The applicability of Mach number independence principle in case of hypersonic thermo-chemical nonequilibrium flow

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Abstract

Above a certain Mach number, several aerodynamic properties become asymptotically independent of the free stream Mach number, such as pressure coefficient, lift and wave-drag coefficients, and flow-field structure. Known as Mach number independence principle, this phenomenon is important for hypersonic flow analysis and hypersonic vehicle designs. Proceeding from the contents of Mach number independence principle, we numerically study the applicability of the law in thermo-chemical nonequilibrium flow by applying Navier-Stokes equation in a five-component system. Sphere-cylinder is chosen as simulation model with radius 5cm considering the double scale law and the free-stream is at H=60km. To study the effects of nonequilibrium flow, we compare it with frozen flow and thermal equilibrium flow. The flows are set as viscous or inviscid. For viscous flow, the wall boundaries are treated by assuming an adiabatic or isothermal wall for laminar flow. Wave-drag, lift and friction coefficient, boundary layer distribution, shock shapes and pressure distribution along the wall are given for several combinations of Mach number and gas states (frozen, thermal equilibrium and thermo-chemical nonequilibrium gas). For sphere-cylinder bodies considered here, Mach number independence principle only exists for wave-drag coefficient in the considering nonequilibrium flow due to the coupled effects of real-gas and expansion.

Keywords: Mach number independence, Oswatitsch, nonequilibrium flow, real-gas effect, expansion effect ;

1. Introduction

The concept of Mach number independence principle appears first in a formal way by Oswatitsch in 1951 [1]. The law explains why above a certain Mach number, some aerodynamic properties become independent of Mach number. For atmospheric re-entry into the earth's atmosphere the flight Mach number will exceed 20. To optimize the design of re-entry vehicles we need some information from the ground tests. However, existing ground test facilities only simulate free-stream conditions in range of about Mach 10. These facilities cannot meet the demand considering a lot of similarity parameters [2]. In this situation, Oswatitsch's Mach number independence principle can be used to overcome the deficiency.

In his original work, by deriving this law Oswatitsch assumed the following:

- Very high free-stream Mach numbers $M_\infty >> 1$ (or $M_\infty \sin\beta >> 1$, β is shock wave angle).
- Calorically perfect gas (γ=constant.).
- Inviscid flow.

In these assumptions the following flow properties are no longer a function of Mach number, as long as the free-stream Mach number is high enough in the test section:

- Pressure coefficient,
- Lift and wave-drag coefficients,
- Flow-field structure (such as shock shapes, stand-off distance and Mach wave patterns).

Oswatitsch extended the principle to include the flow past slender bodies and an extension to include real gas effects and the boundary layer flow has been made by Hayes and Probstein [3]. In a later treatment of the independence principle Oswatitsch himself included the effects of dissociation and ionization [4]. Charters and Thomas et.al [5-7] confirmed the Mach number independence principle for the drag coefficient of blunt bodies, results showing drag coefficient is nearly constant for Mach numbers greater than about 7. Kliche, Mundt and Hirschel [9] investigated the Mach number independence principle for inviscid and viscous flow theoretically. In view of the above mentioned flow properties, they found only negligible differences between inviscid perfect gas and inviscid equilibrium air for a big Mach number region. On the other hand, Schneider's [10] investigations on conical flows seem to prove dependence. His research showed a free-stream velocity dependent behavior of the bow-shock and the lift coefficient. Cox and Crabtree [11] showed the Mach number effect on the shock shapes for the flow over slender cones or wedges. They found for semi-angles less than about 10° ,the difference between the shock angles for M_1 and $M_1=\infty$ is much greater than for larger semi-angles (for which $M_n>2$). Vas, Bogdonoff and Hammitt [12] showed experimentally that for the flow of helium past blunted flat plates, the surface pressure varied with $M^{2.2}$ for a range of Mach numbers between 11 and 17. In conclusion, these investigation reveal some opposing results concerning the influence of real gas effects on Oswatitsch' s independence principle. Thus, some more detailed investigations are necessary.

In the following, we start with governing equations to find the decisive variables on flow-field solutions. This is presented in section 2. The equations indicate that independence principle exists for calorically perfect gas but not for high temperature real gas. Section 3 describes the numerical methods and gives information about studied flow cases. The numerical results are presented in section 4. Considering the high temperature real-gas effect on the shock shapes, wave-drag coefficient, lift coefficient, friction coefficient, pressure coefficient along the wall and boundary layer velocity distribution, we find Mach number independence principle exists only for calorically perfect and thermal equilibrium gas, not for thermo-chemical nonequilibrium gas.

2. Theoretical considerations for calorically perfect gases

The hypersonic Mach number independence principle is more than just an observed phenomenon; it has a mathematical foundation. As outlined above, in this section we will examine the roots of this principle and analysing the dominant parameters of governing equations on flow-field solutions. The non-dimensional continuity, momentum and energy equations in steady flow are as follow:

Continuity: $$\frac{\partial(\overline{\rho u})}{\partial \bar{x}}+\frac{\partial(\overline{\rho v})}{\partial \bar{y}}+\frac{\partial(\overline{\rho w})}{\partial \bar{z}}=0 \quad (2.1)$$

X momentum: $$\overline{\rho u}\frac{\partial \bar{u}}{\partial \bar{x}}+\overline{\rho v}\frac{\partial \bar{u}}{\partial \bar{y}}+\overline{\rho w}\frac{\partial \bar{u}}{\partial \bar{z}}=-\frac{\partial \bar{P}}{\partial \bar{x}} \quad (2.2)$$

Y momentum: $$\overline{\rho u}\frac{\partial \bar{v}}{\partial \bar{x}}+\overline{\rho v}\frac{\partial \bar{v}}{\partial \bar{y}}+\overline{\rho w}\frac{\partial \bar{v}}{\partial \bar{z}}=-\frac{\partial \bar{P}}{\partial \bar{y}} \quad (2.3)$$

Z momentum: $$\overline{\rho u}\frac{\partial \bar{w}}{\partial \bar{x}}+\overline{\rho v}\frac{\partial \bar{w}}{\partial \bar{y}}+\overline{\rho w}\frac{\partial \bar{w}}{\partial \bar{z}}=-\frac{\partial \bar{P}}{\partial \bar{z}} \quad (2.4)$$

Energy: $$\bar{u}\frac{\partial}{\partial \bar{x}}\left(\frac{\bar{P}}{\bar{\rho}^{\gamma}}\right)+\bar{v}\frac{\partial}{\partial \bar{y}}\left(\frac{\bar{P}}{\bar{\rho}^{\gamma}}\right)+\bar{w}\frac{\partial}{\partial \bar{z}}\left(\frac{\bar{P}}{\bar{\rho}^{\gamma}}\right)=0 \quad (2.5)$$

The non-dimensional variables are defined as

$$\bar{x}=\frac{x}{L},\bar{y}=\frac{y}{L},\bar{z}=\frac{z}{L},$$

$$\bar{u}=\frac{u}{V_\infty},\bar{v}=\frac{v}{V_\infty},\bar{w}=\frac{w}{V_\infty},$$

$$\bar{P}=\frac{P}{\rho_\infty V_\infty^2},\bar{\rho}=\frac{\rho}{\rho_\infty}$$

Where L denotes a characteristic length of the flow, and ρ_∞ and V_∞ are the free-stream density and velocity, respectively.

In reality, Eq. (2.5) is the entropy equation whose derivation uses an isentropic process in a calorically perfect gas, P/ρ^γ=const; it can serve as the energy equation in an inviscid, adiabatic flow. Hence, if the entropy of a moving fluid element is constant as stated by Eq. (2.5), then the quantity P/ρ^γ is also constant for the moving fluid element. Any particular solution of these equations is governed by the boundary conditions which are discussed below.

The boundary condition for steady inviscid flow at a surface requires that the flow must be tangent to the surface. Then we have boundary condition:

$$\bar{u}n_x + \bar{v}n_y + \bar{w}n_z = 0 \quad (2.6)$$

The boundary conditions behind the bow shock are given by the oblique shock relations:

$$\frac{P_2}{P_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 \sin^2\beta - 1) \quad (2.7)$$

$$\frac{\rho_2}{\rho_\infty} = \frac{(\gamma+1)M_\infty^2 \sin^2\beta}{(\gamma-1)M_\infty^2 \sin^2\beta + 2} \quad (2.8)$$

$$\frac{u_2}{V_\infty} = 1 - \frac{2(M_\infty^2 \sin^2\beta - 1)}{(\gamma+1)M_\infty^2} \quad (2.9)$$

$$\frac{v_2}{V_\infty} = \frac{2(M_\infty^2 \sin^2\beta - 1)\cot\beta}{(\gamma+1)M_\infty^2} \quad (2.10)$$

Noting that for a calorically perfect gas

$$P_2 / P_\infty = \bar{P}_2(\rho_\infty V_\infty^2)/P_\infty = \bar{P}_2\gamma M_\infty^2 \quad (2.11)$$

Using shock relation Eq. (2.7), equations (2.8) to (2.10) in the limit of high M_∞ goes to

$$P_2 \to \frac{2\sin^2\beta}{\gamma+1} \quad (2.12)$$

$$\rho_2 \to \frac{\gamma+1}{\gamma-1} \quad (2.13)$$

$$u_2 \to 1 - \frac{2\sin^2\beta}{\gamma+1} \quad (2.14)$$

$$v_2 \to \frac{\sin 2\beta}{\gamma+1} \quad (2.15)$$

For blunt bodies and M_∞>>1 these properties behind the shock wave are functions of the shape of the shock angle β and the ratio of specific heats γ only. Moreover, the shape of the bow-shock surface is governed by the body shape and the free-stream parameters. Hence the solution of flow-field depends only on the body shape and the ratio of specific heats, and not on the free-stream Mach number. These are the conclusions of calorically perfect gas inviscid flow.

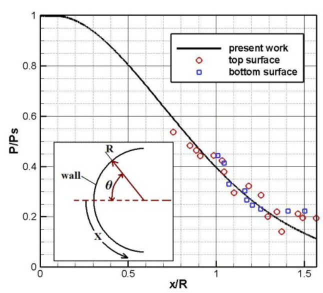
Different from subsonic or supersonic flow, hypersonic flow has its own particular features: thin shock layer, thick boundary layer thickness for viscous flow and high temperature real-gas effect (such as vibrational excitation, chemical reactions and ionization). Through the shock wave, the flow gas starts vibrational excitation and chemical reaction; vibrational excitation will decrease the ratio of specific and chemical reaction increase it. On the other hand boundary layer thickness increasing will influence the wall shape. So in hypersonic flow, we have not sufficient evidence to confirm Mach number independence exists. To know the applicability of the independence principle in very high Mach number, we should realize how the viscosity and thermo-chemical process have an effect on the flow-field solution. The following numerical research is trying giving such an answer to this question.

3. Numerical method, configuration and flow cases

There have been some numerical and theoretical results on the hypersonic Mach number independence for equilibrium gas but haven't been the results for thermo-chemical non-equilibrium air. We know that in the range of Mach 10 to 20, thermo-chemical non-equilibrium processes play an important role on the flow-field and these processes have non-neglectable difference from equilibrium gas. To study the applicability of Mach number independence in thermo-chemical non-equilibrium flow, we choose parameters at altitude 60km as free-stream conditions. Sphere-cylinder model are chosen as research configuration; the radius of sphere is 5cm, and the following cylinder length is equal to the radius. The parameters selected principle follows double scale law. The law indicates that if $\rho_\infty L$ is small, the flow is likely in non-equilibrium state.

In this paper, a numerical code has been developed to obtain steady-state solutions to the equations of fluid motion coupled with the finite-rate chemistry in thermal and chemical non-equilibrium air. It solves the Euler or Navier-Stokes equations for five-component reacting gas flow. Inviscid convective terms are discretized by NND schemes, while viscous fluxes are approximated with a standard centered scheme. The wall can be treated as adiabatic or isothermal wall.

To validate the numerical method, we compare the numerical pressure distribution at Mach 10 with experiments [13]. The comparison is showed as fig. 1. Results show that the method used is accurate.



Fi. 1 comparison of pressure distribution on sphere surface between experimental and present numerical results

4. Results

A brief description of the computed sphere-cylinder flow-field and the effect of Mach number on flow properties are given below. The details of the Mach numbers effect on wave-drag coefficient, lift coefficient, friction coefficient, shock shapes and boundary layer thickness are presented.

Main features of the flow-field are shown in Fig. 2 and Fig. 3 in terms of Mach number contours and pressure distribution at Mach 20. We consider non-equilibrium viscous flow and the wall condition is adiabatic. The prominent features like bow-shock and expansion region can be easily identified. Fig.2 shows the pressure comparison between calorically perfect gas and thermo-chemical nonequilibrium gas. Results indicate that for nonequilibrium gas the flow-field has smaller shock-off distance, meaning thinner shock layer and more strong expansion effect. The hypersonic flow-field is the results of both real-gas effect and expansion effect. The details will be showed in the following section.

4.1. Mach number independence principle applicability for wave-drag coefficient Cd

Fig. 4a shows wave-drag coefficient as a function of Mach number in different gas states (nonequilibrium air, thermal equilibrium air and calorically perfect air) for inviscid flow. Results indicate that Mach number independence approximately exists for thermal equilibrium, calorically perfect and nonequilibrium air. Thermal equilibrium air is such a perfect gas that the vibrational energy will be excited at any low temperature without dissociating reactions. The transformation of translational energy into vibrational energy decreases the ratio of specific heat in the flow-field. At very high Mach number the effect of energy transformation is becoming smaller, so the wave-drag coefficient increasingly tends to a limit value. For nonequilibrium and calorically perfect air the variation tends are identical to ref. [9]. Fig. 5 shows the comparisons among inviscid flow, viscous flow with adiabatic wall and viscous flow with isothermal wall. The effect of viscosity increases the wave-drag coefficient on the whole profile. Resulting from Fig. 5, we know that the effects of viscosity and energy transformation are of the same order. These results may have a little difference from equilibrium flow and big model because flow-field is changed in a different way.

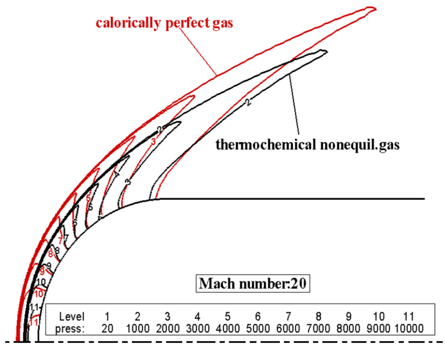


Fig. 2 pressure contours comparison between calorically perfect gas and thermo-chemical non-equilibrium gas

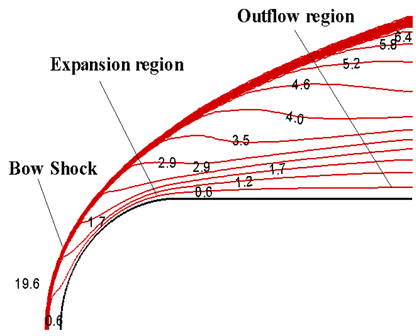
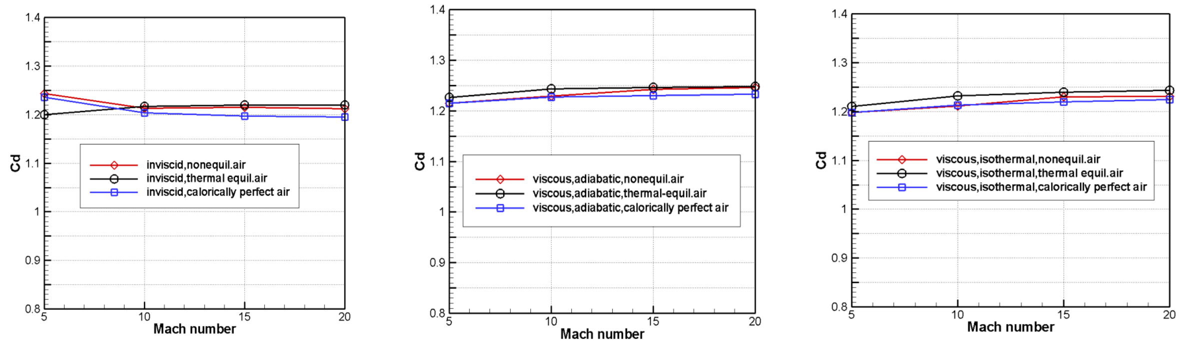


Fig.3 Mach number distribution at Mach 20：viscous non-equilibrium flow for adiabatic wall



(a) (b) (c)

Fig. 4 effect of variation in gas states for a) inviscid flow, b) viscous flow, adiabatic wall, c) viscous flow, isothermal wall on wave-drag coefficient

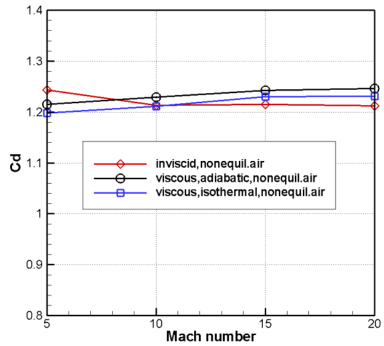


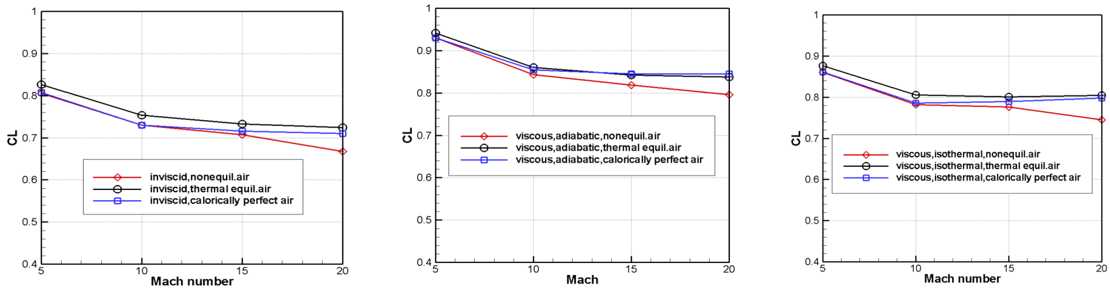
Fig. 5 effect of variation in inviscid flow and flow with adiabatic and isothermal wall for non-equilibrium air on wave-drag coefficient

4.2. Mach number independence principle applicability for lift coefficient CL

Fig. 6a shows the comparison among nonequilibrium air, thermal equilibrium air and calorically perfect air in inviscid flow. The corresponding results in viscous flow are given in Fig. 6b and Fig. 6c. We can conclude that as for lift coefficient, the independence law similarly exists for calorically perfect air and thermal equilibrium air. For thermo-chemical nonequilibrium air, the law doesn’t occur. This is resulting from high temperature real-gas effect and expansion effect and lift coefficient more closely contacts with the two effects than wave-drag coefficient. The real-gas effect and expansion effect will severely change flow-field parameters. The results are given next. Fig. 7 shows the effect of viscosity and wall condition on Mach number independence principle for thermo-chemical nonequilibrium air. For lift coefficient the effects of viscosity and wall condition are of the same order. The variation of lift coefficient with Mach number shows decreasing tend, so the law has no applicability for thermo-chemical nonequilibrium air corresponding to lift coefficient.

4.3. Mach number independence principle applicability for friction coefficient $C\tau$

As outlined above the effect of viscosity cannot be negligible. The friction coefficient can directly reflect the change of boundary layer distribution at the wall due to the effect of viscosity. Fig. 8a and b show the variation of friction with Mach number for nonequilibrium, thermal equilibrium and calorically perfect air with adiabatic and isothermal wall, respectively. Though Mach number independence principle does not exist for friction coefficient, we know wall condition has a severely effect on friction coefficient variation with Mach number. For the nonequilibrium air and adiabatic wall in Fig. 8a, the friction coefficient tiny increases first, and then has a small decrease. This may result from the change of boundary layer distribution. For the nonequilibrium air and isothermal wall in Fig. 8b, the friction coefficient decreases as the Mach number increases. To know how the wall condition has effect on friction coefficient, the boundary layer distribution varying with Mach number are given in the following.



(a) (b) (c)

Fig.6 effect of variation in gas states for a) inviscid flow, b) viscous flow, adiabatic wall, c) viscous flow, isothermal wall on lift coefficient

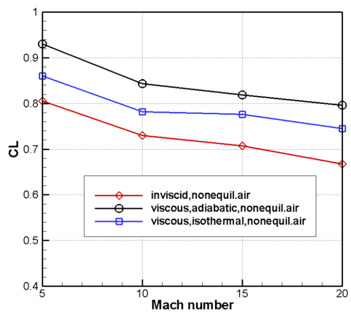
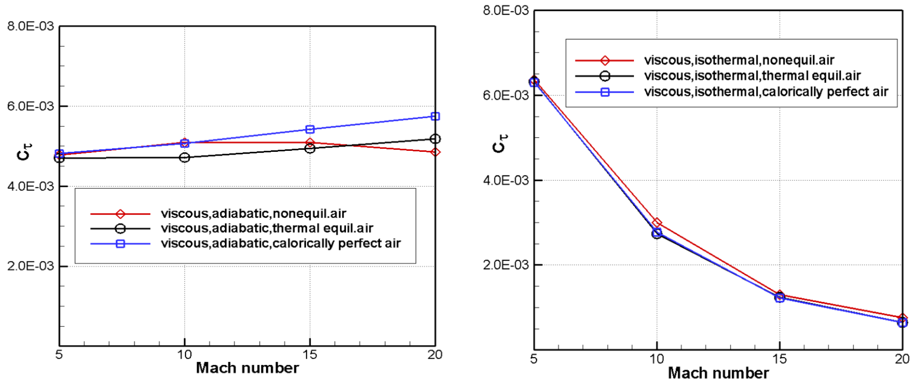


Fig. 7 effect of variation in inviscid and adiabatic, isothermal wall for non-equilibrium air on lift coefficient

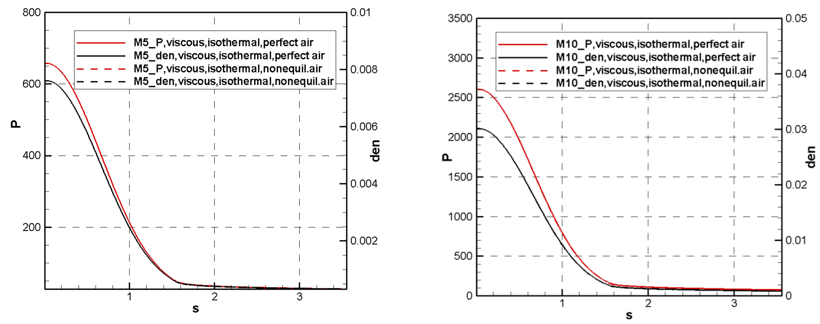
4.4. Mach number independence principle applicability for state parameters along the wall

From the results of section 4.1, wave-drag coefficient changes slowly at high Mach number where the thermo-chemical nonequilibrium process has remarkable effect on flow-field. So it is of interest to know the state parameter variation along the wall with different Mach numbers. Fig. 9 a, b, c, d show the pressure distributions (Mach_p) and density distributions (Mach_den) along the isothermal wall for calorically perfect and thermo-chemical nonequilibrium air. In the Fig. 9, s is the dimensionless length from the stagnation point along the wall. From the state equation $P = (1+\alpha)\rho R_i T$ (α is dissociating degree and R_i is gas constant for specie i) and isothermal wall condition, we have $P \propto (1+\alpha)\rho$. The equation shows that for isothermal wall the pressure and density have the same variation tend. At Mach 20, the nonequilibrium effect gets to the maximum in our considered cases. Results indicate that s<0.7, the pressure for nonequilibrium air is larger than calorically perfect air and s>0.7, result is on the contrary. Due to real-gas effect and expansion effect the pressure distribution along the wall has been changed severely. In conclusion, we should consider the coupled effect of real-gas and expansion for nonequilibrium air.

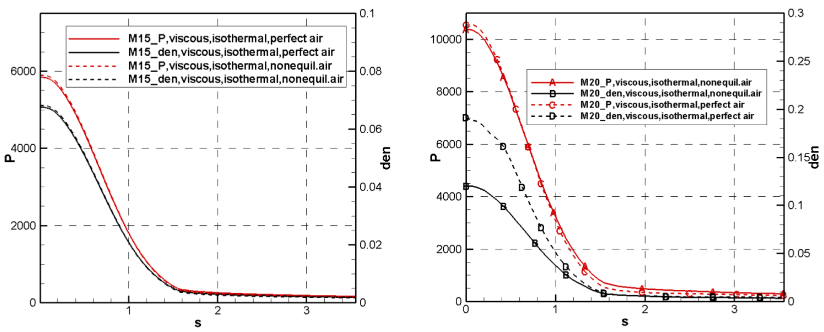


(a) (b)

Fig. 8 effect of variation in gas states for a) adiabatic wall, b) isothermal wall



(a) (b)



(c) (d)

Fig. 9 pressure (Pa) and density (kg/m^3) distribution along the isothermal wall at Mach number a) 5, b) 10, c) 15, d) 20

Fig. 10 a, b, c, d show pressure distributions along the wall for adiabatic wall at Mach 5, 10, 15, 20, respectively. The corresponding density and temperature distributions are given in Fig. 10 a’, b’, c’, d’. From Mach 5 to 20, we can see that the density and temperature distributions change severely and the corresponding pressure distributions a little small. From state equation $P = (1+\alpha)\rho R_i T$ and adiabatic wall, we know $P \propto (1+\alpha)\rho T$. The nonequilibrium flow multiplies the density while diminishes the temperature. The overall pressure distribution along the wall has a small change, though the dissociating degree changes at the same time. This may be the reason of some kinetic parameters changing slowly.

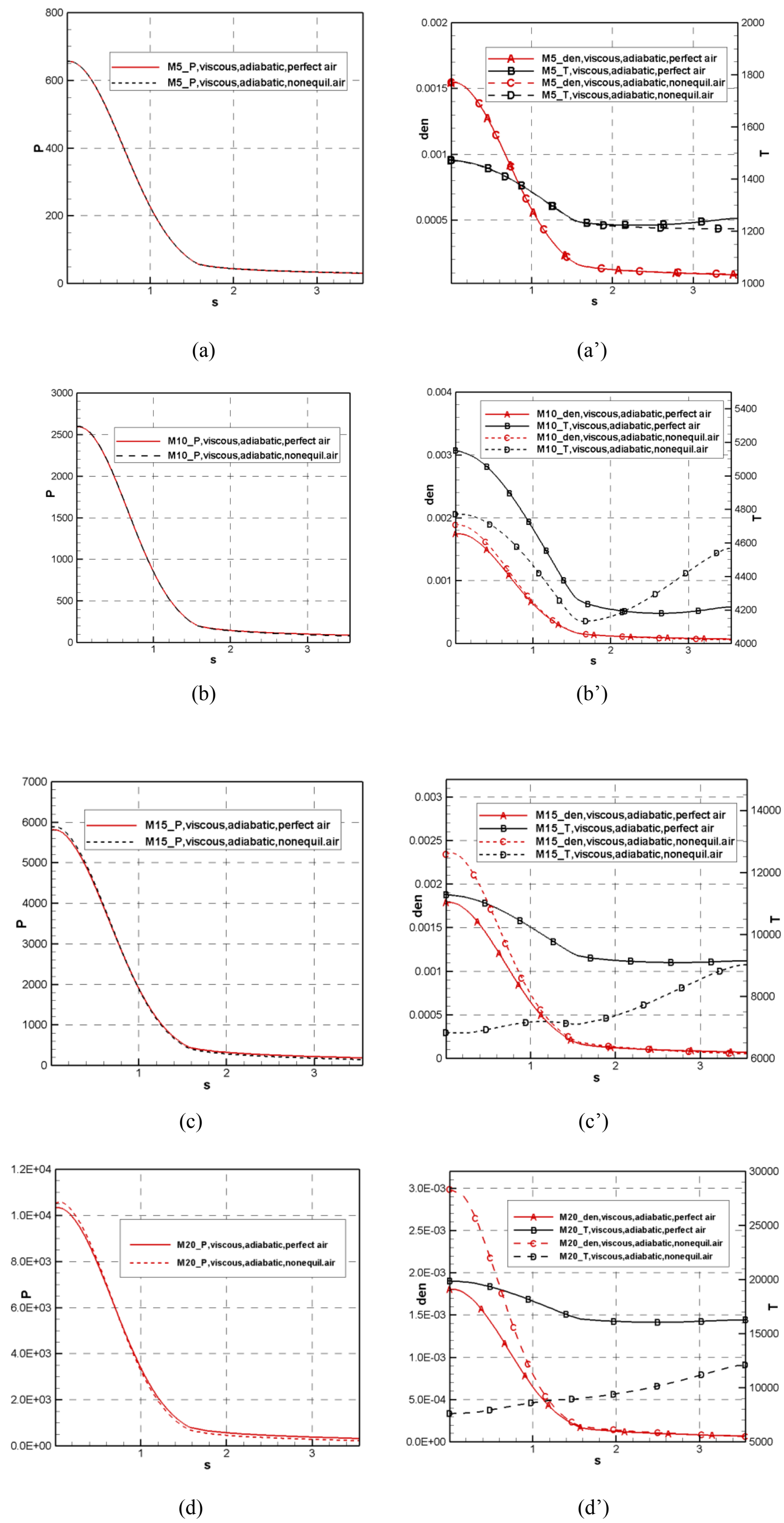


Fig. 10 pressure distribution (Pa) and density (kg/m^3) & temperature (K) distribution along the adiabatic wall at Mach: a and a') 5, b and b') 10, c and c') 15, d and d') 20

4.5. Mach number independence principle applicability for velocity distribution in boundary layer

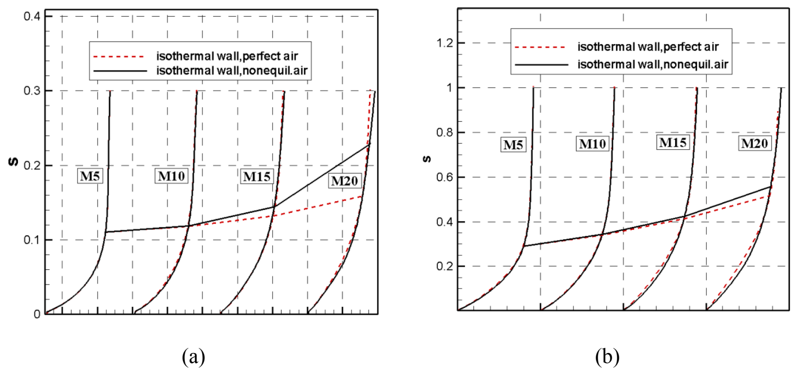


Fig. 11 velocity distribution in boundary layer for isothermal wall at a) expansion region, b) outflow region

The shape of the bow-shock surface is governed by the body shape and the free-stream parameters. The boundary layer thickness modifies the body shape non-negligibly. From the results of wave-drag, lift coefficient and friction coefficient, we know that the effect of viscosity is identical to nonequilibrium effect. Hence, to understand how nonequilibrium process has effect on boundary layer distribution makes a difference. Fig. 11 shows velocity distribution in boundary layer for isothermal wall at expansion and outflow region where s is the dimensionless distance along the wall-normal direction. Results reveal that the difference between calorically perfect air and nonequilibrium air is bigger in the outflow region. Fig. 12 gives the corresponding adiabatic wall results at expansion and outflow region. We know that nonequilibrium effect results in more difference in expansion region. A remarkable distinction between Fig. 11 and Fig. 12 is that the difference of the two kinds of gas is increasing with Mach number in outflow region of Fig. 11, but not in Fig. 12. This phenomenon may explain the results of friction coefficient variation with Mach number at different wall conditions.

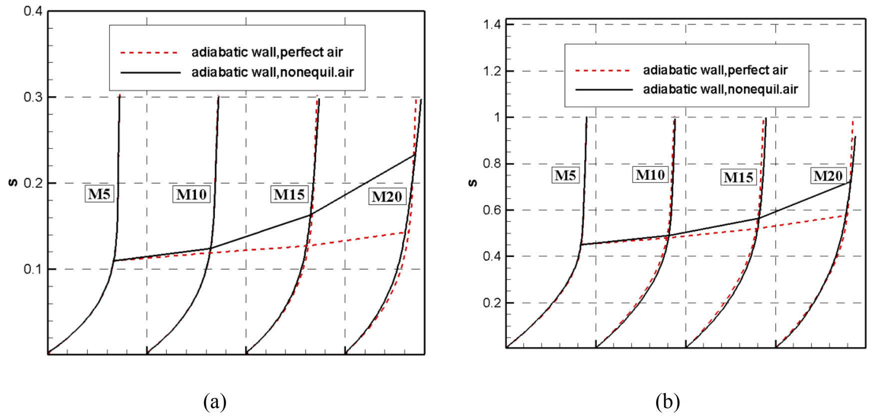
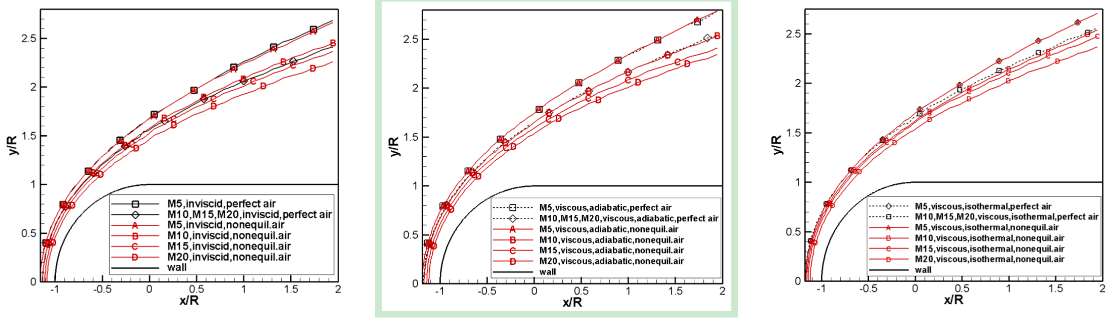


Fig. 12 velocity distribution in boundary layer for adiabatic wall at a) expansion region, b) outflow region

4.6. Mach number independence principle applicability for shock wave shapes

The shape of bow-shock surface is the decisive factor on the solution of shock layer flow-field. In this section, we give the effect of Mach number on shock shapes in different conditions. Fig. 13 a, b, c show variation of shock shapes with Mach number at inviscid flow, viscous flow with adiabatic wall and viscous flow with isothermal wall, respectively. Results indicate that the shock shapes are varying with Mach number, so the

Mach number independence principle doesn't exist for shock shapes. This result has a little difference from the contents of Oswatitsch's law. However, experimental and theoretical results show that the thermo-chemical nonequilibrium effect will change the shape of bow-shock [14-15]. We also give the nonequilibrium air comparison among different wall conditions at Mach 20 in Fig. 14. Comparing line labeled 'A' and 'B' or line labeled 'C' and 'D', we know that whether the flow is viscous is the most great effect on shock shapes. The conditions of adiabatic or isothermal wall have a little small difference.



(a) (b) (c)

Fig. 13 the shock shapes comparison between calorically perfect air and non-equilibrium at different Mach number for: a) inviscid flow, b) viscous flow, adiabatic wall, c) viscous flow, isothermal wall

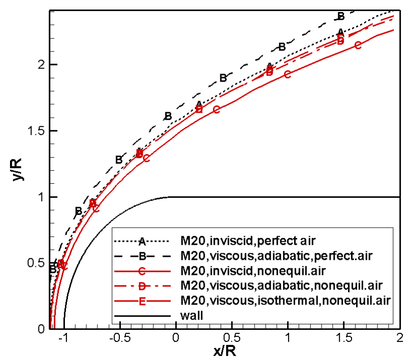


Fig.14 the shock shapes comparison for non-equilibrium air among inviscid (labeled 'C'), adiabatic wall (labeled 'D') and isothermal wall (labeled 'E')

5. Conclusions

Proceeding from the contents of Oswatitsch's Mach number independence, we numerically consider the effect of viscosity, thermal-equilibrium and nonequilibrium on wave-drag coefficient, lift coefficient, friction coefficient, boundary layer, velocity distribution along the wall and shock shapes to research the applicability of independence law. Summing up the results mentioned:

- In viscous flow, Mach number independence principle still exists if it occurs in corresponding inviscid flow. However, the viscosity will change the value of wave-drag coefficient and lift coefficient to some extent and influence the shock wave shapes. Adiabatic and isothermal conditions both have no effect on the applicability of this law.

- Mach number independence principle is true for wave-drag and lift coefficient, shock shapes in calorically perfect gas flow. In nonequilibrium flow, this law occurs just for wave-drag coefficient and not for other parameters we considered due to the coupled effect of real-gas and expansion.
- Friction coefficient and boundary layer distribution do not follow Mach number independence. Their variation tends depend on wall condition (such as adiabatic or isothermal wall) and flow states (such as frozen flow or nonequilibrium flow).

The nonequilibrium effect will increase the thickness of boundary layer while change of boundary layer thickness brings the change of shock wave shapes. The solution of shock layer depends on the shock shapes and gas properties. This is why we consider six parameters to validate the applicability of Mach number independence principle. Comparing the existing results with present work we know that some conclusions have a little difference between nonequilibrium flow and equilibrium flow. It has no sufficient evidence to say the results of present work can be used in other situation. The coupling of high speed flow and thermo-chemical process makes the hypersonic flow a hard problem. There are still many things we need to do.

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