}^p_{e\theta_*}\right\}\big|_{\theta=\pi} \\ = \dfrac{\bar{\alpha}_2K^{n_2}}{E_2}[\dot{\tilde{\varepsilon}}^p_{r_*} - 2\tilde{\varepsilon}^p_{r\theta_*} + 2n_2c_1\tilde{\varepsilon}^p_{r\theta_*}]\big|_{\theta=-\pi} \end{array}\right\} \tag{3.8}$$

The traction continuity on the interface

$$\left.\begin{array}{l} F_1^+(0) = F_1^-(0) \\ \dot{F}_1^+(0) = \dot{F}_1^-(0) \end{array}\right\} \tag{3.9}$$

The displacement continuity on the interface

$$\left.\begin{aligned}&\frac{\bar\alpha_1K^{n_1}}{E_1}(\tilde\varepsilon^p_{r_1})\big|_{\theta=+0}=\frac{\bar\alpha_2K^{n_2}}{E_2}(\tilde\varepsilon^p_{r_*})\big|_{\theta=-0}\\&\frac{\bar\alpha_1K^{n_1}}{E_1}\left\{\dot{\tilde\varepsilon}^p_{r_1}-2[1-(n_1-2)c_1]\tilde\varepsilon^p_{r\theta_1}+2(n_1-1)c_2\tilde\varepsilon^p_{r\theta_0}+2n_1c_3\tilde\varepsilon^p_{r\theta_*}\right\}\big|_{\theta=+0}\\&\quad=\frac{\bar\alpha_2K^{n_2}}{E_2}[\dot{\tilde\varepsilon}^p_{r_*}-2\tilde\varepsilon^p_{r\theta_*}+2n_2c_1\tilde\varepsilon^p_{r\theta_*}]\big|_{\theta=-0}\end{aligned}\right\}\tag{3.10}$$

(3.10) can be further expressed in terms of the stress functions as

$$\left.\begin{aligned}&a_1\ddot F_1^+(0)+a_2\dot F_1^+(0)+a_3F_1^+(0)=\frac{\bar\alpha_2E_1}{\bar\alpha_1E_2}K^{n_2-n_1}a_4+c_3a_5\\&b_1\dddot F_1^+(0)+b_2\ddot F_1^+(0)+b_3\dot F_1^+(0)+b_4F_1^+(0)=\frac{\bar\alpha_2E_1}{\bar\alpha_1E_2}K^{n_2-n_1}b_5+c_3b_6\end{aligned}\right\}\tag{3.10$'$}$$

where the coefficients a_1 to a_5 and b_1 to b_6 are only associated with the first-order fields. Their expressions are given in Appendix A.

Similarly, the third equation of (3.8) can be written as

$$b_1'\dddot F_1(\pi)+b_2'\ddot F_1(\pi)+b_3'\dot F_1(\pi)+b_4'F_1(\pi)=\frac{\bar\alpha_2E_1}{\bar\alpha_1E_2}K^{n_2-n_1}b_5'+c_3b_6'\tag{3.11}$$

where b_1' to b_6' can be found in Appendix A. Analogous to the method used by Xia and Wang[14], we may solve (3.6)–(3.11) to get the third-order stress field

$$\tilde\sigma_{\beta\gamma_1}=\frac{\bar\alpha_2E_1}{\bar\alpha_1E_2}K^{n_2-n_1}\tilde\sigma^{(1)}_{\beta\gamma_1}+\frac{1}{\bar\alpha_2K^{n_2-1}}\tilde\sigma^{(2)}_{\beta\gamma_1}+c_3\tilde\sigma^{(3)}_{\beta\gamma_1}\tag{3.12}$$

where $\tilde\sigma^{(3)}_{\beta\gamma_1}$ bears a relationship with the first-order fields

$$\tilde\sigma^{(3)}_{\beta\gamma_1}=\frac{(n_1+1)}{2}\tilde\sigma_{\beta\gamma_*}\tag{3.13}$$

$\tilde\sigma^{(1)}_{\beta\gamma_1}$ and $\tilde\sigma^{(2)}_{\beta\gamma_1}$ are shown as in Fig.2(b), (c) and Fig.3(b), (c). Here Figs.2 correspond to the case when crack faces open freely while Figs.3 the case when crack faces contact each other without friction.

IV. DISCUSSION

Sum up the computation and analyses of above section, we can obtain the stress field around the interfacial crack tip

$$\begin{aligned}\sigma_{\beta\gamma}=\sigma_oK\Big\{&\Big[\frac{1}{\rho}+c_2(n_1+1)+c_3\frac{(n_1+1)}{2}\rho\Big]\tilde\sigma_{\beta\gamma_*}\\&+\rho\Big[\frac{\bar\alpha_2E_1}{\bar\alpha_1E_2}K^{n_2-n_1}\tilde\sigma^{(1)}_{\beta\gamma_1}+\frac{1}{\bar\alpha_2K^{n_2-1}}\tilde\sigma^{(2)}_{\beta\gamma_1}\Big]\Big\}\end{aligned}\tag{4.1}$$

where eigenvalue c_1 has been determined through the solution of the first-order fields, i.e. $c_1=1/(n_1+1)$. c_2 and c_3 are two arbitrary coefficients. Their values can be decided by the comparison with the numerical full-field solutions of the finite element computations.

Letting both c_2 and c_3 equal to zero will lead to a simple form of the stress field

$$\sigma_{\beta\gamma} = \sigma_o K \left\{ \frac{1}{\rho} \tilde{\sigma}_{\beta\gamma_*} + \rho \left[\frac{\bar{\alpha}_2 E_1}{\bar{\alpha}_1 E_2} K^{n_2-n_1} \tilde{\sigma}^{(1)}_{\beta\gamma_1} + \frac{1}{\bar{\alpha}_2 K^{n_2-1}} \tilde{\sigma}^{(2)}_{\beta\gamma 1} \right] \right\}$$

$$\approx \sigma_o K \left\{ (\frac{1}{r})^{\frac{1}{n_1+1}} \tilde{\sigma}_{\beta\gamma_*} r^{\frac{1}{n_1+1}} \left[\frac{\bar{\alpha}_2 E_1}{\bar{\alpha}_1 E_2} K^{n_2-n_1} \tilde{\sigma}^{(1)}_{\beta\gamma_1} + \frac{1}{\bar{\alpha}_2 K^{n_2-1}} \tilde{\sigma}^{(2)}_{\beta\gamma_1} \right] \right\} \tag{4.2}$$

Provided that the elastic moduli of two interface materials are equal to each other, i.e. $E_1 = E_2$; furthermore, $\nu_1 = \nu_2 = 0.3; \alpha_1 = \alpha_2 = 0.1$; but $\sigma_{o1} \neq \sigma_{o2}$; $n_1 = 5, n_2 = 3$; we have

$$\bar{\alpha}_1 = \alpha_1 = 0.1 \qquad \bar{\alpha}_2 = \alpha_2 \left(\frac{\sigma_{o1}}{\sigma_{o2}} \right)^{n_2-1} = 0.1 \left(\frac{\sigma_{o1}}{\sigma_{o2}} \right)^{n_2-1} \tag{4.3}$$

According to Shih and Asaro[3], the amplitude K is

$$K = \left(\frac{J}{\bar{\alpha}_1 \sigma_o \varepsilon_o L} \right)^{1/(n_1+1)} \tag{4.4}$$

Under the small yielding condition[3]

$$J = \Lambda K_c \bar{K}_c \tag{4.5}$$

where K_c is the complex stress intensity factor, and

$$\Lambda = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) / (2 \cos \mathrm{h}^2 \pi \varepsilon) \tag{4.6}$$

As mentioned above, we have assumed that $E_1 = E_2, \nu_1 = \nu_2$. Therefore $\varepsilon = 0$. It leads to

$$\Lambda = \frac{1-\nu^2}{E} \tag{4.6}$$

If the characteristic length is further taken to be $L = (K_c \bar{K}_c)/\sigma_o^2$, then

$$K = \left(\frac{1-\nu^2}{\alpha_1} \right)^{\frac{1}{n_1+1}} = 1.44 \tag{4.7}$$

Moreover,

$$\frac{\bar{\alpha}_2 E_1}{\bar{\alpha}_1 E_2} K^{n_2-n_1} = \frac{1}{K^2} \left(\frac{\sigma_{o1}}{\sigma_{o2}} \right)^2 , \qquad \frac{1}{\bar{\alpha}_2 K^{n_2-1}} = \frac{10}{K^2} \left(\frac{\sigma_{o2}}{\sigma_{o1}} \right)^2 \tag{4.8}$$

Therefore, (4.2) can be written as

$$\sigma_{\beta\gamma} = 1.44 \sigma_o \left\{ \left(\frac{1}{r} \right)^{1/6} \tilde{\sigma}_{\beta\gamma_*} + \frac{1}{1.44^2} \left(\frac{\sigma_{o2}}{\sigma_{o1}} \right)^2 r^{1/6} \left[\left(\frac{\sigma_{o1}}{\sigma_{o2}} \right)^4 \tilde{\sigma}^{(1)}_{\beta\gamma_1} + 10 \tilde{\sigma}^{(2)}_{\beta\gamma_1} \right] \right\} \tag{4.9}$$

In what follows, two different cases will be considered: (1) $\sigma_{o2}/\sigma_{o1} = 2$; (2) $\sigma_{o2}/\sigma_{o1} = 5$.

The angular distributions of $\alpha_{\beta\gamma}/(1.44\sigma_0/r^{\frac{1}{\varepsilon}})$ for above two cases are shown by Fig.4 and Fig.5, respectively. Here Fig.4(a) and Fig.5(a) correspond to $r = 10^{-8}$ while Fig.4(b) and Fig.5(b) to $r = 10^{-6}$. Moreover, Fig.6(a), (b) present the radial distributions of $\sigma_{\beta\gamma}/(1.44\sigma_o r^{1/6})$ for two cases. They indicate that the leading singular term of the stress solution (corresponding to the first term of (4.9)) will dominate within the range of $r < 10^{-8}$.

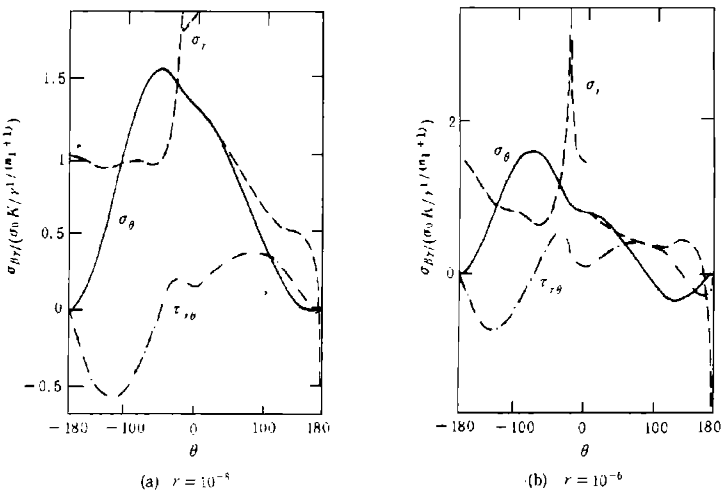


Fig.4 Angular distribution of the stresses for the traction free condition on the crack faces ($\sigma_{02}/\sigma_{01} = 2$)

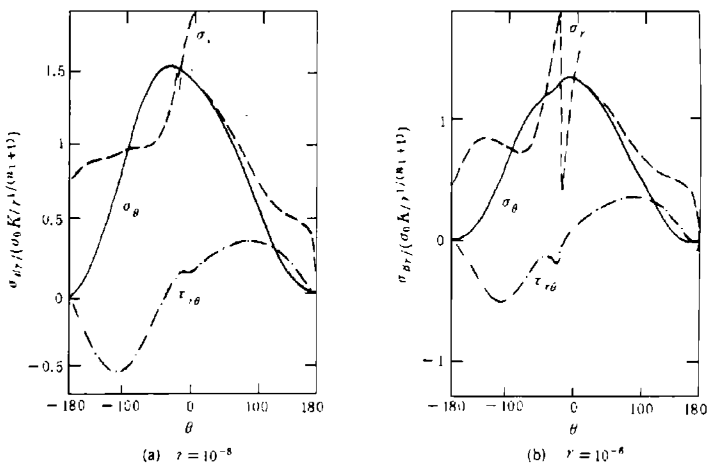


Fig.5 Angular distribution of the stresses for the traction free condition on the crack faces ($\sigma_{02}/\sigma_{01} = 5$)

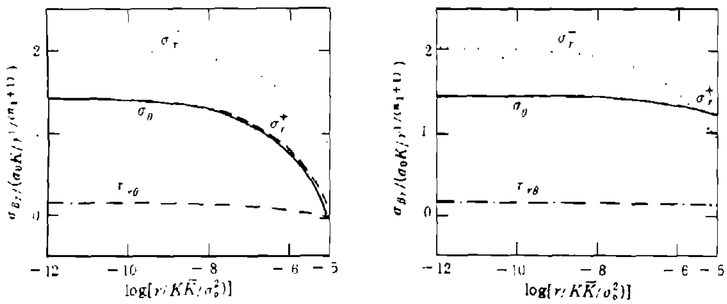
Based on the principle that the second term has to be smaller than the leading singular term $(\frac{1}{r})^{1/6}\tilde{\sigma}_{\beta\gamma_*}$ in (4.8), we can estimate the rational existing zone of the stress solution (4.9)

$$(1) \quad r < 2.57 \times 10^{-6} \qquad \text{for} \qquad \sigma_{o2}/\sigma_{o1} = 2 \tag{4.10}$$

$$(2) \quad r < 1 \times 10^{-6} \qquad \text{for} \qquad \sigma_{o2}/\sigma_{o1} = 5 \tag{4.11}$$

It can be found that the extent of the rational zone will change for different values of the ratio σ_{o2}/σ_{o1}. But generally speaking, the separable form of near-tip fields for the interfacial crack discussed here only dominate within a very small range ($r < 10^{-6}$). This result coincides with that of Shih[11].

Compared with the above discussed case (i.e. the crack faces open freely), the range in which the asymptotic solutions dominate will be much larger for the frictionless contact condition on the crack faces. This feature can be seen clearly in Fig.3.



(a) $\sigma_{o2}/\sigma_{o1} = 2$ (b) $\sigma_{o2}/\sigma_{o1} = 5$

Fig.6 Radial distribution of the stresses for the traction free condition on the crack faces ($\theta = 0°$, $n_1 = 5$, $n_2 = 3$)

V. CONCLUSIONS

In this paper an asymptotic expanding method of a regular series is introduced to investigate the interfacial crack in two different elastic power-law hardening materials. An exactly higher-order analysis is carried out for its near-tip stress strain fields. The solutions including first three terms are derived for asymptotic series of the stress field, with two different boundary conditions on the crack faces being considered: (1) traction free and (2) frictionless contact.

Our analyses show that along the interface ahead of the crack tip the stress fields of the upper part will keep co-order continuous with those of the lower part. But for the displacement fields, the first-, the second-order fields of the upper part are equal to zero on the interface; the third-order field of the upper part will be continuous with the first-order one of the lower part. These co-order continuity of the stress field and cross-order continuity of the displacement field are important features of the near-tip stress strain fields of the interfacial crack in two dissimilar elastic-plastic materials.

According to our investigations, the solutions of the fully continuous, first-order stress field can be only obtained for two specific mixity parameter M^p. For $n_1 = 5, n_2 = 3$, they are $M^p_{\text{open}} = 0.937284$, $M^p_{\text{contact}} = -0.515412$.

The second-order stress field is simply equal to $c_2(n_1+1)$ times the first-order stress field, i.e. $\tilde{\sigma}_{\beta\gamma_0} = c_2(n_1+1)\tilde{\sigma}_{\beta\gamma_*}$. For the third-order stress field, its solution is composed of three parts (see (3.12)).

In the case of $n_1 = 5, n_2 = 3$, the rational existing range of the asymptotic series solutions is estimated for the traction free condition on the crack faces. It is shown that if $\sigma_{o2}/\sigma_{o1} = 2,\ r < 2.57 \times 10^{-6}$; and if $\sigma_{o2}/\sigma_{o1} = 5,\ r < 1 \times 10^{-6}$.

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APPENDIX A

Note that $n = n_1$ for upper part while $n = n_2$ for lower part.

$$\left.\begin{aligned} \tilde{\varepsilon}^p_{r_0} &= A_1(\tilde{\sigma}_{r_6} - \tilde{\sigma}_{\theta_0}) + A_2\tilde{\tau}_{r\theta_0} \\ \tilde{\varepsilon}^p_{r\theta_0} &= \frac{A_2}{2}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0}) + A_3\tilde{\tau}_{r\theta_0} \end{aligned}\right\} \tag{A.1}$$

$$\left.\begin{aligned} \tilde{\varepsilon}^p_{r_1} &= A_1(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + A_2\tilde{\tau}_{r\theta_1} + B_1 \\ \tilde{\varepsilon}^p_{r\theta_1} &= \frac{A_2}{2}(\tilde{\sigma}_{r_1} - \tilde{\sigma}_{\theta_1}) + A_3\tilde{\tau}_{r\theta_1} + B_2 \end{aligned}\right\} \tag{A.2}$$

where

$$\left.\begin{aligned} A_1 &= \frac{3^{(n+1)/2}}{4}\tilde{g}^{(n-3)/2}[(n-1)\tilde{s}^2_{r*} + \tilde{g}^2] \\ A_2 &= \frac{3^{(n+1)/2}}{4}\tilde{g}^{(n-3)/2}[2(n-1)\tilde{s}_{r_*}\tilde{\tau}_{r\theta_*}] \\ A_3 &= \frac{3^{(n+1)/2}}{4}\tilde{g}^{(n-3)2}[2(n-1)\tilde{\tau}^2_{r\theta_*} + \tilde{g})] \end{aligned}\right\} \tag{A.3}$$

$$\tilde{g} = \tilde{s}^2_{r_*} + \tilde{\tau}^2_{r\theta_*} \qquad \tilde{s}_{r_*} = \frac{1}{2}(\tilde{\sigma}_{r_*} - \tilde{\sigma}_{\theta_*})$$

$$\left.\begin{aligned} B_1 = \frac{3}{2}\Big\{&\frac{n-1}{2}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-3)/2}[\frac{3}{4}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0})^2 + 3\tilde{\tau}_{r\theta_0}]\tilde{s}_{r_*} \\ &+ \frac{n-1}{2}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-3)/2}\tilde{\sigma}_{e_0}\tilde{s}_{r_0} + \frac{(n-1)(n-3)}{8}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-5)/2}\tilde{\sigma}^2_{e_0}\tilde{s}_{r_*}\Big\} \\ B_2 = \frac{3}{2}\Big\{&\frac{n-1}{2}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-3)/2}[\frac{3}{4}(\tilde{\sigma}_{r_0} - \tilde{\sigma}_{\theta_0})^2 + 3\tilde{\tau}^2_{r\theta_0}]\tilde{\tau}_{r\theta_*} \\ &+ \frac{n-1}{2}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-3)/2}\tilde{\sigma}_{e_0}\tilde{\tau}_{r\theta_0} + \frac{(n-1)(n-3)}{8}\left(\tilde{\sigma}^2_{e_*}\right)^{(n-5)/2}\tilde{\sigma}^2_{e_*}\tilde{\tau}_{r\theta_*}\Big\} \end{aligned}\right\} \tag{A.4}$$

In (3.6)

$$\left.\begin{aligned} D_1 &= A_1 \\ D_2 &= 2\dot{A}_1 - (1+c_1)A_2 - \frac{1}{2}c_bA_2 \\ D_3 &= \ddot{A}_1 - 2(1+c_1)\dot{A}_2 - (2+c_1)c_1A_1 - c_aA_1 - c_b[\frac{1}{2}\dot{A}_2 - (1+c_1)A_3] \\ D_4 &= -[(1+c_1)\ddot{A}_2 + 2(2+c_1)c_1A_1] + c_a(1+c_1)A_2 + c_b[(1+c_1)\dot{A}_3 + \frac{1}{2}(2+c_1)c_1A_2] \\ D_5 &= -(2+c_1)c_1\ddot{A}_1 + c_a(2+c_1)c_1A_1 + \frac{1}{2}c_b(2+c_1)c_1\dot{A}_2 \end{aligned}\right\} \tag{A.5}$$

$$\left.\begin{aligned} D_a &= 2n[(n-1)c_1-1]\tilde{\varepsilon}^p_{r_*} - 2n\tilde{\varepsilon}^p_{r\theta_*} \\ &\quad -\{2(\ddot{A}_1F_* + 2\dot{A}_1\dot{F}_* + A_1\ddot{F}_*) + \ddot{A}_2\dot{F}_* + 2\dot{A}_2\ddot{F}_* + A_2\dddot{F}_*\} \\ &\quad +c_a(2A_1F_* + A_2\dot{F}_*) + c_b(\dot{A}_2F_* + A_2\dot{F}_* + \dot{A}_3\dot{F}_* + A_3\ddot{F}_*) \\ D_b &= \ddot{\tilde{\varepsilon}}^e_{r_*} + c_1\tilde{\varepsilon}^e_{r_*} - c_1(1-c_1)\tilde{\varepsilon}^e_{\theta_*} - 2(1-c_1)\dot{\tilde{\varepsilon}}^e_{r\theta_*} \end{aligned}\right\} \tag{A.6}$$

where

$$c_a = (n-2)c_1[(n-2)c_1-2]\ , \qquad c_b = 2[1-(n-2)c_1]$$

In (3.10')

$$\left\{\begin{aligned} a_1 &= A_1\big|_{\theta=+0} \\ a_2 &= -(1+c_1)A_2\big|_{\theta=+0} \\ a_3 &= -(2+c_1)c_1A_1\big|_{\theta=+0} \\ a_4 &= \tilde{\varepsilon}^p_{r_*}\big|_{\theta=-0} \\ a_5 &= (2F_*A_1 + \dot{F}_*A_2)\big|_{\theta=+0} \end{aligned}\right. \tag{A.7}$$

$$\left\{\begin{aligned} b_1 &= A_1\big|_{\theta=+0} \\ b_2 &= -[(1+c_1)A_2 + \langle 1-(n_1-2)c_1\rangle A_2 - \dot{A}_1]\big|_{\theta=+0} \\ b_3 &= -\{(2+c_1)c_1A_1 + (1+c_1)\dot{A}_2 - 2[1-(n_1-2)c_1](1+c_1)A_3\}\big|_{\theta=+0} \\ b_4 &= -\{(2+c_1)c_1A_1 - [1-(n_1-2)c_1](2+c_1)c_1\}\big|_{\theta=+0} \\ b_5 &= \{\tilde{\varepsilon}^p_{r_*} - 2\tilde{\varepsilon}^p_{r\theta_*} + 2n_2c_1\tilde{\varepsilon}^p_{r\theta_*}\}\big|_{\theta=-0} \\ b_6 &= \{2n_1\tilde{\varepsilon}^p_{r\theta_*} + A_2\ddot{F}_* + \langle 2A_1 + \ddot{A}_2 - 2[1-n_1-2)c_1]A_3\rangle \dot{F}_* \\ &\quad +\langle 2\dot{A}_1 - [1-(n_1-2)c_1]A_2\rangle A_2F_*\}\big|_{\theta=+0} \end{aligned}\right. \tag{A.8}$$

The expressions of b'_1 to b'_6 in (3.11) are the same as (A.8) except that values taken at $\theta = +0$ and -0 should be taken at $\theta = \pi$ and $-\pi$, respectively.