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COMPUTER SIMULATION OF CREEP DAMAGE AT CRACK TIP IN SHORT FIBRE COMPOSITES

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ABSTRACT: Creep damage at crack tip in short fibre composites has been simulated by using the finite element method (FEM). The well-known Schapery non-linear viscoelastic constitutive relationship was used to characterize time-dependent behaviour of the material. A modified recurrence equation was adopted to accelerate the iteration. Kachanov-Rabotnov's damage evolution law was employed. The growth of the damage zone with time around the crack tip was calculated and the results were shown with the so-called "digit photo", which was produced by the printer.

KEY WORDS: creep damage, visco-elasticity, finite element method, short fibre composite, computer simulation

I. INTRODUCTION

The non-linear visco-elastic properties with consideration of creep damage of random short fibre composites have been studied experimentally in a previous paper[1]. The Schapery's constitutive equation was used to describe the creep behaviour and the constitutive parameters were determined experimentally. In this paper, computer simulation with FEM of creep damage at crack tip in the material will be presented. Of various visco-elastic constitutive relations[2], both linear and non-linear, the Schapery's equation is regarded to be appropriate to describe creep behaviour of the short fibre composites.

Short fibre composites, typified by sheet moulding compound (SMC) and chopped strand mat (CSM) reinforced plastics[1], possess the mechanical properties of small failure strain and vulnerability to damage, and exhibit nonlinearity of viscoelasticity.

In order to accelerate our computation, the Brinson's recursion relation[3] was modified, and a new recurrence equation was proposed.

Then, the FEM analysis was performed. The eight node quadrilateral isoparametric element was adopted in the computation. At the crack tip, the so-called collapsed triangular quarter point singular elements were patched to fit the stress singularity at crack tip. The damage evolution during creep at crack tip was simulated using Kachanov-Rabtonov's law. And finally, the evolutions and redistributions of stress and damage zone were obtained and illustrated with so called "digit photograghs", which were produced by the printer.

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II. SCHAPERY EQUATION

The Schapery nonlinear visco-elastic constitutive relation is given as[1,4]

$$\varepsilon(t) = A_0 g_0 \sigma + g_1 \int_0^t \triangle A(\psi - \psi') \frac{\partial(g_2\sigma)}{\partial\tau} \mathrm{d}\tau \tag{1}$$

where A_0 and $\triangle A$ are instantaneous elastic compliance and creep compliance, respectively, and the reduced time ψ is defined as

$$\psi = \int_0^t \frac{\mathrm{d}t'}{a_\sigma} \qquad \psi' = \int_0^\tau \frac{\mathrm{d}t'}{a_\sigma} \tag{2}$$

where g_0, g_1, g_2 and a_σ are dimensionless stress-dependent material parameters. When the input stress σ is low, $g_0 = g_1 = g_2 = a_\sigma = 1$, Eq. (1) is reduced to the equation of Boltzmann superposition principle. It is seen that in Eqs.(1) and (2) there are one function of time and four functions of stress which have to be evaluated experimentally.

For many kinds of materials, $\triangle A(\psi)$ can be approximated by a power function

$$\triangle A(\psi) = S_1 \psi^n \tag{3}$$

where S_1 and n are constants and independent of applied stress level and creep time. So, we also have three material constants A_0, S_1, n to be determined experimentally.

III. BRIEF DESCRIPTION OF THE MATERIAL

The material to be simulatied is chopped strand mat (CSM) glass fibre reinforced F-grade epoxy. The composite plaque was manufactured by hand-lay-up method. The nominal thickness is 2.75mm. The length of the chopped glass fibre ranges from 40mm to 60mm, and the weight fraction of glass fibre is about 30%. It is in-plane isotropic.

The experimental procedure and results have been presented in Ref.[1]. The material constitutive parameters are summarized as follows:

$$g_0 = 1$$
$$g_1 = (0.00692\sigma/\sinh(0.00692\sigma)$$
$$g_2 = (0.01102\sigma)/\sinh(0.01102\sigma)$$
$$a_\sigma = (0.00830\sigma)/\sinh(0.00830\sigma)$$
$$n = 0.141$$
$$S_0 = 74.532\mu\varepsilon/\mathrm{MPa}$$
$$S_1 = 12.500\mu\varepsilon/\mathrm{MPa}/\mathrm{min}^{0.141}$$

In calculating g_1, g_2 and a_σ, the absolute value of σ should be used.

IV. DAMAGE EVOLUTION LAW

In the previous paper[1], the constitutive theory (1) was used to characterise creep behaviour of chopped strand mat glass fibre reinforced epoxy composite (CSM-GRP). The results show clearly that the creep damage plays a significant role in creep behaviour of the material. At crack tip (or notch tip), the effect of damage is more serious.

In the simulation of creep damage evolution, the influence of damage on stress distribution has to be taken into account and the nominal stress σ has to be replaced by the *real stress $\tilde{\sigma}$, which is related to damage parametor D by the following equation*

$$\tilde{\sigma} = \frac{\sigma}{1-D} \tag{4}$$

During creep, the damage grows according to the Kachanov-Rabotnov's damage evolution law[7]

$$\dot{D} = (\frac{\sigma}{R})^r (1-D)^{-k} \tag{5}$$

where D can be defined in many ways. Here we define

$$D = 1 - \frac{E_D}{E} \tag{6}$$

E_D is damaged modulus. In Eq.(5), R, r and k are all material constants, and can be determined with creep experiments.

Under creep condition, the load is kept constant. Simple integration of Eq.(5) leads to

$$(1-D)^{k+1} = 1 - (k+1)(\sigma/R)^r t \tag{7}$$

The experimental procedure is: loading at high rate to a certain stress level σ_i, followed by creep for a time duration t_c, and then unloading to $\sigma = 0$ as fast as possible.

The elastic strain in the loading stage is

$$\varepsilon_0 = \sigma/E \tag{8}$$

and the elastic strain in the unloading stage is

$$\varepsilon_{t_c}^- - \varepsilon_{t_c}^+ = \sigma/E_D \tag{9}$$

According to Eq.(6), we have[5]

$$D(\sigma_i, t_c) = 1 - \varepsilon_0/(\varepsilon_{t_c}^- - \varepsilon_{t_c}^+) \tag{10}$$

Several loading-creep-unloading experiments were conducted with different stress levels σ_i. Applying the least square method to Eq.(7), the three parameters r, k and R were determined. The results are as follows

$$R = 210\text{MPa}$$

$$r = 25$$

$$K = 6$$

V. RECURRENCE RELATIONSHIP

In the vicinity of the crack tip, the stress changes with time during creep. When the hereditary integration Eq.(1) is used to calculate creep strain in the zone near crack, an analytic solution can not be obtained. Thus, we can only resort to numerical integration method to obtain approximate solution. In Ref.[3], Brinson et al proposed the following recurrence relation

$$\begin{aligned}\varepsilon_N = \; & A_0 g_0^N \sigma^N + g_1^N S_1 \{ g_2^1 \sigma^1 (\psi_N)^N + (g_2^2\sigma^2 - g_2^1\sigma^1)(\psi_N - \frac{t_1}{a_\sigma^1})^n + \cdots \\ & + (g_2^N\sigma^N - g_2^{N-1}\sigma^{N-1})(\psi_N - \psi_{N-1})^n \}\end{aligned} \quad (11)$$

$$\psi_N = \sum_{k=1}^{N} \frac{t_k - t_{k-1}}{a_\sigma^k} \qquad \psi_{N-1} = \sum_{k=1}^{N-1} \frac{t_k - t_{k-1}}{a_\sigma^k} \quad (12)$$

The above relation has a shortcoming, i.e., when calculating strain at time t_N the values of stress field and the parameters g_0, g_1, g_2 and a_σ of all the steps from t_0 to t_{N-1} must be used. Thus, with increase of integration steps, the computing speed will be getting slower and slower. Moreover, this method needs a very large computer memory.

In this paper, we propose a modified recurrence equation. Making the binomial expansion of $(1 - \frac{\psi_{N-1}}{\psi_N})$ and combining the result with Eq.(11), we obtain

$$\begin{aligned}\varepsilon_N = \; & g_0^N A_0 \sigma^N + g_1^N g_2^N S_1 \sigma^N \psi_N^n \\ & + g_1^N S_1 \psi_N^n \sum_{p=1}^{m} (-1)^p \frac{n(n-1)\cdots(n+1-p)}{p!} A_N^p\end{aligned} \quad (13)$$

$$A_N^p = (\frac{\psi_{N-1}}{\psi_N})^p (A_{N-1} + g_2^N \sigma^N - g_2^{N-1}\sigma^{N-1}) \quad (14)$$

where $\psi_N = \psi_{N-1} + (t_N - t_{N-1})/a_\sigma^N$. It can be seen that calculating ε_N using Eqs.(13) and (14) needs only values of stresses and parameters at the step of t_{N-1}. Therefore, Eqs.(13) and (14) have greatly simplified the calculation.

Comparison of computation shows that the calculation speed of recurrence relations (13) and (14) is far higher than that of (11) and (12) and the accuracy of the new relations is fairly good.

VI. FINITE ELEMENT METHOD FORMULATION

1. The Initial Strain Method

In the numerical simulation of creep damage of crack tip in the non-linear viscoelastic material, the initial strain method was used. The concept of initial strain can be shown by the following equation

$$\{\sigma\} = [C](\{\varepsilon\} - \{\varepsilon_0\}) \quad (15)$$

where $\{\sigma\}, \{\varepsilon\}, \{\varepsilon_0\}$ are stress, strain and initial strain vectors, respectively; $[C]$ is material elastic matrix. In the following, we will treat the visco-strain as the initial strain.

Simple derivation of minimizing the potential function leads to the matrix equation:

$$[K]\{U\} = \{R\} + \{R_\varepsilon\} \quad (16)$$

$$[K]=\sum_e K_e=\sum_e\int_{v_e}[B]^{\mathrm{T}}[C]_e[B]\mathrm{d}v_e \tag{17}$$

$$\{R\}=\sum_e\{R_e\} \tag{18}$$

$$\{R_\varepsilon\}=\sum_e\{R_{\varepsilon_0 e}\}=\sum_e[B][C]_e\{\varepsilon_0\}\mathrm{d}v_e \tag{19}$$

where $\sum_e$ means making summation through each elements, $[K]$ represents stiffness matrix, $[B]$ strain matrix, $[R]$ and $[R_\varepsilon]$ are node force and initial node force vectors, respectively, and $\{U\}$ node displacement vector. In our case, the initial strain can be calculated as

$$\varepsilon_0=\varepsilon^v=g_1\int_0^t S_1:(\psi-\psi')^n\frac{\partial g_2\sigma}{\partial\tau}\mathrm{d}\tau \tag{20}$$

Here the Gauss quadrature technique as well as the recurrence Eqs.(13) and (14) are used.

2. The Damage Stiffness Matrix

When material in some area is damaged, the material modulus E should be replaced by E_D. Making use of isotropic damage assumption, the stiffness matrix of material in this area becomes

$$[K]_e^D=\int_{v_e}[B]^{\mathrm{T}}[D_D][B]\mathrm{d}v_e \tag{21}$$

$$[D_D]=\frac{E_D}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \dfrac{1+\nu}{2}\end{bmatrix} \tag{22}$$

3. Finite Element Mesh Pattern

The specimen with an edge crack is schematically shown in Fig.1. The marked portion with shadow lines was simulated with FEM.

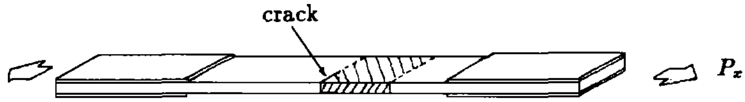

Fig.1. Schematic picture of the cracked specimen

The finite element mesh pattern is shown in Fig.2(a), which represents the marked part of the tensile specimen. The line segment EO is the crack and point 0 is the crack tip.

To simulate the stress (and strain) singularity at the crack tip, the so-called quarter node point elements are used[6]. The nodal positions can be determined using the following equations and refering to Fig.2(b).

$$r_A=\frac{r_B}{4}\qquad r_C=\frac{(\sqrt{r_B}+\sqrt{r_D})^2}{4} \tag{23}$$

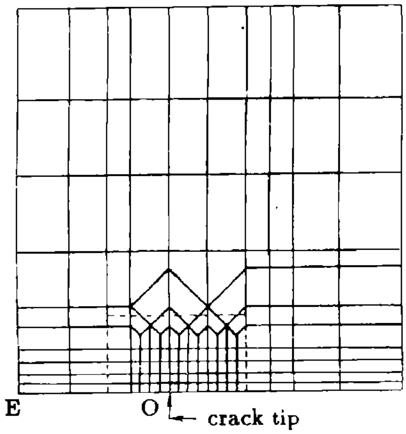

Fig.2(a) FEM mesh pattern.

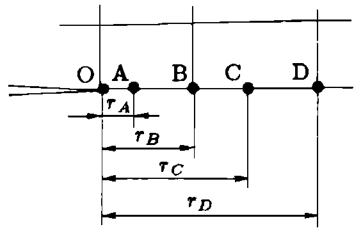

Fig.2(b) Quarter point element

4. Iterating Computations

Because Eq.(1) is non-linear and creep damage is coupled with creep deformation, the computation has to be performed with a step by step procedure, and in every step iteration has to be performed to get satisfactory results. For the creep strain, the following convergence criterion is assumed, that is when the relative difference of R value obtained at two consecutive iterations is less than a preselected small value, the iteration in this step is regarded as convergent.

VII. COMPUTATIONAL RESULTS

The computational results are shown in Figs.3–5. In Fig.3, we can see that the distribution of σ_x in the ligament varies with time, and there is a low stress zone in the vicinity of the crack tip due to the damage. Figure 4 illustrates the extension of damage factor D with time. Fig.5(a) and Fig.5(b) are two digital photograghs produced by the printer, which indicate the growth of damage zone. It is clear that the creep damage zone is confined in the vicinity of the crack tip and develops with time although the applied load is kept constant.

Creep crack growth in CT specimen has been simulated in Ref.[7]. The method and procedure bear some similarity with the present paper.

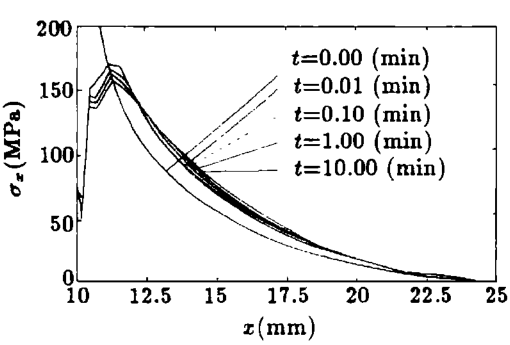

Fig.3 Distribution of σ_x in the ligament at different times

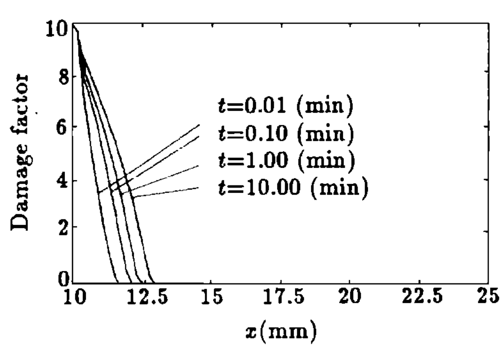

Fig.4 Damage distribution in the ligament at different times

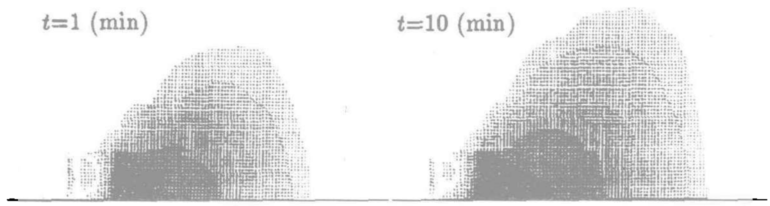

Fig.5(a) Damage zone at t=1.0min Fig.5(b) Damage zone at t=10min

VIII. CONCLUSIONS

1. This paper presents a computer simulation of creep damage in the vicinity of crack tip in the short fibre composite. The Schapery equation and the Kachanov-Rabotnov's damage evolution law are used. The 8-node quadrilateral element is adopted in the FEM simulation. Initial strain approach is used to deal with the nonlinearity of the constitutive relation and a modified recursive relation is proposed. The results of the computation seems to be reasonable.
2. The recursion formulas proposed in this paper are superior to the Brinson's recursion equations and can be used for other cases of hereditary integration.

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