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Effect of surface reflectivity on photonic Doppler velocimetry measurement

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Abstract

While photonic Doppler velocimetry (PDV) is becoming a common diagnostic for tracking velocity in shock physical experiments, its validity on measuring surfaces with different reflectivity is not studied. This paper investigates the effects of surface reflectivity on PDV measurement for tracking back free surface velocity in laser shock processing. Credible measurement results for coarse polished surfaces with low reflectivity are obtained, whereas fine polished surfaces with relatively high reflectivity lead to heterodyne fringes with high frequency and corresponding unreasonably fast velocities. This phenomenon reported in the paper is somewhat inconsistent with the general view that PDV has remarkable robustness to large changes in surface reflectivity. The reason might be ascribed to multiple reflections of light, which cause the generation of multiple Doppler shifts. The mixing of the reference light and those Doppler-shifted lights brings out high frequency heterodyne fringes resulting in high velocity. Low surface reflectivity is better suited for PDV measurements.

Keywords: photonic Doppler velocimetry, surface reflectivity, back free surface velocity, multiple Doppler shifts

(Some figures may appear in colour only in the online journal)

1. Introduction

Optical velocimetry is a powerful diagnostic technique for measuring dynamic behavior of materials under shock loading conditions [1, 2]. Being compact, simple to reconstruct and robust to non-ideal experimental conditions [3, 4], PDV (photonic Doppler velocimetry), also known as 'heterodyne velocimetry' [2, 5], has been widely used for tracking particle velocities in shock physics experiments including laser induced shock, explosives and gas guns [2, 6–8].

PDV is essentially a fiber-based fast Michelson interferometer [6], where interference fringes are yielded by mixing Doppler-shifted light reflected from a moving surface of interest with a beam of reference light. The frequency difference between the Doppler-shifted light and the reference light, named the beat frequency, f_{beat}, can be recorded using a detecting system with a high enough bandwidth. The frequency of the interference fringes is proportional to the velocity of the moving surface, and the velocity can be derived by extracting the power spectrogram of the interference fringes [7, 9]. The velocity of the moving surface can be obtained by the relationship $u = f_{\text{beat}} \cdot \lambda/2$ [2, 6], where λ is the wavelength of the reference light.

With the velocity information of the moving surface being encoded in the time domain, it is possible for a PDV system to record multiple velocities simultaneously, thus making PDV an exciting and powerful diagnostic tool to measure multiple velocities in shock-induced phenomena such as phase transitions and ejection [6, 8, 10, 11]. Generally, the multiple velocities can be distinguished in discrete velocity PDV measurements because the extra velocities add new and indiscernible features to the power spectrogram. However, undesired multiple velocities often give rise to difficulty in data processing and interpretation of experimental phenomena. These multiple velocities are usually caused by multiple reflections at the surface or interfaces of a PDV system. For example, when using an optical window to maintain a high pressure state in a sample during shock experiments, multiple velocities due to Fresnel reflections are easily tracked by a PDV system [10, 12]. Another case is that multiple reflections take place between the moving surface of interest and the end-surface of the PDV probe or interfaces of the PDV system while measuring a surface with high reflectivity, which has not been studied in the literature and is the focus of this paper. In PDV measurements, subsequent reflections in the PDV system are persistent and become weaker with each transit. Each reflection causes additional Doppler shifts in frequency and consequently yields new velocity signals in the heterodyne fringes. It may not be a problem for a surface with relatively low reflectivity because the multiple Doppler shifts are too weak to reduce the signal contrast. This issue can be severe when the measured surface has high reflectivity, such as polished or metal coated surfaces. Although the light becomes weaker after each transit, the multiple Doppler shifts might still reduce the signal contrast, making the signal ambiguous and leading to an unreasonable measured velocity. Although PDV is generally known to have remarkable robustness to large changes in surface reflectivity, its validity on measuring surfaces with different reflectivity is not systematically studied in the literature.

The paper investigates the effect of surface reflectivity on PDV measurement. Back free surface velocities of aluminum specimens with different reflectivity, which are induced in laser shock processing (LSP) experiments, are measured with a developed PDV system [13, 14]. Section 2 describes our PDV system and related data processing methods. Section 3 depicts LSP experimental and PDV measurement results for specimens with different reflectivity. Section 4 discusses the effects of surface reflectivity on PDV measurements.

2. PDV function

2.1. PDV system

As depicted in figure 1, we developed a PDV system that basically follows the configuration reported by Strand *et al* [2]. It consists of a continuous wave (CW) laser with narrow linewidth, a fiber optic circulator, a high bandwidth detector, an optical collimating lens probe and an oscilloscope with high bandwidth and sample rate. The laser and the detector are connected to the first and the third ports of the circulator respectively, and the probe, which is used to provide reference light and collect Doppler-shifted light reflected from the moving surface of interest, is connected to the second port of the circulator.

A high power 1550 nm CW distributed feedback laser with a polarization maintaining fiber (CQF938 series, provided by JDS Uniphase Corporation) is used. The laser is operated at a maximum power of 40 mW with a linewidth of about 200 kHz. A photodiode detector (InGaAs PIN, provided by New Focus Inc.) with bandwidth 12 GHz is employed. The heterodyne signals are recorded with an oscilloscope (WaveMaster 808Zi, provided by Lecory Inc.) operating with a bandwidth of 8 GHz and a sampling rate of 40 GS s^{-1} for each channel. An optical

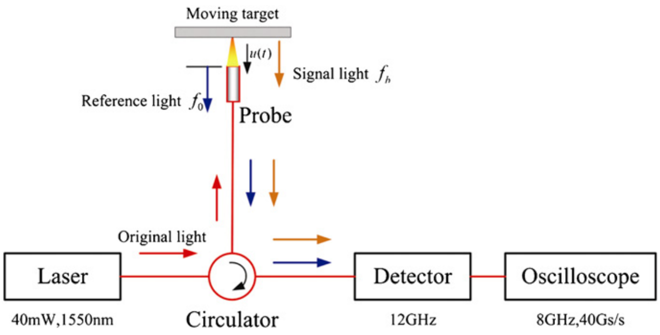

Figure 1. Schematic of the PDV system.

collimating lens probe with back reflection (−13 dB) and a work distance of 15 mm is used to provide reference light and collect Doppler-shifted light. The study of Valenzuela *et al* [8] shows that the best signals are achieved when the reference signal is about −10 dB as compared to the reflected surface probe signal from the unperturbed target. So it is expected to get good signals in our experiments. The end-surface of the probe is coated with a thin layer of anti-reflection coating, reducing the reflection of Doppler-shifted light at the end-surface of the probe to about −20 dB. To distinguish the interference signals at the very beginning of the back surface motion in LSP experiments, the digital oscilloscope is triggered by the temporal domain of a Q-switched high power laser pulse (laser source of LSP) with a Si-biased detector.

2.2. Data processing method

Heterodyne signals recorded by the oscilloscope need to be mathematically transformed from the time domain to the frequency domain for the computation of velocity spectrogram. Short time Fourier transform (STFT) and continuous wave transform (CWT) are commonly used methods in time–frequency analysis for PDV measurements. Our previous study showed that CWT has a better temporal resolution and frequency resolution than STFT for tracking fast-changing low particle velocity in LSP [9]. Hence, CWT is taken to extract velocity history in the experiments. The practical procedure for analyzing the particle velocity by the CWT analysis program is also given in [9].

3. Effect of surface reflectivity

3.1. Experimental details

The experimental setup for measuring back free surface velocity in LSP is illustrated in figure 2. The target material is 2024 aluminum with thickness of 1.0 ± 0.01 mm. The shocked surface of each target is glued with a 60 μm thick Al foil as an absorption layer, confined by 4 mm thick K9 glass against the laser irradiation. The K9 glass is fully clamped with the target without a cushion at the back surface by a specially designed fixture. A Q-switched high power Nd:YAG laser of 2.5 J per shot with FWHM of about 10 ns is utilized in LSP experiments. The temporal and spatial profiles of the laser are near-Gaussian in distribution. Once the high power density

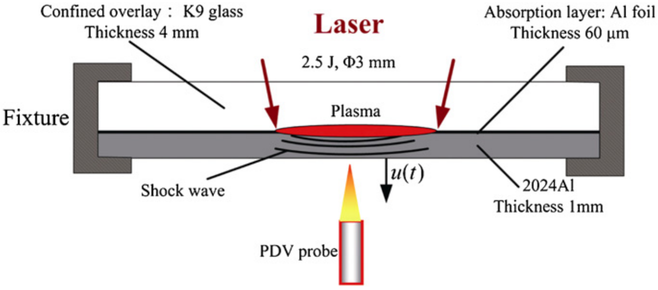

Figure 2. Experimental setup for measuring back free surface velocity induced by plasma pressure in LSP.

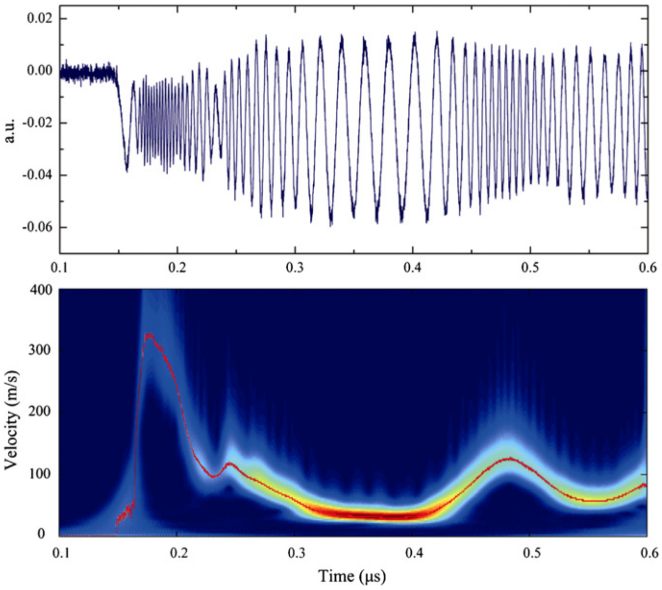

Figure 3. Typical PDV results for surface with low reflectivity of a 2024 aluminum specimen.

laser irradiates the absorption layer, high density plasma is generated. The adiabatic expansion of the heated plasma in the confined region between the target and the transparent overlay creates pressure pulse with high amplitude and short duration, inducing shock wave propagation in the target and causing back free surface velocity once the pressure pulse reaches the back surface. In experiments, the irradiated surfaces of targets are finely polished to a roughness of about 42 nm (measured with atomic force microscopy), and the back free surfaces are polished to different roughness values to obtain various surface reflectivity and its effects on PDV measurements are investigated. The roughness of the back free surfaces with low reflectivity is about 600 nm, and the roughness of the back surfaces with high reflectivity is about 42 nm measured with atomic force microscopy.

3.2. PDV results

3.2.1. Results for surfaces with low reflectivity. Typical heterodyne fringes measured from the surface of low reflectivity (surface roughness 600 nm) and the calculated surface velocity are shown in figure 3. According to the principle of PDV, the instantaneous velocity is proportional to the heterodyne frequency. Therefore, the density of heterodyne fringes indicates the magnitude of the surface velocity; the closer two adjacent fringes are, i.e., the higher the beat frequency, the faster the velocity at that moment is. Due to the reflection and transmission of the shock wave within the target, the velocity is oscillated periodically, and the time duration between the adjacent peak velocities is about twice the transmission duration of the shock wave propagating through the target. The zero point at the horizontal coordinate is the onset of the Nd:YAG laser pulse. The first two interference fringes, starting at about 145.8 ns, are caused by the elastic precursor wave, indicating that the propagation of the elastic wave reaches the back free surface at this time. The surface velocity caused by the elastic precursor wave is about 50 m s^{-1}. At 177.2 ns, the surface velocity reaches the first peak value of about 325.4 m s^{-1}, indicating that the first shock wave reaches the back surface. At 482.2 ns, the surface velocity is about 126.1 m s^{-1} when the shock wave reaches the back surface again. The time between the adjacent peak velocities is about 305.0 ns.

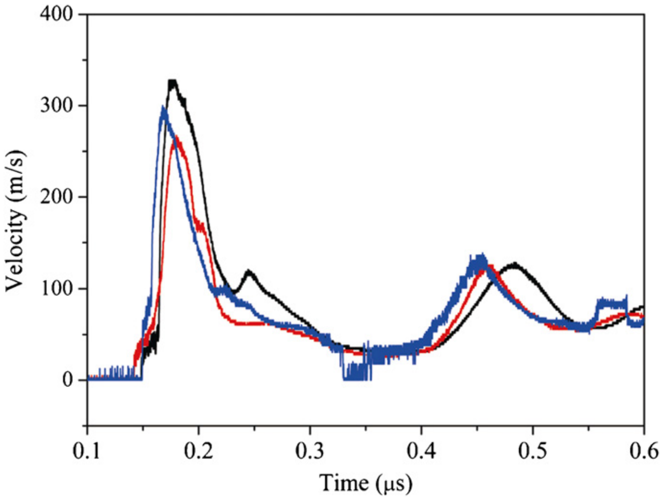

Figure 4. Measurement results for surfaces with low reflectivity of 2024 aluminum specimen. The differences in arriving time and peak velocity are caused by small differences in sample thickness and laser induced peak plasma pressure in each experiment.

More experiments on surfaces with low reflectivity are conducted and the extracted velocities are given in figure 4. The measured velocities have slight difference in arrival time, peak value and time duration, which is caused by small differences in sample thickness and laser induced peak plasma pressure in each experiment. Nevertheless, the velocity profiles agree with each other, indicating the repeatability of the PDV measurement for measuring surfaces with low reflectivity.

3.2.2. Results for surfaces with high reflectivity. Typical heterodyne fringes measured from the surface with high reflectivity (surface roughness 42 nm) and the calculated surface velocity are shown in figure 5. The time between adjacent velocity peaks is the same as the measurements of surfaces with low reflectivity. However, the density of the heterodyne fringes for the surface with high reflectivity is much higher than that of the measurements for the surfaces

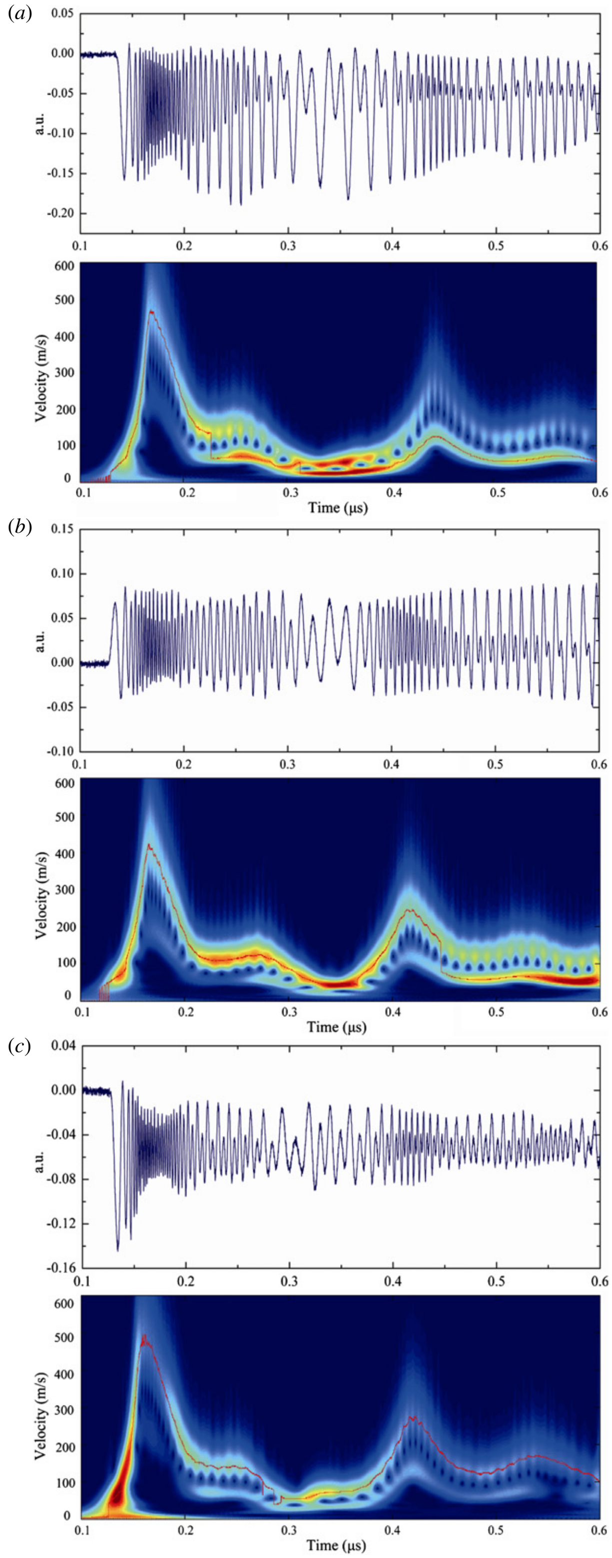

Figure 5. Typical PDV results for surface with high reflectivity of a 2024 aluminum specimen. The heterodyne fringes are a mix of two series of interference fringes with different frequency, leading to high density or high frequency of the heterodyne fringes and unreasonably fast back free surface velocity.

with low reflectivity. It seems that the heterodyne fringes are a mix of two series of interference fringes with different frequency, which leads to high density or high frequency of the heterodyne fringes. As a result, the corresponding back free surface velocity extracted with the same method is much faster. In view of figure 5(*a*), the peak velocity reaches up to 474.3 m s^{-1} at about 168.1 ns. However, the second peak velocity reached at 473.7 ns is about 123.6 m s^{-1}, which is the same as the measurements of surface with low reflectivity. In addition, the contrast of the power spectrum is greatly reduced when measuring the surface with high reflectivity. The power spectrogram analyzed by the CWT method is much wider across the extracted velocity curve, indicating that the velocity with maximum probability at each time is relatively uncertain in the spectrogram compared with the measurements of surfaces with low reflectivity. At the beginning, the velocity is extracted along the upper boundary of the spectrogram, whereas it goes along the lower boundary after 226.1 ns, which leads to a jump in velocity at 226.2 ns. In figures 5(*b*) and (*c*), the same phenomenon is captured. The heterodyne fringes and corresponding velocity profiles as depicted in figures 5(*a*)–(*c*) agree with each other, in which the phenomena of high density of heterodyne fringes and fast velocity are evident in the measurements, demonstrating that surfaces with high reflectivity will generate heterodyne fringes with high frequency and essentially lead to fast velocities for PDV measurements.

4. Discussion

The experimental results show that measured surface reflectivity has a great impact on PDV measurement. High frequency of heterodyne fringes and fast velocity are observed for measuring surfaces with high reflectivity. Here, theoretical analysis is performed to distinguish the credible results of PDV, and then the mechanism of unreasonable results is also discussed.

4.1. Theoretical analysis of maximum velocity in LSP

The maximum velocity of back free surface is determined by the pressure applied on the shocked surface and its attenuation in the target. A coupling analysis method for laser induced shock pressure has been developed, by which the peak shock pressure can be calculated for various laser power intensity [15, 16]. In our experiments, the peak laser power intensity is 2.02 GW cm^{-2} and its induced peak shock pressure is about 4.71 GPa. For a 2024 aluminum target with infinite thickness, the attenuation behavior of laser induced shock pressure can be analyzed and it follows the scaling law [15],

$$p/p_m = 0.67\exp\left[-\frac{H/(2R)}{0.71}\right] + 0.25\exp\left[-\frac{H/(2R)}{0.12}\right] + 0.09, \qquad (1)$$

where H is thickness of target and R is radius of laser spot. From equation (1), the peak pressure at depth 1 mm is attenuated to 3.12 GPa. According to the Rankin–Hugoniot relation [17, 18], for the uniaxial strain state, the compressive pressure,

p, is related to the particle velocity as equation (2) while p exceeds the Hugoniot elastic limit (HEL) of the material,

$$p = \rho_0(C_0 + Su)u + \tfrac{2}{3}Y_0, \tag{2}$$

where $\rho_0 = 2.77 \times 10^3$ kg m^{-3} is the initial material density, $C_0 = 5.33 \times 10^3$ m s^{-1} is the sound velocity at zero pressure, $S = 1.34$ is the empirical material parameter, u is the particle velocity, $Y_0 = 265$ MPa is the yield stress for the 2024 aluminum material and HEL is related to Poisson's ratio μ and dynamic yield stress Y_0, HEL $= (1-\mu)Y_0/(1-2\mu) =$ 547 MPa [18, 19].

By equation (2), the peak particle velocity at depth 1 mm, u, is about 162 m s^{-1} for an infinite thickness target. Therefore, the maximum velocity at back free surface, u_{surf}, that is approximately twice u, is about 324 m s^{-1} by taking into account the reflection of stress waves at the back free surface, which indicates that the PDV measurements for surfaces with low reflectivity are credible, whereas the measured velocities for surfaces with high reflectivity are unreasonably fast.

4.2. Multiple velocities in PDV measurements

PDV is essentially a fiber-based fast Michelson interferometer. The mixing of reference light and Doppler-shifted light yields heterodyne fringes. Multiple Doppler shifts induced by multiple reflections in the system will lead to heterodyne fringes with high frequency. In PDV measurements, multiple reflections can take place between the probe's end-surface and the moving surface. Additionally, there might be some other sources of reflection in the system, such as fiber-to-fiber couplers that lead to obvious multiple frequencies for high surface reflectivity measurements. Here, we take the multiple reflections between the probe's end-surface and the measured surface of interest as an example. As shown in figure 6, the intensity of the light becomes weaker after each reflection. Eventually, the reference light with the frequency f_0 is combined with the series of multiple Doppler shifts with frequencies f_1, f_2, etc. This is similar to the observations of Ao and Dolan [12] and Jenson *et al* [10] when using a window in PDV measurements. Generally, the effect of multiple reflections between a PDV probe's end-surface and measured surface of interest, which involves multiple Doppler shifts, can be distinguished for a measuring surface with low reflectivity because the intensities of multiple Doppler shifts are too weak to reduce signal contrast. However, the influence of multiple reflections reduced the signal contrast for a measuring surface with high reflectivity as illustrated in figures 5(*a*)–(*c*). The added new features of those multiple reflections are discernible especially for measuring fast velocities, which makes the power spectrum ambiguous for extracting correct results.

Consider the mixing of reference and multiple Doppler-shifted signals,

$$E_i = A_i \cos(\omega_i t + \varphi_i), \quad i = 0, 1, 2, \ldots, N \tag{3}$$

where A_i is amplitude, ω_i is angular frequency, $\varphi_i = -2\pi\Delta l/\lambda_i + \varphi_{0i}$ is phase, λ_i is wavelength and φ_{0i} is initial phase. After each Doppler shift, the frequency has

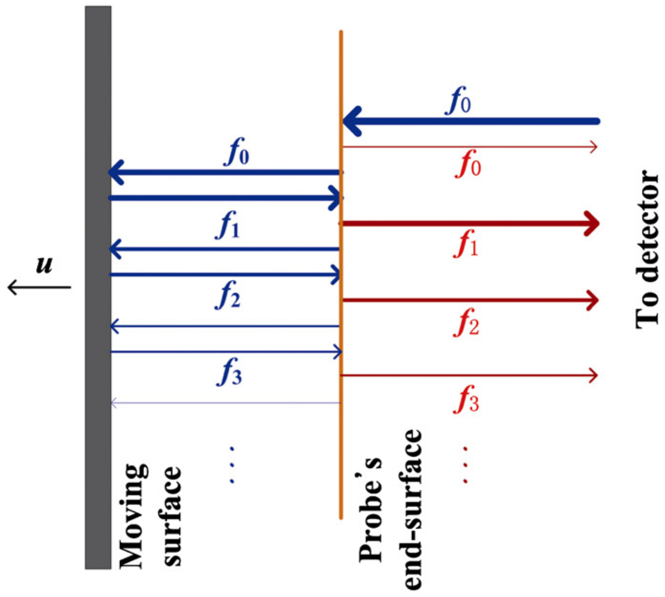

Figure 6. Multiple reflections between PDV probe's end-surface and moving surface. The intensity of the light becomes weaker after each reflection. The Doppler shift in frequency occurs with each reflection at the moving surface.

an increment of about $2u/\lambda$. The output electric field is a superposition of those N optical signals,

$$E = \sum_{i=1}^{N} E_i \tag{4}$$

which leads to the following time averaged output intensity:

$$\begin{aligned} I(t) &= I_0(t) + \sum_{i=1}^{N} I_i(t) \\ &\quad + 2\sum_{i=1}^{N} \sqrt{I_0(t)I_i(t)} \cos[\bar{\omega}_{0i}(t-\bar{t}) - \bar{\varphi}_{0i}(t)], \\ &\quad + 2\sum_{i\neq j} \sqrt{I_i(t)I_j(t)} \cos[\bar{\omega}_{ij}(t-\bar{t}) - \bar{\varphi}_{ij}(t)] \end{aligned} \tag{5}$$

where $I_i(t)$ is time averaged intensity of the individual optical signal, $\bar{\omega}_{0i}$ and $\bar{\varphi}_{0i}$ are angular frequency difference and phase difference between Doppler-shifted signals and reference signal, respectively, and $\bar{\omega}_{ij}$ and $\bar{\varphi}_{ij}$ are angular frequency difference and phase difference between Doppler-shifted signals, respectively. The output electrical signal from a linear detector is

$$\begin{aligned} D(t) &= 2\sum_{i=1}^{N} A_{i0} \cos[\bar{\omega}_{0i}(t-\bar{t}) - \bar{\varphi}_{0i}(t)] \\ &\quad + 2\sum_{i\neq j} A_{ij} \cos[\bar{\omega}_{ij}(t-\bar{t}) - \bar{\varphi}_{ij}(t)]. \end{aligned} \tag{6}$$

Theoretically, there are $N(N+1)/2$ peaks in the electrical output signals. For surface with low reflectivity, the intensities of multiple Doppler shifts are so weak that their added new intensities are very low and the interference fringes are indiscernible. The influence of those added features on the power spectrum can be eliminated by data reduction. However, as shown in figures 5(*a*)–(*c*), the intensities added by the multiple Doppler shifts are discernible in the interference fringes especially for fast velocities when measuring surfaces with high reflectivity. The signal contrast is reduced sharply and it is hard to extract the correct result in the power spectrum, obtaining unreasonably fast velocities in PDV measurements. From this point, a surface with low reflectivity is useful to diminish the influence of multiple Doppler shifts and consequently obtain credible PDV results. Additionally, Ao and Dolan [12] found that a wedge window can significantly reduce velocity ringing caused by multiple Fresnel reflections for PDV measurement with an optical window. Similarly, a probe with slightly wedged end-surface might be useful if the multiple reflections occur between the probe's end-surface and the measured surface, by which the multiple reflections are not collected by the PDV probe. It will be implemented in our experiments to investigate if credible results can be obtained. In addition, with this method we can also confirm if multiple reflections take place between the probe and the measured surface. Moreover, the effect of surfaces' reflectivity is qualitatively investigated in the present research. Further validation of PDV for measuring surfaces with quantitative reflectivity is expected in the future.

5. Summary

The paper investigates the effect of surface reflectivity on PDV measurement. While measuring back free surface velocity in LSP, credible measurement results for surfaces with low reflectivity are achieved by a PDV system, whereas surfaces with high reflectivity lead to heterodyne fringes with high frequency and unreasonably fast velocities. This phenomenon is inconsistent with the general view of PDV measurements that they have remarkable robustness to large changes in surface reflectivity. The reason might be ascribed to multiple reflections of light, which could take place between PDV probe's end-surface and measured surface of interest or interface of the PDV system. Each reflection at the moving surface experiences a Doppler shift in frequency. For measuring surfaces with high reflectivity, the multiple Doppler shifts reduce the signal contrast of the power spectrum and consequently lead to unreasonably fast velocity. This problem can be avoided by reducing surface reflectivity. In addition, a probe with slightly wedged end-surface might also be helpful if the multiple reflections take place between probe's end-surface and measured surface, which will be further investigated in the future.

Acknowledgments

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