ACTA MECHANICA SINICA, Vol.8, No.1, February 1992
Science Press, Beijing, China
Allerton Press, INC., New York, U.S.A.

ISSN 0567 - 7718

FLOW STRUCTURES AND FORCE CHARACTERISTICS FOR FLAT PLATE IN OSCILLATORY FLOWS WITH *Kc* NUMBER FROM 2 TO 40 AND IN COMBINED FLOWS*

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ABSTRACT: The evolution of wake structures and variation of the forces on a flat plate in harmonic oscillatory and in-line combined flows are obtained numerically by improved discrete vortex method. For the oscillatory oncoming flow cases, when Kc number varies from 2 to 40, the vortex pattern changes from a "harmonic wave" shaped (in a range of small Kc numbers) to a slight inclined "harmonic wave" shaped (in a range of moderate Kc numbers), then to inclined vortex clusters with an angle of 50° to the oncoming flow direction (at $Kc=20$), at last, as Kc number becomes large, the vortex pattern is like a normal Karman vortex street. The well predicted drag and inertia force coefficients are obtained, which are more close to the results of Keulegan & Carpenter's experiment as compared with previous vortex simulation by other authors. The existence of minimum point of inertia force coefficient C_m near $Kc=20$ is also well predicted and this phenomenon can be interpreted according to the vortex structure. For steady-oscillatory in-line combined flow cases, the vortex modes behave like a vortex street, exhibit a "longitudinal wave" structure, and a vortex cluster shape corresponding to the ratios of U_m to U_0 which are of $O(10^{-1})$, $O(1)$ and $O(10)$, respectively. The effect on the prediction of forces on the flat plate from the disturbance component in a combined flow has been demonstrated qualitatively. In addition to this, the lock-in phenomenon of vortex shedding has been checked.

KEY WORDS: flow structure, force coefficients, oscillatory flow, combined flow, that plate, discrete vortex method

I. INTRODUCTION

As the oncoming flow is oscillating with varying periods and amplitudes or is combined with different flow components, the wake flows of bluff body will show a series of non-linear behavior on vortex dynamics and dynamical response which have been studied insufficiently up to now. The flow past a flat plate, where separation can be assumed to be fixed at two edges and the crucial problem of predicting the unsteady separation in usual bluff body flows can be avoided, offers a good example to study in detail the non-linear relations between the varying oncoming flow and wake characteristics as well as forces on the body. It is interesting to know what the evolution process of vortex structure is, and correspondingly, how the force on the flat plate varies when Kc number varies from small to large and the ratio of the oncoming flow components changes. We believe that these studies are helpful to understanding the mechanism of viscous forces caused by wave-current on offshore structures. Graham[1] and other authors have made quite a lot of calculations about oscillatory flow past a plate. Although the drag coefficient was well predicted, the inertia force coefficient was rather lower than the experimental results.[2] Lian's results[3] failed to predict the tendency of inertia force coefficient when $Kc>10$. Keulegan & Carpenter's experimental results[2] showed that the inertia force coefficient goes down steeply to a nadir for Kc number between 10 — 20. However, the numerical simulation up to date has not yet predicted such an important phenomenon. Thus, making further numerical simulation is necessary.

Following Ling and Luo's work[4], this paper develops an improved discrete vortex method for simulating the vortex flows about a flat plate in harmonic oscillatory flows and in some typical in-line combined flows. For the former cases, more attention will be paid to the periodical

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Received 5 March 1991 revised 1 August 1991

* The project supported by National Natural Science Foundation of China & LNM, Institute of Mechanics, CAS

vortex shedding, the evolution of vortex pattern and the dynamic characteristics of the flat plate with Kc number varying from 2 to 40. For the latter cases, the three ratios of U_m to U_0, *i.e.* $O(10^{-1})$, $O(1)$, $O(10)$, will be considered.

II. METHOD

Let $U_\infty(t)=U_0+Kc/Vr\,\sin\left(\frac{\pi}{2}\,Vr^{-1}\,t\right)$ express the dimensionless oncoming flow velocity. The parameters are selected according to each oncoming flow case as follows:

$$\left.\begin{array}{lll} U_0=1, & Kc/Vr=0 & \text{for uniform flow} \\ U_0=0, & Vr=Kc & \text{for harmonic oscillatory flow} \\ U_0=1 & & \text{for steady-harmonic oscillatory in-line combined flow} \end{array}\right\} \quad (1)$$

where, Kc is Keulegan-Carpenter number, $Kc=U_mT/4C$, U_m, the amplitude of the oscillatory flow velocity, T, oscillatory flow period, $4C$, the length of the flat plate, t, dimensionless time (the time normalized by $4C/U_c$), U_c is a characteristic velocity, $U_c=U_0$ for uniform and combined flow cases and $U_c=U_m$ for oscillatory flow cases, and, Vr is the reduced velocity, $Vr=U_0T/4C$, U_0, uniform flow velocity.

The Joukowski transformation

$$z=\zeta+1/\zeta \quad (2)$$

is used to transform the flow around the flat plate in physical plane (z) into the transformed plane (ζ).

The nascent vortex strength and position are determined simultaneously using Kutta condition and a relation among the nascent vortex strength Γ_0, the distance from the flat plate edge to the nascent vortex δ_0 and the time step Δt, given in detail in [4] [5][6]. i.e.,

$$\left.\frac{dW}{d\zeta}\right|_{\zeta=\pm1}=0 \quad (3)$$

$$\Gamma_0=8.0\,\delta_0^2/\Delta t \quad (4)$$

where, W is the complex-velocity potential, and $dW/d\zeta$ is the complex-velocity.

The convection of kth vortex can be determined using $V_k=dz/dt$ at time t. For oscillatory oncoming flow cases, the first order scheme in time is strongly suggested, which is based on the fact that when the oncoming flow changes its direction, the second order scheme, which overweights the historical effect on the current flow, will bring serious error into the calculated results. While, for the uniform oncoming flow case, the second order scheme in time is used in the present calculation.

In the discrete vortex method, some parameters have to be determined. These parameters include the nascent vortex position at the first step of calculation, the time interval and the radius of vortex core. Kamemoto & Bearman[7], Sarpkaya[8] and Kiya et al.[6] have extensively investigated the selection of these parameters. By consulting the value recommended by Kiya et al.[6], and after preliminary calculations, we choose the following parameters: the nascent vortex position, $\delta_0=0.16C$, the time step $\Delta t=0.64$ for the uniform flow case. The time step for the oscillatory flow cases in dependent on Kc number of the oncoming flow, that is, $\Delta t=T/60$, $T/100$ and $T/150$ in the ranges of $Kc\leqslant5$, $5<Kc\leqslant15$ and $Kc>15$, respectively. The similar choice for the time step is made for combined flow cases by replacing Kc with Vr.

In order to avoid the extremely large velocities induced by the vortices which approach too close to each other or to the flat plate, a vortex core model is adopted. According to Kiya et al.[6], $\sigma=0.1C$ is used.

For the uniform flow case, as a compensation for diffusion and three dimensional deformation of vortex, the vortex decay is introduced in the present discrete vortex model, and the decay rate suggested by Chein & Chung[5] is used:

$$\Gamma(t)=\Gamma_0\left[1-\exp\left(-\frac{a\cdot Re}{\Delta t}\right)\right] \quad (5)$$

where, $\Gamma(t)$ is the vortex strength at time t and Γ_0 is the strength when it is created, $a \cdot Re$ is determined based on the experimental results and $a \cdot Re = 30$ is used in the present calculation. It is noted that the above vortex decay rate satisfies the N-S equation exactly for a single rectilinear viscous vortex. On the other hand, no vortex decay model is needed for the oscillatory flow cases because some experimental results have shown that the vortices are highly two dimensional.

The in-line force acting on the flat plate contains two parts, which is contributed by the variation of the unsteady oncoming flow and induced by vortices. The latter can be obtained by Blasius theorem which can be expressed in terms of strengths, velocities and positions of all vortices in the flow field, thus, the total in-line force coefficient can be expressed as

$$C_f = 2\pi \sin\alpha \frac{\partial U_x}{\partial t} + \sum_{l=1}^{2} (-1)^{l-1} \sum_{k=0}^{N_l} \Gamma_{kl} Re\left[\left(\frac{1}{2} + \frac{1}{\zeta_{kl}^2 - 1}\right) V_{kl}\right] \tag{6}$$

where, $C_f = F / \left(\frac{1}{2} \rho U_c^2 \cdot 4C\right)$ is the dimensionless in-line force coefficient, α, the angle of attack. Γ, the vortex strength. V, the complex-velocity in physical plane.

In terms of Morison formula, the in-line force coefficient, denoted by C_{fM}, can be expressed as

$$C_{fM} = C_d \frac{U_x(t) | U_x(t) |}{U_c^2} + 2\pi C C_m \frac{dU_x/dt}{U_c^2} \tag{7}$$

where C_d is the drag coefficient and C_m is the inertia force coefficient.

Therefore, we can obtain C_d and C_m values using the least square method from Eq. (7) by assuming that C_d and C_m are independent of time t.

III. NUMERICAL RESULTS AND DISCUSSION

3.1 Global Features of Uniform Flow over the Flat Plate

To check the present method and computation program, we have first calculated the separated flow from a flat plate normal to the uniform oncoming flow. The global calculated features of the flow and some comparisons are listed in Table 1.

Table 1 Comparison of gross features of uniform flow over a plate

	Sarpkaya *	Kiya et al. *	Chein et al.*	Present results	Experiment * results	Present results (Lock-in)
$\overline{C_d}$	– – –	2.4 — 2.8	2.8	2.81	1.96	3.10
$\overline{\frac{\partial\Gamma}{\partial t} / U_c^2}$	1.0	0.9	1.05	1.10	1.10	1.14
St	0.154	0.14 — 0.16	0.14	0.15	0.15	0.17

* Data taken from reference[5]

It is shown that the present results are in fair agreement with previous vortex simulation and experiments. Particularly, the drag force coefficient $\overline{C_d}$ is close to those given by some previous numerical simulation while $\overline{\frac{\partial\Gamma}{\partial t} \Big/ U_c^2}$ and Strouhal number St are consistent with those taken from the experiments.

3.2 Variations of the Drag and the Inertia Force Coefficients of the Flat Plate in Oscillatory Flows

Our calculation gives the variation of the drag and the inertia force coefficient in a wide range of Kc numbers, i.e., $2 \leqslant Kc \leqslant 40$. As shown in Fig.1, the calculated drag coefficient C_d decreases from a high value rapidly in the region of $Kc < 10$, then C_d varies slowly as Kc number increases, and as Kc number becomes large, it approaches monotonously to the value in the uniform oncoming flow case. The present results are in good agreement, not only qualitatively but also quantitatively, with Keulegan & Carpenter's experimental results[2] and other previous numerical simulation.

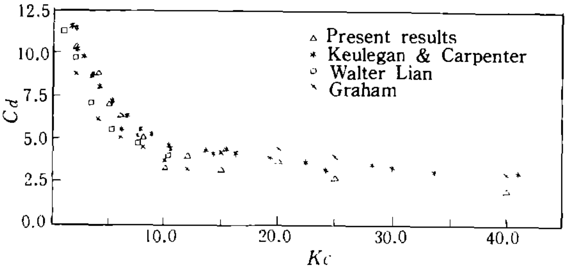


Fig.1 C_d vs Kc numbers for oscillatory flow

Unlike drag coefficient C_d, the inertia force coefficient C_m varies with Kc numbers non-monotonously, as shown in Fig.2. The results show that the C_m rises for $Kc<8$, and goes down steeply as Kc number increases, it has a maximum value and a minimum value near $Kc=8$ and $Kc=20$, respectively, the minimum value of C_m is approximately 0.83. The numerical results give us a better prediction of the inertia force coefficient than those of the other numerical results and well compared, even quantitatively, with the experimental results. In the experiment, C_m exhibits a minimum value $C_m=0.95$ near $Kc=16.3$. The phenomenon is related to the special vortex pattern which will be discussed later.

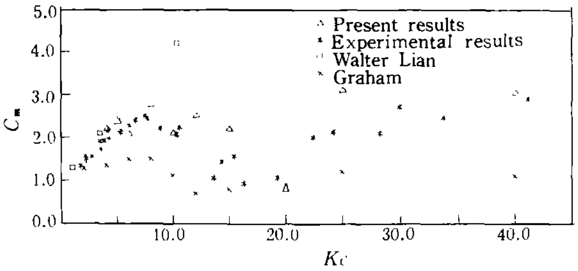


Fig.2 C_m vs Kc numbers for oscillatory flow

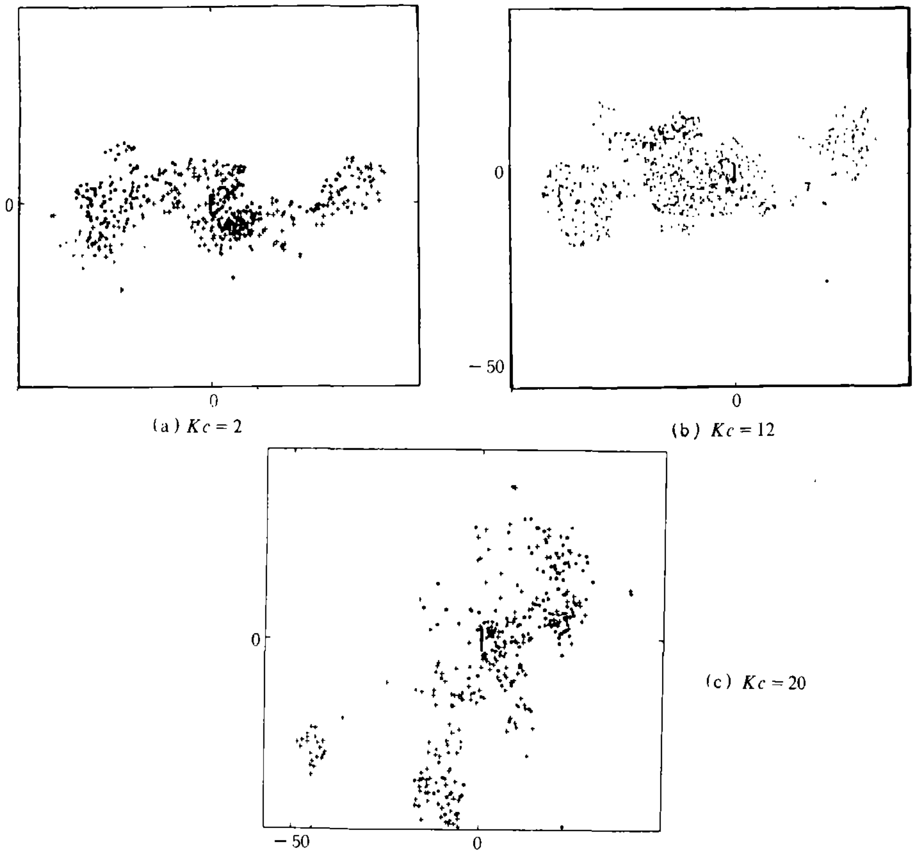
3.3 Evolution of Wake Vortex Pattern of the Flat Plate with *Kc* Numbers in Oscillatory Flows

Ling & Luo[4] has reported some new patterns of vortex shedding and pairing in wake flow of a flat plate in early stages of oscillatory oncoming flow for small Kc number. Now, we pay more attention to the study of long time flow structures (Fig.3) and the variation of forces on the plate (Fig.4, Fig.5), present the evolution of the vortex patterns illustrating the non-linear behavior of vortex dynamics when Kc number increases.

Case A small Kc numbers

In the early stages at $Kc=2$, the flow is basically symmetric, which can be recognized from the variations of the vorticity shedding rate from both edges of the plate versus time in Fig. 4 (the solid line and the dash line illustrate the vorticity shedding rate from the upper and lower edges of the plate, respectively). After a considerable time, the vortex cloud is distributed inhomogeneously but confined to a band area parallel mostly to the oscillatory flow direction, specifically, the vortices array themselves like a " harmonic wave " structure inside the band. In the later time, the whole vortex cloud diffuses and the band becomes wider, as shown in Fig.3a.

The increase of the vortex points and the dispersion of vortex cloud band affect the generation of successive nascent vortex directly. The vortex shedding rate curve is relatively smooth in the earlier two periods, and burrs occur more and more in the later time. In the range of small



(a) $Kc=2$

(b) $Kc=12$

(c) $Kc=20$

Fig.3 The vortex pattern in oscillatory oncoming flow

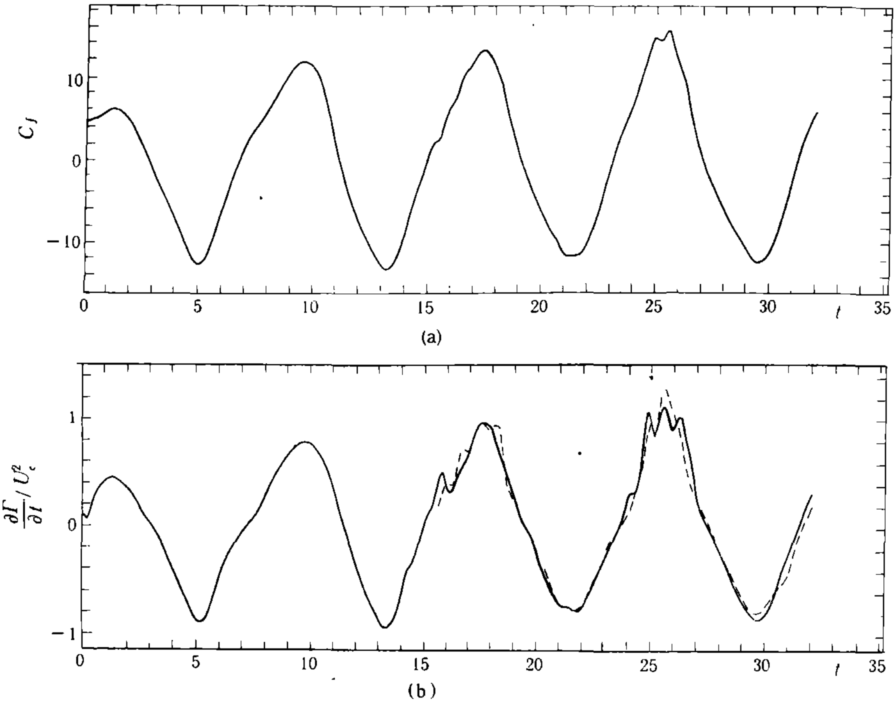


Fig.4 The in-line force and vorticity shedding rate vs time for oscillatory flow case $Kc=2$

Kc numbers the vortex cloud encloses the plate and makes drag coefficient high. This corresponds to the results shown in Fig.1.

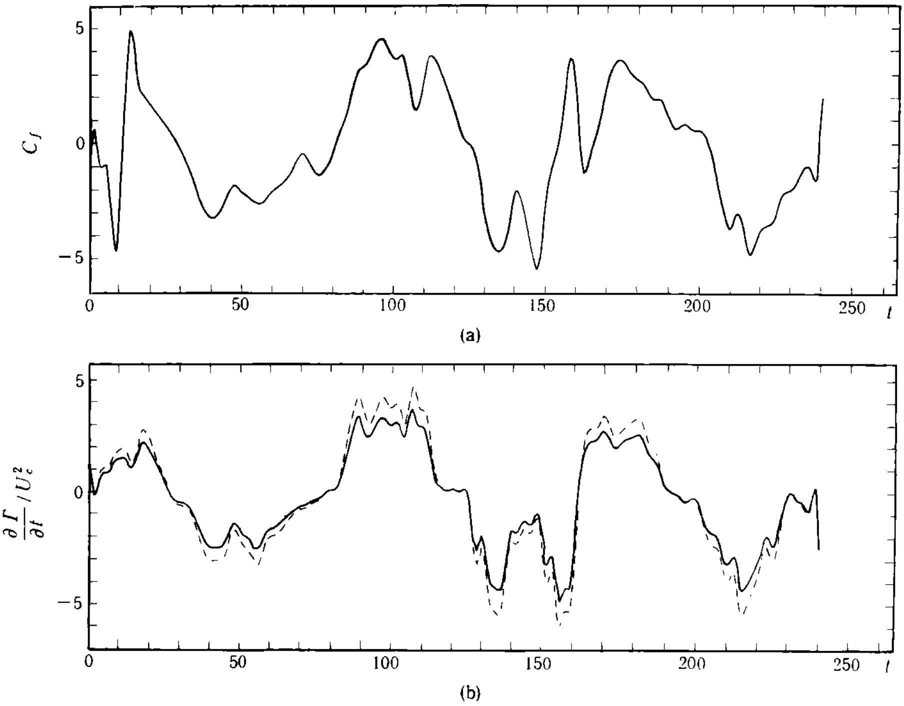


Fig.5 The in-line force and vorticity shedding rate vs time for oscillatory flow case $Kc=20$

Case B moderate *Kc* numbers

Fig.3b shows the vortex pattern at $Kc=12$. It is very similar to that in the case of small *Kc* numbers, but the " harmonic wave " vortex cloud band turns slightly its direction away from the oncoming flow direction.

As *Kc* number increases, particularly, at $Kc=20$, the calculation yields a new vortex structure. The vortex cloud is no longer distributed as a wavy shape and some dense vortex blobs occur. As a whole, the vortex cloud is diagonal to the flat plate and the angle between the vortex band and the oscillatory flow direction is about 50 degrees. Fig.3c shows the marked inclined vortex pattern. In the later flow, unlike the case of small *Kc* numbers, several concentrated vortex clouds appear and are distributed regularly in an inclined direction to the flat plate. Correspondingly, the inertia force coefficient C_m at this *Kc* number reaches a minimum value as shown in the curve of C_m versus *Kc* numbers in Fig.2. It is, therefore, shown that a distinct inclination of vortex cloud will lead to a big decrease of the inertia force coefficient of the flat plate. The phenomenon is analogous to that in the case of circular cylinder in oscillatory flow, which occurs at $Kc=15$, and correspondingly the lift force reaches a maximum value.

Case C large *Kc* numbers

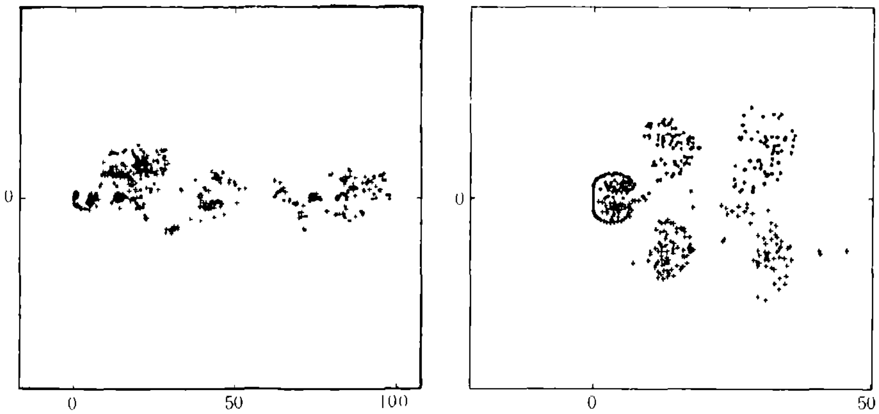
In this case, the parameter $Kc=40$ is concerned. The vortices form a simple vortex street before the oncoming oscillatory flow changes its direction. The vortex street is parallel to the direction of flow, and behaves like a Karman vortex street in the uniform flow. In fact, the uniform flow is the limit of oscillatory flow when *Kc* number approaches infinity.

3.4 Flow Features of the Flat Plate in Steady-Oscillatory in-Line Combined Flows

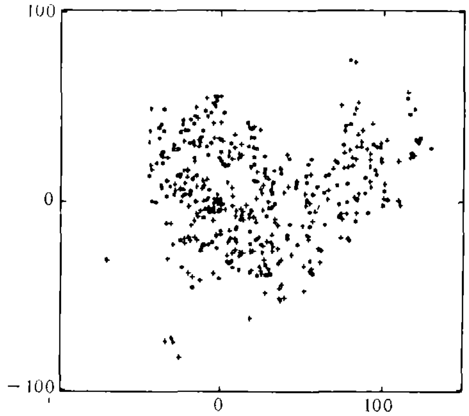
As mentioned before, three cases of in-line combined flows are calculated and the main results are presented as follows.

Case A $U_m/U_0=O(10^{-1})$

In this case, the steady flow component is dominant and the oscillatory flow component is considered as a small disturbing flow. As examples, the cases when $U_m/U_0=0.1$ and $U_m/U_0=0.5$ are calculated and the global features of flow field obtained behave like those in pure uniform oncoming flow case, e.g. the vortex pattern in each cases is a Karman vortex street in the down stream of the flow (Fig.6a).



(a) $U_m/U_0=0.5$ (b) $U_m/U_0=1$ $Kc=4$



(c) $U_m/U_0=10$ $Kc=40$

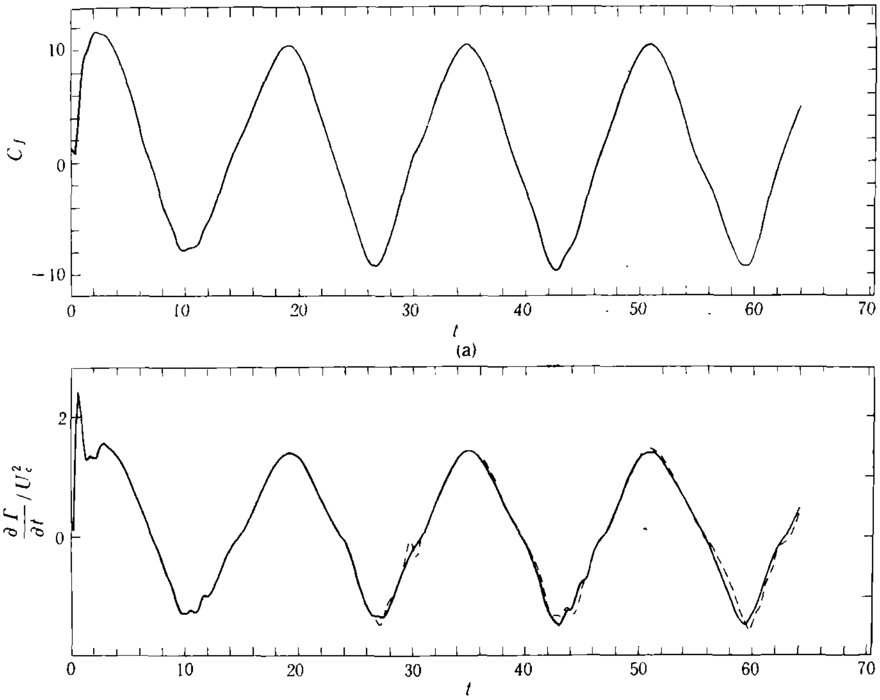
Fig.6 The vortex pattern in combined oncoming flow

However, in the present case, an important phenomenon called lock-in of vortex shedding will occur if the frequency and amplitude of disturbing oscillatory flow satisfy some specific conditions. With reference to Armstrong & Barnes[9], we have computed a case which satisfies the lock-in condition, where the specific parameters $U_0=1$, $U_m=0.015$, $Kc=0.0525$ are used. For comparing with those results in pure uniform oncoming flow case, the same vortex decay model is introduced. The calculated results show that the vorticity shedding rate rises by approximately 4% (from 1.10 in pure uniform flow case to 1.14 in the lock-in flow case) and the drag coefficient rises by more than 10% (from 2.81 to 3.10). More results are given in Table 1, which are in agreement with the experiments[9].

Case B $U_m/U_0=O(1)$

In this case, both steady and oscillatory oncoming components are equally important. For $U_m/U_0=1$ and $Kc=4$, it is obvious that the wake vortex pattern has regular periodic and symmetric features. The shedding vortices form a "3" shaped structure like a longitudinal wave propagating in the downstream direction (see Fig.6b). The "wave" period calculated is $T=16$ which is equal to the oscillatory flow period. Another window to see the periodicity is in C_f curve, where the variation of C_f is smooth and has a normal wave shape (Fig.7).

With the same ratio of the two flow components but with Kc number doubled (i.e. $Kc=8$), the calculation yields the pattern where the vortex shedding is still periodic, but takes place alternatively rather than simultaneously at different edges of the plate. This causes the shed vortices to adhere each other transporting in downstream.



(a)

(b)

Fig.7 The in-line force and vorticity shedding rate vs time for combined flow case $U_m/U_0=1$, $Kc=4$

Case C $U_m/U_0=O(10)$

In this case, the oscillatory flow component is dominant, the wake pattern is like a vortex cloud in general, which is analogous to that in a pure harmonic oscillatory flow. At $Kc=40$, the vortices confine themselves in a triangle region around the plate, it seems as Vee-shaped (Fig.6c). At $Kc=4$, the vortex pattern also displays as a cloud cluster but the detailed structure is different.

Table 2

C_d, C_m of the flat plate in steady-oscillatory in-line combined flows

U_m/U_0	0	0.5	1	1	10	10	∞
K_c	– –	10	4	8	4	40	4
C_d	1.8— 2.0	2.05	2.25	2.09	8.08	2.54	8.85
C_m	– –	– –	1.84	2.24	2.24	2.51	2.25

The present calculation gives the values of C_d and C_m in the combined flow cases. The results indicate that C_d in the combined flow is higher in general than that in the pure uniform flow (C_d=1.8 — 2.0) (see Table 2). In small Kc number cases, adding a steady flow component as a disturbing flow, the C_d will decrease but the C_m will change only a little. In large Kc number cases, adding a steady flow as a disturbing component will hardly affect the C_d and the C_m. Therefore, in large Kc number cases, as the oscillatory flow component is dominant, the effect of steady flow disturbing can be neglected, while in small Kc number cases, special attention should be paid to the effect of the disturbing steady flow.

IV. CONCLUSION

1. The calculation results give the evolution of new wake vortex patterns of the flat plate in

oscillatory and combined flows, respectively, when the Kc number varies from small to large and the ratio of oncoming flow components changes, the results show the non-linear behavior of wake vortex dynamics and of forces on the body.

2. The calculated force coefficients about the flat plate in oscillatory flow are in good agreement with the Keulegan & Carpenter's experiments. Specially, the inertia force coefficient C_m versus Kc numbers is better predicted than that in all previous simulation.
3. The present improved discrete vortex method and numerical scheme are effective for simulation of long time flow behavior when the wake reverses around the body with sharp edges.

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