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A Pipe-Soil Interaction Model for Anti-Rolling Pipeline On-Bottom Stability on a Sloping Sandy Seabed

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ABSTRACT

The stability of a pipeline on the continental slope involves a complicated interaction between the pipe and the sloping seabed. Based on the Coulomb passive earth pressure theory, a theoretical pipe-soil interaction model is proposed for the instability of the anti-rolling pipeline on a sloping sandy seabed. The mechanisms of shallowly-spreading slippage and deeply-spreading slippage are discussed respectively. The theoretical model is verified with the existing experimental results. Parametric study indicates that the slope angle has much influence on the ultimate lateral soil resistance of the pipeline on a sloping sandy seabed.

KEY WORDS: Sloping seabed; pipe-soil interaction; lateral stability; theoretical solution.

INTRODUCTION

One of the key issues for the submarine pipeline design is the lateral stability analysis. To avoid the occurrence of pipeline lateral instability, the surrounding soil must provide enough lateral soil resistance to balance the hydrodynamic loads.

Before 1970s, the traditional Coulomb friction theory was employed to calculate the lateral soil resistance to the pipeline in waves. However, the model tests by Lyons et al. (1973) showed that the Coulomb friction theory is not appropriate for describing the complicated pipe-soil interaction. Karal (1977) employed the upper bound theorems of classical plasticity theory to predict the lateral soil resistance, idealizing the pipe as a rigid wedge indenter. Till now, several empirical pipe-soil interaction models have been proposed to predict the ultimate lateral soil resistance to the pipeline in waves. In the pipe-soil interaction model by Wagner et al. (1989), it was assumed that the ultimate soil lateral resistance is the sum of the sliding resistance component and the soil passive resistance component. In the energy-based pipe-soil interaction model (Brenodden et al., 1989), the aforementioned soil passive resistance component is relative to the work done by pipe during its movement. Note that in those empirical models, the direct combination of the sliding resistance and the so-called soil passive resistance component lacks theoretical basis.

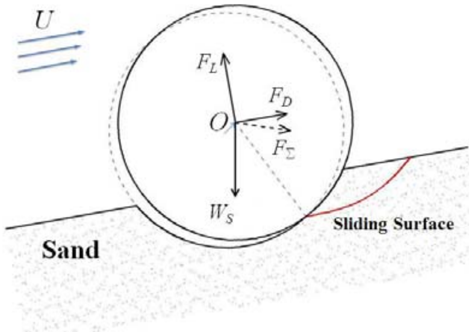
To simulate the interaction of a shallowly-embedded pipeline with the calcareous sand, a non-associated bounding surface model was developed by Zhang et al. (2002) on the basis of the theory of plasticity and the experimental data obtained from a series of centrifugal tests. Gao et al. (2011) investigated the pipe-soil interaction mechanism with a 1g mechanical-actuator facility for the steady flow-induced instability of a pipeline partially embedded in the sandy soil. Youssef et al. (2013) further carried out centrifuge modeling of the pipe-soil interaction behavior under equivalent wave and current loading. Flow-pipe-soil interaction mechanisms for the pipeline on-bottom stability in waves or steady current have also been studied with oscillatory-flow tunnel or water flume (e.g., Gao et al., 2002, 2007; Teh et al, 2003). A selective literature review on physical modeling of pipeline on-bottom stability was given by Gao et al. (2012).

The aforementioned studies focused mainly on pipeline on-bottom stability on the horizontal seabed. With more and more oil and gas reservoirs having been found at the continental slopes, the stability of pipelines on a sloping seabed attracts increasing attention of engineering designers and researchers. But the effect of seabed slope angle on the pipeline on-bottom stability is far from being well understood. Recently, a newly-designed pipe–soil interaction facility and a flow-structure-soil interaction flume have been utilized by Gao et al. (2012) for full-scale physical modeling of the pipeline instability on a sloping sand-bed, including the downslope instability and the upslope instability.

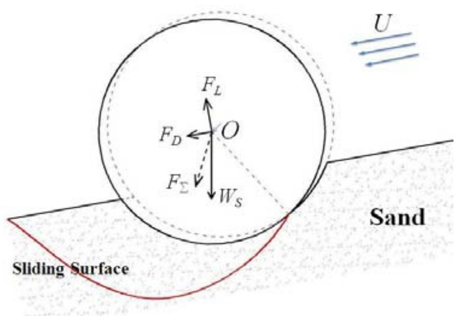
In this paper, the lateral soil resistance of the pipeline embedded in a sloping sand-bed is analyzed theoretically on the basis of Coulomb passive earth pressure theory. A parametric study is then performed to investigate the influential factors for the critical lateral soil resistance.

PIPE-SOIL INTERACTION MODEL

For a pipeline installed on the seabed, an initial embedment into the soil usually occurs due to the action of its submerged weight. To balance the hydrodynamic loads in the ocean currents or waves, the pipeline may push the soil in front of it and finally the internal slippage in the soil is triggered with the development of soil plastic deformation. The pipeline thereafter loses lateral on-bottom stability. Two types of internal slippages for pipe-soil interaction mechanism will be discussed in this study, i.e. Type I: shallowly-spreading slippage, and Type II: deeply-spreading slippage (see Fig. 1).



(a)



(b)

Fig. 1 Illustration of pipeline instability on a slopping seabed:
(a) Shallowly-spreading slippage; (b) Deeply-spreading slippage

Evaluation of lateral soil resistance to pipeline

Coulomb passive earth pressure theory

Coulomb passive earth pressure theory is one of the classical earth pressure theories in the geotechnical engineering. The basic assumption is as follows: while the retaining wall extrudes the soil, the soil reaches its limit equilibrium state; the sliding surface is a plane through the bottom corner (see Fig. 2). The passive earth pressure E_1 is expressed as: (see **Craig, 1998**)

$$E_1 = \frac{1}{2}\gamma H^2 K_P \tag{1}$$

where

$$K_P = \frac{\cos^2(\varphi+\xi)}{\cos^2\xi\cdot\cos(\xi-\delta)\left[1-\sqrt{\dfrac{\sin(\delta+\varphi)\cdot\sin(\varphi+\alpha)}{\cos(\xi-\delta)\cdot\cos(\xi-\alpha)}}\right]^2} \tag{2}$$

in which K_p is the passive earth pressure coefficient, γ is the buoyant unit weight of the soil, H is the height of the wall, φ is the internal friction angle of the soil, δ is the friction angle between the wall and the soil, ξ is the angle of the wall and the vertical line, α is the slope angle to the horizontal line. Note: for the values of both ξ and α, the clockwise is negative and counterclockwise positive.

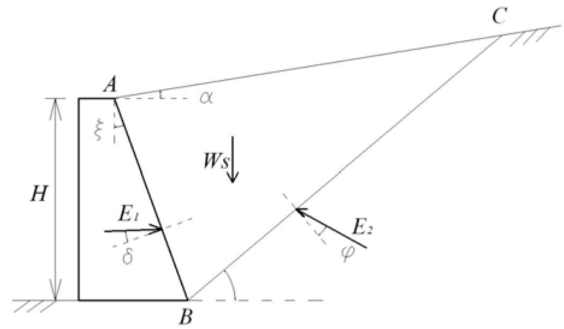
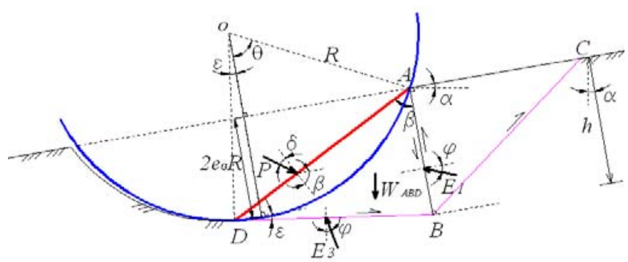


Fig. 2 Coulomb passive earth pressure at the retaining wall

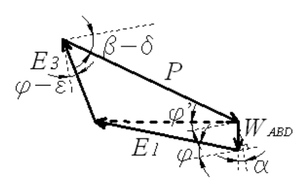
Unlike the aforementioned conventional retaining wall, the pipe-soil interface is circular. To be able to theoretically evaluate the lateral earth resistance to the partially-embedded pipeline, the circular pipe-soil interface (arc AD) is simplified as the interface plane-AD (see Fig. 3). As the actual pipe-soil interface is circular, the interfacial friction angle δ should be relative to the pipe embedment while losing lateral stability. It is empirically assumed that

$$\delta = \delta_0 e^{\tan[k_1(\theta+\varepsilon)]} \tag{3}$$

where $\delta_0(=\arctan\mu)$ is the conventional interfacial friction angle of the plane interface (μ is the coefficient of the plane wall-soil interface). k_1 is an empirical parameter relative to the embedment ratio e_G (the ratio of the embedment to the pipe diameter). With the assumption of exponent of the angle ($\theta+\varepsilon$), while the pipe embedment approaching zero (i.e. $e^{\tan[k_1(\theta+\varepsilon)]}\to 1.0$), $\delta\approx\delta_0$. The parameters θ and ε are shown in Fig.3.



(a)



(b)

Fig. 3 (a) The double-triangle slippage form; (b) Equilibrium polygon of the soil wedge-ABD

Fig. 3 gives the double-triangle slippage form and the corresponding equilibrium polygon of the soil wedge-ABD. In this figure, the interface plane-AC is the surface of the sloping seabed, and α is the slope angle. For the upslope instability, α is positive, and for the downslope instability negative. In the constructed double-triangle slippage form, the plane-AB is perpendicular to the soil surface; plane-BD is tangential with pipe surface at the pipe bottom, and plane-BC is the Coulomb failure interface. The soil wedge-ABD is regarded as rigid. When plane-AD (together with the soil wedge-ABD) protrudes from the underlying soil, the soil wedge-ABC moves upward relative to the soil wedge-ABD. Thus the soil wedge ABC is subject to a downward force at the interface AB, and the interfacial friction angle at interface AB is taken as the internal friction angle of the soil. According to (1) and (2), the passive earth pressure on interface plane-AB is

$$E_1 = \frac{1}{2}\gamma(h\cos\alpha)^2 K_{\rm p} \tag{4}$$

where

$$K_{\rm p} = \frac{\cos^2(\varphi+\alpha)}{\cos^2\alpha\cdot\cos(\alpha-\varphi)\left[1-\sqrt{\dfrac{\sin 2\varphi\cdot\sin(\varphi+\alpha)}{\cos(\alpha-\varphi)}}\right]^2} \tag{5}$$

W_{ABD} is the submerge weight of the soil in the wedge-ABD. The angle φ' in Fig. 3(b) can be calculated with

$$\tan\varphi' = \frac{E_1\sin\varphi - W_{ABD}\cos\alpha}{E_1\cos\varphi + W_{ABD}\sin\alpha} \tag{6}$$

Considering the static equilibrium of the soil wedge ABD, the passive earth pressure on plane-AD is obtained as the following form:

$$P = \frac{1}{2}\gamma h^2\cos^2\alpha K_{\rm P}\frac{\cos(\varphi-\varepsilon+\varphi')\cdot(\cos\varphi+t\sin\alpha)}{\cos\varphi'\cdot\cos(\beta-\delta+\varphi-\varepsilon)} = \frac{1}{2}\gamma h^2 K_{\rm P}K_{\rm R} \tag{7}$$

where K_R is lateral resistance coefficient, i.e.

$$K_R = \frac{\cos^2\alpha\cdot\cos(\varphi-\varepsilon+\varphi')\cdot(\cos\varphi+t\sin\alpha)}{\cos\varphi'\cdot\cos(\beta-\delta+\varphi-\varepsilon)} \tag{8}$$

in which

$$t = \frac{W_{ABD}}{E_1} = t(\theta,\ \varepsilon) \tag{9}$$

The component of P along the slope (i.e. the lateral soil resistance F_R) and the one perpendicular to the slope (i.e. the supporting force F_S) are expressed as follows respectively, i.e.

$$F_R = P\cos(\beta-\delta) \tag{10a}$$

$$F_S = P\sin(\beta-\delta) \tag{10b}$$

Shallowly-spreading slippage and deeply-spreading slippage

In the on-bottom stability design of a submarine pipeline, one of the main purposes is to obtain the submarged weight of the pipeline for given environments. In the steady current parameter with certain velocity U, the drag force F_D and the lift force F_L on the pipeline can be calculated by the Morison equation, i.e.

$$F_D = \frac{1}{2}C_D\rho D U^2 \tag{11a}$$

$$F_L = \frac{1}{2}C_L\rho D U^2 \tag{11b}$$

where ρ is the mass density of the water, D is the outer diameter of the submarine pipeline (D=2R).

For the case of ε =0, i.e. no additional settlement occurs while the pipeline losing stability (see Fig. 4), if the value of e_G is given, the parameters θ, β_0 and h_0 can be obtained:

$$\theta = \arccos(1-2e_G) \tag{12}$$

$$\beta_0 = \frac{\pi}{2} - \frac{1}{2}\arccos\left(1-2e_G\right) \tag{13}$$

$$h_0 = e_G D \tag{14}$$

The passive earth pressure on pipe Eq. (7) is simplified as

$$P_0 = \frac{1}{2}\gamma' h_0^{\,2}K_{\rm P}K_{\rm R}\Big|_{\substack{\varepsilon=0\\ h=h_0}} \tag{15}$$

Thus, for the critical value of the pipeline settlement ($2e_G R = h_0$) for the case of ε =0 (i.e. no additional settlement occurs while the pipeline losing lateral stability), the corresponding critical lateral soil resistance (F_{R0}) and the supporting force (F_{S0}) can be calculated with (10a) and (10b).

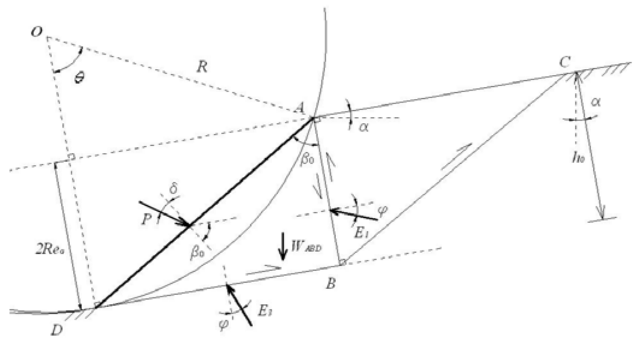


Fig. 4 The double-triangle slippage for the pipeline instability without additional settlement (ε =0)

To appropriately evaluate the critical lateral resistance, the criterions for the various soil slippage types need to be established for certain submerged weight of the pipeline on the sloping seabed. In the following analysis, when the design drag force is given, the submerged weight can be calculated for an appropriate types of soil slippage, i.e. Type I: shallowly-spreading slippage, or Type II: deeply-spreading slippage.

Type I —Shallowly-spreading slippage: $F_D \le F_{R0} + (F_{S0}+F_L)\tan\alpha$

As aforementioned, when the initial pipline settlement is given, the critical lateral soil resistance (F_{R0}) and the supporting force (F_{S0}) can be determined on the basis of Coulomb passive earth pressure theory and the plane interface assumption.

A comparison is made firstly between the values of the drag force "F_D" and "$F_{R0} + (F_{S0}+F_L)\tan\alpha$". If $F_D \le F_{R0} + (F_{S0}+F_L)\tan\alpha$, to keep the balance of the forces in the direction of seabed surface, the pipeline embedment should get shallower for a lower value of the lateral soil resistance. As shown in Fig. 5, with the decrease of the pipeline embedment, the corresponding contact angle (θ) gets a decrease of ε. The plane-BD keeps parallel to the seabed surface. As the point-D gets closer to the point-A, the lateral soil resistance is reduced. Meanwhile, β, δ and h are changed as $\beta = \beta_0 - \varepsilon/2$, $\delta = \delta_0 e^{[\tan k_1(\theta-\varepsilon)]}$, $h_1 = 2e_G R - R(1-\cos\varepsilon)$ respectively. An appropriate value of ε is then calculated to satisfy the following equation, i.e.

$$F_D = F_{RI} + (F_{SI} + F_L)\tan\alpha \tag{17}$$

where F_{RI} and F_{SI} are the values of F_R and F_S calculated with (7), (10a), (10b) for the Type-I soil slippage respectively. K_R (see Eq. (7)) is then simplified as

$$K_R = \frac{\cos^2\alpha\cdot\cos(\varphi+\varphi')\cdot(\cos\varphi + t\sin\alpha)}{\cos\varphi'\cdot\cos(\beta-\delta+\varphi)} \tag{18}$$

The submerged weight of the pipeline is obtained as

$$W_S = (F_L + F_{SI})\sec\alpha \tag{19}$$

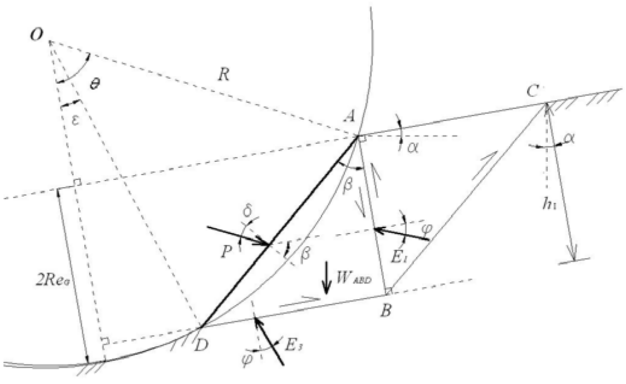


Fig. 5 The double-triangle slippage form for a shallowly-spreading slippage

Type II—Deeply-spreading slippage: $F_D > F_{R0} + (F_{S0} + F_L)\tan\alpha$

Similarly, if $F_D > F_{R0} + (F_{S0} + F_L)\tan\alpha$, to keep the balance of the forces in the direction of seabed surface, the pipeline embedment should get deeper for a larger value of the lateral soil resistance. As shown in Fig. 6, with the increase of the pipeline embedment, the corresponding contact angle (θ) gets an increase of ε . The following two cases are considered.

Case (i): $\varepsilon \leqslant \theta$: As shown in Fig. 6(a), as point-D gets farther away from point-A, the soil lateral resistance is increased. The value of P becomes larger with increasing ε . The plane-BD is tangential with pipe surface. β , δ and h are changed as $\beta = \beta_0 + \varepsilon/2$, $\delta = \delta_0 e^{[\tan k_1(\theta+\varepsilon)]}$, $h_{\mathrm{II}} = R\cos\varepsilon - (R - 2e_G R) + [R\sin\varepsilon + R\sqrt{1-(1-2e_G)^2}]\tan\varepsilon$ respectively. An appropriate value of ε is then calculated to satisfy the following equation, i.e.

$$F_D = F_{RII} + (F_{SII} + F_L)\tan\alpha \tag{20}$$

where F_{RII} and F_{SII} are the value of F_R and F_S calculated with (7), (10a), (10b) for the case (i) of Type-II soil slippage respectively. The submerged weight of the pipeline is then obtained as $W_S = (F_L + F_{SII})\sec\alpha$.

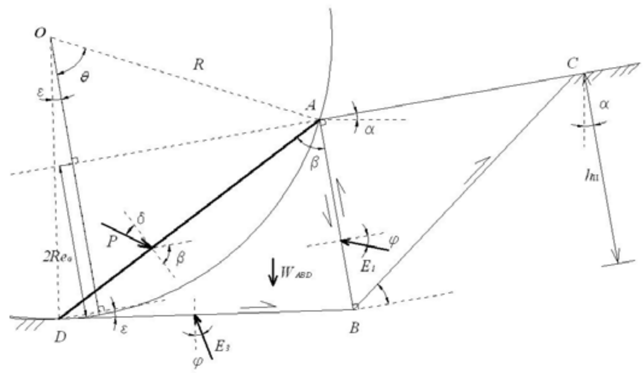
Case (ii): $\varepsilon > \theta$: The plane-BD is assumed parallel to the tangent through point-E (see Fig. 6(b)). Now, both β and δ become constants: $\beta = \beta_0 + \theta/2 = \pi/2$, $\delta = \delta_0 e^{\tan 2k_1\theta}$. h is changed as

$$h'_{\mathrm{II}} = R\cos\varepsilon - (R - 2e_G R) + [R\sin\varepsilon + R\sqrt{1-(1-2e_G)^2}]\tan\varepsilon - R[1-\cos(\varepsilon-\theta)]\csc(\frac{\pi}{2}-\varepsilon)$$

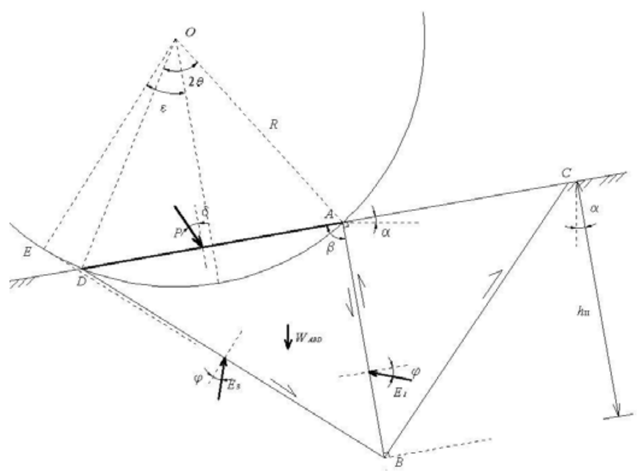
An appropriate value of ε can be calculated to satisfy the following equation:

$$F_D = F'_{RII} + (F'_{SII} + F_L)\tan\alpha \tag{21}$$

where F'_{RII} and F'_{SII} are the values of F_R and F_S calculated with (7), (10a), (10b) for the case (ii) of Type-II soil slippage respectively. The submerged weight of the pipeline is then obtained as $W_S = (F_L + F'_{SII})\sec\alpha$.



(a) Case (i): $\varepsilon \leqslant \theta$



(b) Case (ii): $\varepsilon > \theta$

Fig. 6 Two cases of double-triangle slippage form for the deeply-spreading slippage

Note that, if the value of e_G is quite small (e.g. smaller than 0.05), the soil slippage form usually is of deeply-spreading type. As the pipeline embedment is very shallow, the contribution of soil slippage to the lateral resistance is negligible. Thus the critical lateral soil resistance is recommended to be evaluated with the conventional Coulomb friction theory:

$$W_S = \frac{F_D + \mu F_L}{\sin\alpha + \mu\cos\alpha} \tag{22}$$

Comparison with the existing mechanical-actuator tests

As stated in the Introduction, the pipe-soil interaction mechanism has been recently investigated with 1g mechanical-actuator facilities for on-bottom stability of a pipeline partially embedded in the sandy soils. The facility for modeling the pipeline instability on a sloping sand-bed can be referenced in Gao et al. (2012).

Fig.7 gives the comparison between the prediction of the present pipe-soil interaction model and the results of the existing mechanical-actuator tests. The parameter k_1 in Eq.(3) is chosen as $k_1 = 5.5e_G^2 - 3.05e_G + 0.79$ according to the experimental results. For various values of test pipeline diameter and dimensionless submerged weight of the pipeline ($G = W_S / \gamma' D^2$), the prediction of the proposed model are in good agreement with the results of the mechanical-actuator test (see Fig. 7). As indicated in this figure, the predicted data lie within about 10% error range from most of the experimental results.

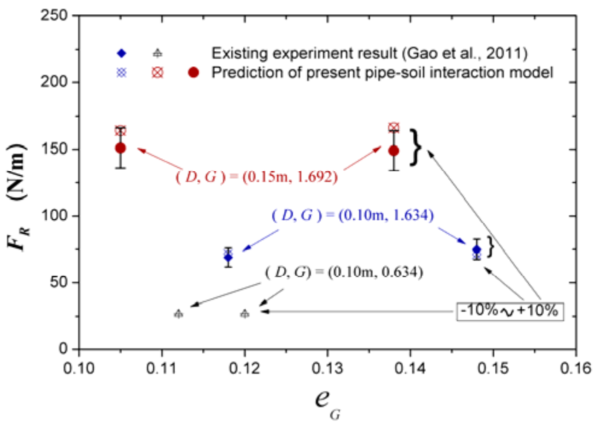


Fig. 7 Comparison between the prediction of the present pipe-soil interaction model and results of the mechanical-actuator tests on the horizontal sand-bed (μ =0.4, φ =26.7°)

EFFECT OF THE ANGLE OF SLOPING SEABED

Based on the proposed pipe-soil interaction mode, both the upslope and downslope instability of the pipeline on a sloping sandy seabed is examined. For the upslope-instability, the pipe is moving upward along the sloping seabed (α is positive); and for the downslope-instability, the pipe is moving downward (α negative).

The coefficient of critical lateral-soil-resistance (η_α) is defined as the ratio of the critical lateral-soil-resistance ($F_D - W_S \sin\alpha$) to the corresponding pipe-soil contact force ($W_S \cos\alpha - F_D \tan\theta'$) perpendicular to the surface of the sloping seabed while the pipe losing lateral stability (see Gao et al., 2012), i.e.

$$\eta_\alpha = \frac{F_D - W_S \sin\alpha}{W_S \cos\alpha - F_D \tan\theta'} \qquad (23)$$

in which $\theta' = \arctan\left(F_L / F_D\right)$.

Fig. 8 shows the variations of the coefficient of critical lateral soil resistance with the slope angle for various values of pipeline embedment. In this figure, the P_0 line is the demarcation line between shallowly-spreading slippage (Type-I) and the deeply-spreading slippage (Type-II). For the same values of the initial embedment-to-diameter ratio (e_0) under the examined soil and pipeline conditions (D=0.5 m, W_S=1.568kN/m, μ =0.3, φ =30°), the deeply-spreading slippage (Type-II) tends to occur in the downslope-instability process and the shallowly-spreading slippage (Type-I) in the upslope-instability process. For the small value of the initial embedment-to-diameter ratio (e_0=0.1), with the slope angle increasing from -30° to 30°, the value of η_α decreases slightly for the downslope instability (α negative), then increases for the upslope instability (α positive). For the larger value of e_0, the value of η_α increases with the increase of the slope angle.

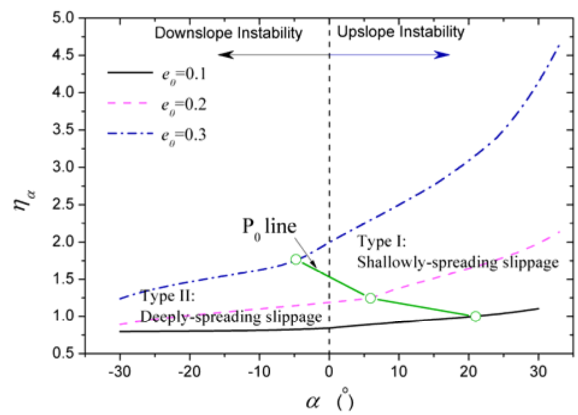


Fig. 8 The variations of the coefficient of critical lateral soil resistance with the slope angle for various values of pipeline embedment (D=0.5 m, W_S=1.568kN/m, μ =0.3, φ =30°)

CONCLUDING REMARKS

More and more deepwater oil and gas reservoirs have been or to be explored at the continental slopes. The on-bottom stability of the pipeline on a sloping seabed is one of the key issues for the offshore geotechnical design of submarine pipelines.

Most of the existing pipe-soil interaction models are empirical ones for the horizontal seabed conditions, which were mainly based on the results of pipe-soil interaction experiments.

In this paper, on the basis of the Coulomb passive earth pressure theory, a theoretical pipe-soil interaction model is proposed to predict the ultimate soil resistance for the partially-embedded pipeline on a sloping sandy seabed. In the proposed model, the shallowly-spreading slippage (Type I) and the deeply-spreading slippage (Type II) are described theoretically. The model is verified with the existing mechanical-actuator tests. A parametric study is performed, which indicates the slope angle has much effect on the pipeline on-bottom stability.

ACKNOWLEDGEMENTS

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