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## Vibration behavior and response to an accidental collision of SFT prototype in Qiandao Lake (China)

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### Abstract

This article presents free vibration analysis of the submerged floating tunnel (SFT) prototype, which has been designed to be built in Qiandao Lake (China). As an approximation the supporting effect of the tethers is omitted in the calculation of beam-like bending vibrations. As a case study, the response of the SFT prototype to an accidental collision by an object like a sinking boat is examined. The deformation and the stress of the SFT are computed and the safety assessment is given. Finally, by using the plastic hinge model, the energy absorption capacity of the aluminium alveolate layer is estimated.

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*Keywords:* submerged floating tunnel (SFT); SFT prototype; free vibration; accidental oscillation; plastic hinge model

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### 1. Introduction

Submerged floating tunnel (SFT), also named Archimedes Bridge, is a kind of transportation passage floating and submerged within water to bridge water banks, which takes the advantage of buoyancy and is tethered to the foundation and the shores. SFT is a new alternative solution for crossing sea strait, lake or other water sound. As an innovative transportation technology, SFT will become attractive competing with traditional techniques with its economical and environmental advantages [1]. However at present, there is still not an actual SFT being built in the world. In recent years, the Sino-Italian Joint Laboratory of Archimedes Bridge (SIJLAB) has made lots of efforts in the design of the first SFT prototype in Qiandao Lake of China and has performed relevant simulations and experiments [2–6]. A description on structural analysis of the SFT prototype in Qiandao Lake is given in a recent paper [4]. It is noted that the tube structure of the SFT prototype is complicated with three layers (sandwich structure), and particularly the external layer is an alveolate-shaped aluminium extrusion shell as shown in Figs. 1 and 2. In another paper to be presented in this symposium [7], we have dealt with the homogenization of structural properties of the SFT and made the strength analyses of the SFT under actions of water wave and current, earthquake. In the present article, analysis on the free vibration behaviour of it will be carried out. Furthermore, the

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reliability of the SFT prototype under an accidental collision by an object is assessed and the energy absorption capacity of the external aluminium alveolate layer is estimated.

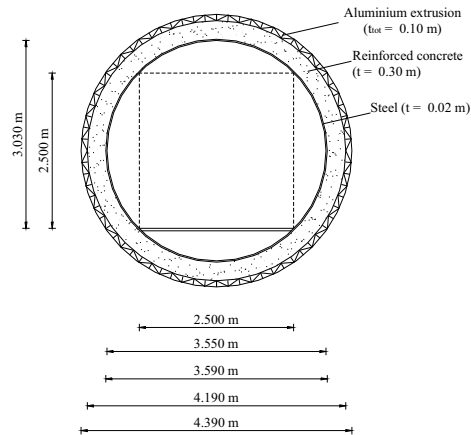


Fig. 1. Cross section of SFT prototype showing three-layered structure [2]

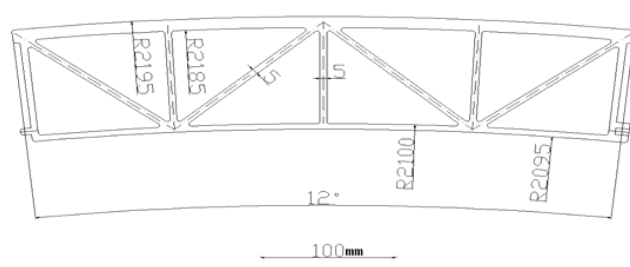


Fig. 2. Profile of the aluminium alveolate layer and its detailed structure [2]

## 2. Free vibration analysis of SFT

In the previous article [7], the homogenized models of the SFT prototype have been obtained, in which the three layered SFT was replaced by a single layered tube. In so doing, the bending stiffness of it has not been changed. The effective modulus is equal to  $3.22 \times 10^{10}$  Pa and the bending stiffness is equal to  $35.25 \times 10^{10} \text{ N} \cdot \text{m}^2$ . By using these data, the bending vibration behavior like a beam can be analyzed. As we know that the SFT is tethered to the lake foundation by several steel wires. They must give serious influence to the vibration behavior of the SFT. However, in the present paper, we want to give some insight into the free vibration of the SFT in an analytical form, so the supporting effect of the cables has been omitted.

### 2.1. Transversal bending vibration

The free vibration of the SFT prototype is analyzed by using the formula of beam vibration in Ref. [8]. The fundamental natural frequency of the SFT prototype with two ends simply supported without added mass and live load is

$$f_1 \Big|_{\text{simply-simply}} = \frac{\pi}{2} \sqrt{\frac{J_{\text{total}}}{m_e L^4}} = 2.72 \text{ Hz} \quad (1)$$

where,  $m_e$  is mass per meter and equals to 11.46kg/m). With consideration of added mass,  $m_d$  (=15.14kg/m), due to surrounding water and live load, the frequency is

$$f_1 \Big|_{\text{simply-simply}}^{\text{plus add mass}} = \frac{\pi}{2} \sqrt{\frac{J_{\text{total}}}{(m_e + m_d)L^4}} = 1.78 \text{ Hz} \quad (2)$$

In Eqs. (1) and (2),  $L$  is length of the SFT and equals to 100m. The frequencies of SFT prototype with two ends fixed for three situations (with and without added water mass and live load) are also calculated. The results are summarized in Table 1, in which the results of simply-simply and fixed-fixed supported cases are included.

Table 1. Natural frequency of SFT prototype

Support condition	Frequency, Hz	Frequency, Hz (Plus added mass)	Frequency, Hz (Plus overload and added mass)
Simply-simply	2.72	1.78	1.72
Fixed-fixed	5.25	3.47	3.35

## 2.2. Rotational vibration of SFT prototype

As mentioned in Ref. [2], the two ends of SFT prototype left free for rotation. For this circumstance, the rotational vibration should be paid some attention. The SFT prototype is assumed as a rigid tube with two ends pinned and endowed with three groups of cables. The formula for rotational vibration frequency is

$$f_\rho = \frac{1}{2\pi} \sqrt{\frac{k_\rho}{I_\rho^{\text{total}}}} \quad (3)$$

where  $k_\rho$  is the torsion resisting spring constant provided by the cables about axis of the tube, and  $I_\rho^{\text{total}}$  is the mass inertia moment. For the rotational vibration,  $I_\rho^{\text{total}}$  is simply the sum of those of three layers. For each layer, its mass inertia moment is calculated by the formula:

$$I_\rho^k = \frac{l}{2} m_k (r_k^2 + R_k^2) \quad (4)$$

where  $m_k$  is the mass of  $k^{\text{th}}$  layer,  $r_k$  and  $R_k$  are inner and outer radius of  $k^{\text{th}}$  layer, and  $l$  is the length of prototype.

The values of weight per meter of the three layers are:

$$W_{st} / m = 1.75 \times 10^3 \text{ kg / m} \quad (5a)$$

$$W_{\text{con}} / m = 9.17 \times 10^3 \text{ kg / m} \quad (5b)$$

$$W_{\text{al}} / m = 0.907 \times 10^3 \text{ kg / m} \quad (5c)$$

So the results of the mass inertia moment per meter of the three layers are:

$$I_{\rho}^{st} = \frac{m^{st}(r^2 + R^2)}{2} = 5.58 \times 10^3 \text{ kg} \cdot \text{m} \tag{6a}$$

$$I_{\rho}^{con} = \frac{m^{con}(r^2 + R^2)}{2} = 34.90 \times 10^3 \text{ kg} \cdot \text{m} \tag{6b}$$

$$I_{\rho}^{al} = \frac{m^{al}(r^2 + R^2)}{2} = 4.18 \times 10^3 \text{ kg} \cdot \text{m} \tag{6c}$$

Then by summing up the above results, the total mass inertia moment per meter is

$$I_{\rho}^{total} = I_{\rho}^{st} + I_{\rho}^{con} + I_{\rho}^{al} = 44.66 \times 10^3 \text{ kg} \cdot \text{m} \tag{7}$$

Here, the effective mass density pertinent to this case can be derived and the total mass inertia moment  $I_{\rho}^{total}$  can be rewritten as

$$I_{\rho}^{total} = \frac{1}{2} m^{eff} (r_{SFT}^2 + R_{SFT}^2) \tag{8}$$

where  $m^{eff}$  is the effective mass per meter of the equivalent homogenized SFT prototype. From (8) the effective mass density corresponding this vibration mode is obtained as

$$\rho^{eff} = \frac{m^{eff}}{A_{SFT}} = 2.15 \times 10^3 \text{ kg} / \text{m}^3 \tag{9}$$

Obviously, this effective mass density value is smaller than that for the case of global beam bending vibration. Therefore, for different vibration modes, different effective mass density should be used. The spring constants  $k_{\rho}^k$  resisting rotation of the SFT endowed by the three tether groups about the tube axis can be obtained by using structural mechanics. The eight tethers have different lengths and inclined angles. We calculate their spring constants one by one, and the results are listed in Table 2. The 8 tethers with labels as AB, CD...QP are schematically shown in Fig.3 [4]. It is seen that the longer tethers provide smaller spring constants, and that the tethers with larger inclined angle give smaller constants. The data in Table 2 are normalized with  $10^6 \text{ N} \cdot \text{m}$ .

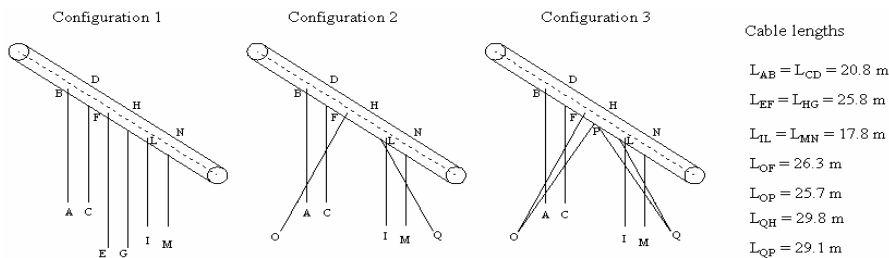


Fig. 3. Cable configurations considered for the prototype [4]

Table 2. Spring constants of eight tethers

Tether	AB	CD	IL	MN	OF	QH	OP	QP
$k_{\rho}^k, 10^6 \text{N} \cdot \text{m}$	97.2	97.2	120.6	120.6	57.7	63.3	28.4	23.8

In the following, the calculation is only carried out for the cables of configuration 3. By substituting the constants of eight tethers in Eq. (3), the rotational natural frequency is

$$f_r = \frac{1}{2\pi} \sqrt{\frac{\sum_{i=1}^8 k_{\rho}^i}{I_{\rho}^{\text{total}}}} = \frac{1}{2\pi} \sqrt{\frac{k_{\rho}^{\text{AB}} + k_{\rho}^{\text{CD}} + k_{\rho}^{\text{OF}} + k_{\rho}^{\text{QH}} + k_{\rho}^{\text{IL}} + k_{\rho}^{\text{MN}} + k_{\rho}^{\text{OP}} + k_{\rho}^{\text{QP}}}{I_{\rho}^{\text{total}}}} = 5.76 \text{Hz} \quad (10)$$

Because of the non-symmetry of the tethers about the prototype longitudinal axis, under the action of residual buoyancy, the prototype will have certain side leaning angle. The calculation results identify that the inclined angle is not large and the influence can be ignored.

### 2.3. Flexural vibration of circular ring

As for the case of local bending problem, the equivalent bending rigidity of the SFT panel was obtained in [7]. In Table 3, the results of effective modulus and bending stiffness corresponding to local bending equivalency and those corresponding to global bending equivalency are given. It should be pointed that the local bending effective stiffness is the bending stiffness in unit width. This bending stiffness will be utilized in the calculation of the breath vibration of the SFT.

Table 3. Effective bending stiffness and modulus in two approaches [7]

Local bending equivalence effective stiffness of panel, Pa · m <sup>3</sup>	Global bending equivalence effective stiffness of beam, N · m <sup>2</sup>	Effective modulus by local bending equivalence, Pa	Effective modulus by global bending equivalence, Pa
$223.3 \times 10^6$ <sup>5</sup>	$35.25 \times 10^{10}$ $29.69 \times 10^{10}$ [*]	$3.62 \times 10^{10}$	$3.42 \times 10^{10}$ $4.05 \times 10^{10}$ [*]

<sup>5</sup> The effective stiffness of the local bending equivalence is bending stiffness in unit width. [\*] Ignore aluminium layer.

As a typical dynamic problem, the prototype can vibrate like a ring and the vibration may be in tensile mode or in flexural mode. For a thin cylindrical shell, the flexural vibration is more important than the tensile one. For this case, Ref. [8] gives a formula to calculate its natural frequency, which is

$$f_i = \frac{1}{2\pi} \sqrt{\frac{E^{\text{eff}} I (1 - i^2) i^2}{\rho A r^4 (1 + i^2)}} \quad (11)$$

where,  $i$  is the number of half wave of vibration mode. When  $i = 2$ , the value of frequency will be the smallest one and the mode is an ellipse. For this case, the effective modulus is that for local bending mode and which has been given as  $E^{\text{eff}} = 3.62 \times 10^{10}$  Pa (please refer to Table 3. So the fundamental frequency is

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{36EI}{5\rho Ar^4}} = 44.2 \text{Hz} \quad (12)$$

### 3. Safety assessment of SFT prototype under accidental impact loading

When an accidental collision by an object (e.g. a sinking ship) happens, the SFT prototype will suffer a dangerous damage. In this circumstance, the safety must be considered and the assessment on the reliability of the SFT prototype is necessary.

#### 3.1. Local response

For this problem, the local bending stiffness of the panel given in Table 3 (being  $223.3 \times 10^6$  Pa) is used. As a case study, the response to a collision of a sinking boat of 50 tons impacting on the SFT prototype with a velocity of 1.0m/s was studied in the thesis of Hui [9]. The SFT prototype has been simplified and equivalent to a single layered cylinder of steel, whose thickness is  $h = 0.23$  m. So the local bending stiffness  $D$  is  $D = 234N \cdot m$ . The effective mass density  $\bar{\rho}$  is  $3341\text{kg/m}^3$ . It is noticed that the value of  $\bar{\rho}$  is far small compared to that of steel. So, this method of homogenizing the composite prototype equivalent to a single-layered structure of steel (or concrete) is not very good, since its effective mass density is contradictory to its real value. By using the commercial code of ANSYS/LS-DYNA, the maximum deflection and the maximum Mises stress for both cases of horizontal and vertical downward impacts were computed. And two circumstances were considered: with and without consideration of added mass of the surrounding fluid. The results are listed in Table 4, in which only horizontal impact results are included. For the vertical impact, the results are similar.

Table 4. Deformation and stress of SFT prototype due to collision

	Without added mass	With added mass
Deflection, mm	6.34	5.57
Mises stress, MPa	41.4	46.0

It can be pointed that the propagation velocity of shear stress wave of SFT prototype approximately equals to  $c_s = 2.52$  km/s. Owing to the time when the impact stress and the deflection reach their maximum values is less than 2ms, the effect of the stress wave reflection can be ignored. From Table 4, the Mises stress is much smaller than the design strength of the steel of S235. Thus, it is concluded that this impact intensity may not cause local failure of the SFT prototype.

#### 3.2. Total response of SFT prototype as a beam

With the assumption that the collision is totally plastic, after collision the sinking object will move continuously together with the SFT prototype. Based upon the energy conservation principle, the dynamic energy will totally change to the bending strain energy of SFT prototype. Due to that the weight of the tube is balanced by buoyancy, the displacement caused by the weight can be ignored. The dynamic energy is:

$$T = \frac{1}{2} \frac{\delta_{dy}^2}{k_{ben}} \quad (13)$$

where,  $\delta_{dy}$  is the maximum displacement, and  $k_{ben}$  is the coefficient of bending stiffness. By omitting the effect of shear deformation, the static bending displacement of simply supported beam under a force acting at central point can be calculated by the formula:

$$\delta_{ben} = \frac{pl^3}{48EI} = 0.029\text{m} \quad (14)$$

Then, we obtained that  $k_{ben} = 5.9 \times 10^8 \text{ N} \cdot \text{m}$ . The dynamic energy of colliding object is

$$T = \frac{1}{2} MV^2 = 25 \times 10^3 \text{ N} \cdot \text{m} \quad (15)$$

From above result and Eq. (11), the maximum impact deflection is

$$\delta_{dy} = \sqrt{2T \times k_{ben}} = 0.054 \text{ m} \quad (16)$$

This value is very close to the vertical displacement under earthquake mentioned in Ref. [2].

### 3.3. Estimation of energy absorption of aluminium alveolate extrusion panel

As stated in Ref. [2, 4] that the main function of the aluminium alveolate layer is to absorb collision energy. As a case study, the absorption capacity of colliding energy of aluminium alveolate layer is estimated. For this problem, the mass of impacting object is 50tons, and the colliding velocity is 1m/s. The dynamic energy is  $T = 0.25 \times 10^5 \text{ J}$ . It is known that the failure stress of aluminium alveolate is 240MPa [2]. By assuming that that collision damage zone is  $\Omega = 0.2\text{m} \times 0.3\text{m}$ , namely the damage area  $\Omega$  is 0.3m long along axial direction and 0.2m wide in circumferential direction, the dynamic energy absorbed by external skin of the aluminium alveolate layer can be estimated using the well known plastic hinge hypothesis. The length of the plastic hinge is that of boundary of the damage zone  $\Omega$ , which is  $\Gamma = 2 \times 0.3 + 2 \times 0.2 = 1.0\text{m}$ . Along  $\Gamma$ , the distribution of bending stress through thickness of the skin is shown in Fig. 4. The bending moment of unit length is

$$M_{unit} = \sigma_{yk} \times \left(\frac{h}{2}\right)^2 = 0.6 \times 10^4 \text{ N} \quad (17)$$

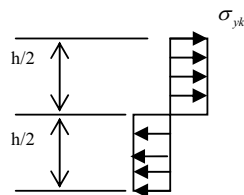


Fig. 4. Schematic showing stress distribution of a plastic hinge

It is assumed that the bending angle of the skin is  $\pi/2$ . Then the plastic work in unit length of plastic hinge is:

$$W_{unit} = \frac{\pi}{2} M_{unit} = 0.94 \times 10^4 \text{ N} \quad (18)$$

From Fig. 5, it is noted that two hinges will form during the impact just like metal forming. The total work done by the two plastic hinges is obtained as

$$W_{skin} = 1.88 \times 10^4 \text{ J} \quad (19)$$

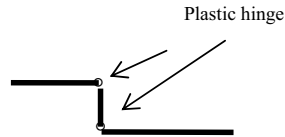


Fig. 5. Two plastic hinges formed in upper skin

Moreover, the straight and slant stiffening webs of 0.02m width will also be crushed, and the process will absorb considerable amount of energy. Roughly speaking, in the width of  $\Gamma$  (0.2m), there are two straight webs and two slant webs. The total web length in plane parallel to the skin surface is  $l_{web} = 2 \times [(0.1 - 0.015) + 0.13485] = 0.44\text{m}$ . Under the compressive force of collision, the webs will buckle, and form plastic hinges like schematic shown in Fig. 6. The bending angle is  $\pi$ . Since the work done by the plastic hinge of webs for unit length is

$$M_{unit} = \sigma_{yk} \times \left( \frac{0.005}{2} \right)^2 = 0.15 \times 10^4 \text{ N} \quad (20)$$

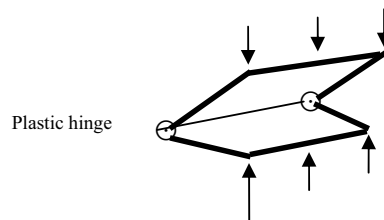


Fig. 6. Schematic showing plastic hinge of stiffening webs

Therefore, the work done by these webs is  $W_{web} = \pi M_{unit} l_{web} = 0.15 \times 10^4 \times \pi \times 0.44 = 0.21 \times 10^4 \text{ J}$ . Summing up the two parts of energy absorption by skin and webs, we have

$$W = W_{skin} + W_{web} = 0.21 \times 10^5 \text{ J} \quad (21)$$

From the above description, it is concluded that the aluminium alveolate layer has considerable capacity of absorbing accidental collision energy. For this particular case, the energy absorbed is larger than 80% of the impacting energy.

#### 4. Concluding remarks

An analysis on the free vibration behavior of it has been carried out. As a case study, the deformation and the safety assessment are evaluated for the collision by an object like a sinking boat. Finally, the energy absorption capacity of the external aluminium panel embraces large capacity of absorbing impact energy.

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