PAPER REF: 2618

TEMPORARILY THERMAL RELAXATION OF RESIDUAL STRESS

Chen-Wu Wu(*)
Institute of Mechanics, Chinese Academy of Sciences, Beijing, China
(*)*Email:* chenwuwu@imech.ac.cn

ABSTRACT

The temporarily thermal relaxation of residual stress is analyzed for an initially stressed solid subjected to non-uniform heating. It is revealed that the redistribution of the initial (residual) strain will be developed by the non-uniform temperature elevation, as which leads to the non-uniform reduction of the material stiffness. Therefore, the total observable displacement field is determined by the initial stresses, the thermal softening effect and the thermal expansion effect. By eliminating the contribution of the thermal expansion from the total observable displacements, the incremental displacement field resulted from the temporarily thermal relaxation of the initial stresses is obtained.

Keywords: residual stress, thermal, relaxation, temporarily

INTRODUCTION

Generally speaking, aside from some subtle methods (like Thermo-elastic Stress Analysis: TSA), the common methods for evaluating the residual stresses can be divided into two categories, i.e., mechanical and physical methods. The mechanical methods measure the released strain after relaxing the residual stress through some mechanical means (Schajer, 2010; Withers 2001a), such as the method of hole-drilling, splitting, sectioning and layer removing etc., which are generally considered to be destructive; The physical methods measure directly the lattice constant deviation due to residual stress through some diffraction test (Withers 2001a), such as electron diffraction, x-ray diffraction and neutron diffraction etc.

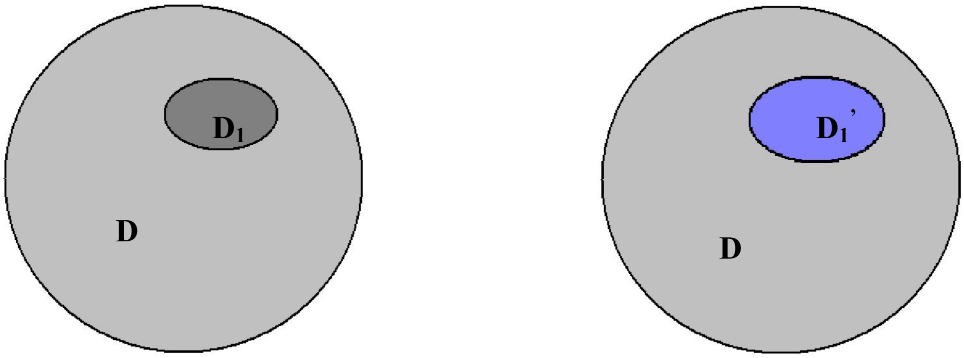
A concept of temporarily thermal relaxation is demonstrated in this article as an alternative to residual stress assessment. According to Hooke' law, the internal stress in a self-equilibrium solid is accompanied with elastic strain (Withers 2001b), of which the magnitude is correlated to the stiffness of the material. Moderate temperature elevation of the solid will lead to the reduction of its stiffness, i.e., the elastic modulus E (Fernandes, 1973; Pitarresi G, 2003). Such non-uniform thermal softening will lead to the redistribution of the initial strain field, which will contribute a displacement increment to the total observable displacement in the initially stressed solid subjected to thermal loading. This article is focused on separating the contribution of the temporarily thermal relaxation of the initial (residual) stress to the total observable displacement.

THEORETICAL DESCRIPTION

Consider a solid domain, within which some eigen strain field arises during manufacturing, thus there is a residual stress field $\boldsymbol{\sigma}_0$ accompanied with the initial eigen strain field as shown in Fig.1 (a). Of course, such residual stress/ strain should develop a displacement field of the material points, although we could not observe it just through its post-manufacturing state.

According to the theory of elastic solid, there exist relevant equations controlling the deformation and stress distribution.
First of all, the equations of equilibrium (Landau LD, 1986) for a static/ quasi-static solid can be expressed as
$\sigma_{ij,j} = 0$, $i = 1,2,3$ (1)
where $\boldsymbol{\sigma}$ is the two order stress tensor.
Secondly, the Hooke's law for the isotropic solid of temperature-dependent elastic parameters is
$\sigma_{ij} = 2\mu(\theta)\gamma_{ij} + \lambda(\theta)\gamma_{kk}\delta_{ij} - \beta(\theta)\theta\delta_{ij}$, $i, j = 1,2,3$, (2)
where $\mu(\theta)$ and $\lambda(\theta)$ are temperature-dependent Lame constants, θ represents temperature, $\boldsymbol{\gamma}$ is the two order strain tensor, $\beta(\theta) = (3\lambda(\theta) + 2\mu(\theta)) \times \int_0^\theta \alpha(T)dT$ is the temperature-dependent thermo-mechanical coefficient with $\alpha(\theta)$ being the thermal expansion coefficient and δ_{ij} represents the Kronecker' delta.
Moreover, the components of the strain tensor $\boldsymbol{\gamma}$ can be related to the displacement vector $\mathbf{u}$ with
$\gamma_{ij} = (1/2)(u_{i,j} + u_{j,i})$, $i, j = 1,2,3$. (3)
Now, consider the initial state with only the self-equilibrium residual stress being acted on the solid, of which the temperature is uniform and identical to the ambient temperature. This indicates that the Lame constants will be identical everywhere within the whole solid and no thermal expansion arises. So the Hooke's law (2) reduces to
$\sigma_{ij} = 2\mu\gamma_{ij} + \lambda\gamma_{kk}\delta_{ij}$, $i, j = 1,2,3$, (4)
Therefore, one can substitute (3) and (4) into (1) and obtain
$\mu\Delta u_i + (\lambda + \mu)e_{,i} = 0$, $i = 1,2,3$ (5)
with the operator
$\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial x_3^2$, (6)
and the volume strain
$e = u_{i,i}$, $i = 1,2,3$. (7)
For a given initially state of self-equilibrium residual stress, there is a displacement $\mathbf{u}_0$ field relative to the stress free state.
Once the solid is partially heated and a non-uniform temperature field arises within it, the temperature of every material point can be expressed as function of the space coordinates
$\theta = \theta(\mathbf{x})$. (8)
Therefore, the temperature-dependent Lame constants and thermo-mechanical coefficients of every material point will depend actually on its space position, that is
$\mu(\theta) = \mu(\theta(\mathbf{x})) = \mu(\mathbf{x})$, (9)
$\lambda(\theta) = \lambda(\theta(\mathbf{x})) = \lambda(\mathbf{x})$, (10)
and
$\beta(\theta) = \beta(\theta(\mathbf{x})) = \beta(\mathbf{x})$, (11)
Thus, one can again substitute (2) and (3) into (1) and obtain
$\mu(\mathbf{x})\Delta u_i + (\lambda(\mathbf{x}) + \mu(\mathbf{x}))e_{,i} + \mu_{,j}(u_{i,k} + u_{k,i}) + \lambda_{,i}u_{k,k} - \beta(\mathbf{x})\theta_{,i} - \beta_{,i}\theta = 0$, $i = 1,2,3$. (12)
In other words, the initial state (as sketched in Fig.1 (a)) is determined by the eigen strain and the elastic constants of that state, while the present state (as sketched in Fig.1 (b)) is determined both by the eigen strain, the present elastic constants and the non-uniform thermal

expansion. By assuming that the thermal expansion coefficient is zero all along, one can understand that the displacement field $\mathbf{u}_0$ developed by the initial residual stress will be changed by the non-uniform drifting of the Lame constants due to the non-uniform temperature elevation. Because, different stress and deformation will be developed by the same eigen strain for the two states of different elastic constants.



(a) (b)

Fig.1 Sketch of (a) the initially strained solid and (b) the redistribution of the initial displacement

That is, after the non-uniform temperature elevation is introduced into the structure by partial heating, the displacement relative to the virtual stress free state would be

$\mathbf{u} = \mathbf{u}_0 + \mathbf{u}'$. (13)

Where, the item $\mathbf{u}'$ represents the displacement field of the present state relative to the initial state of the solid under the only action of the self-equilibrium residual stresses. Obviously, it is observable in the thermal loading experiment, therefore can be measured and analyzed by many techniques, like combining the Digital Image Correlation method and thermo-elastic analysis. Furthermore, this observable displacement vector $\mathbf{u}'$ is composed of two parts. Namely,

$\mathbf{u}' = \mathbf{u}_{TE} + \mathbf{u}_0^r$. (14)

In the right side of equation (14), the first part $\mathbf{u}_{TE}$ is developed by the non-uniform thermal expansion and the second part $\mathbf{u}_0^r$ is induced by the thermal relaxation of the initial residual stress, which is determined both by the initial displacement vector $\mathbf{u}_0$ and the elastic constants, i.e. $\mu(\theta)$ and $\lambda(\theta)$, of both states of ambient temperature and elevated temperature.

NUMERICAL MODELING AND RESULTS

The titanium alloy of Ti-6Al-4V is considered in this theoretical investigation, of which the temperature dependent thermo-mechanical parameters are listed in Table 1. Besides, the density of the material is fixed to be 4470kg/m^3 throughout the analysis.

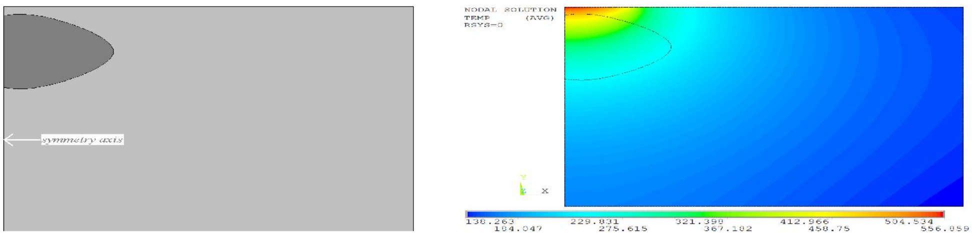
Table 1 Temperature-dependent thermo-mechanical parameters

θ/℃	α/(1e-6)	θ/℃	E/GPa	v	θ/℃	K/(w/m℃)	θ/℃	C/(J/kg℃)
24	4.90	25	110.00	0.35	23	7.30	3	562.30
539	5.69	93	105.00	0.35	94	7.67	94	558.10
861	5.74	204	99.10	0.36	139	7.99	204	584.10
		316	93.00	0.35	172	8.35	316	618.40
		427	86.90	0.37	227	9.10	539	726.00
		538	70.60	0.38	538	13.53	762	868.30
							871	958.80

In table1, the symbols θ represents temperature, α is the thermal expansion coefficient, $E=\mu(3\lambda+2\mu)/(\lambda+\mu)$ is the elastic modulus, $\nu=\lambda/2(\lambda+\mu)$ is the Poisson's ratio, K is the thermal conductivity and C is the specific heat capacity.

The axis-symmetry solid used for analysis is shown in Fig. 2 (a), for which the radius is 30mm. Within the axis-symmetry solid an ellipsoidal homogeneous inclusion of dimension 4mm×4mm×8mm is introduced to make the initial stress field. In detail, the inclusion overlaps the matrix at their interface and the overlapping width equals 0.002 times the local radius of the inclusion everywhere. Then, the action of the eigen strain can be realized by utilizing the contact arithmetic for the interactions between the grid nodes at the outer boundary of the inclusion and that of the corresponding grid nodes at the inner boundary of the matrix.

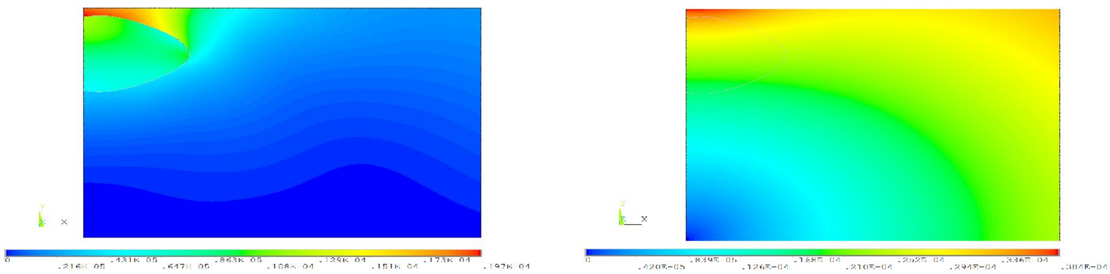
A laser beam heating is simulated in the computation with the beam radius being 5e-3 and average heat flux being 0.9MW/m^2 on the beam covered surface. After being heated for 1second, the temperature profile developed in the specimen by such non-uniform heating is shown in Fig. 2(b). It is indicated that the maximum temperature is about 550℃ when being heated for 1 second and arises at the center of the heated region.



(a) (b)

Fig.2 (a) The axis-symmetry model and (b) temperature profile produced by local heating

The resultant displacement field resulted only from the initial residual stress, corresponding to the initial eigen strain, is shown in Fig. 3 (a). Of course, such resultant displacement field can only be presented as scalar one due to the fact that the real direction of the resultant displacement would be different everywhere. One can see that the great magnitudes of the resultant displacement arise around the surface layer region above the inclusion. The resultant displacement field developed only by the thermal expansion in the same solid is shown in Fig. 3 (b), for which the eigen strain has been artificially deleted in the finite element model. Again, the maximum resultant displacement owe to the thermal expansion also appears at the centre of the heated region.

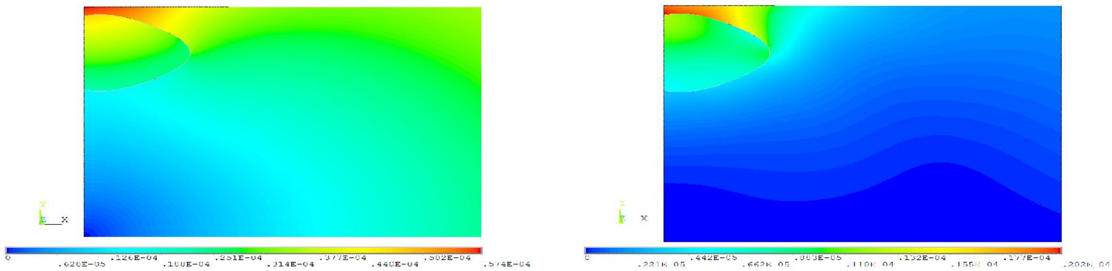


(a) (b)

Fig.3 Displacement developed by (a) only residual stress or (b) only thermal expansion

The synthesized displacement field induced by the actions of initial residual stress, i.e. the initial eigen strain, the effects of thermal expansion and thermal softening is shown in Fig. 4 (a). Now, if the material is formed with some inclusion as being assumed above, the displacement induced by the initial eigen strain could not be directly observed during the present thermal experimental process. However, the initial displacement field accompanied with the initial eigen strain should be disturbed and redistributed. In other words, an incremental displacement field would be induced by the non-uniform thermal softening of the material, which is intrinsically determined by the initial stress and the temperature dependent elastic constants of the material. Therefore, if the thermal expansion effect is eliminated with setting the thermal expansion coefficient as 0, the relaxation of the residual stress due to the thermal softening of the material can be computed.

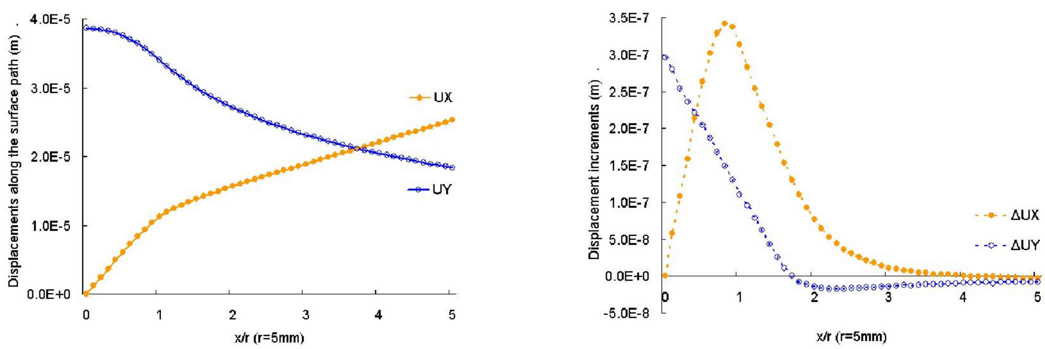
For further demonstration, Fig. 4 (b) shows the displacement field developed by the initial eigen strain and thermal reduced elastic constants with the thermal expansion effect being eliminated. That is to say, the displacement field displayed in Fig. 4 (b) is determined by the initial eigen and the present elastic constants of the temperature elevated state. Thus, one can obtain the incremental displacement field by make vector subtracting between the displacement shown in Fig.4 (b) and that shown in Fig. 3 (a). Considering the fact that only the surface displacement can be measured experiment, the displacement and radial strain mapped onto the surface path along the radial direction are drawn herein.



(a) (b)

Fig.4 Displacement (a) developed both by residual stress and thermal loading and (b) for the imaginary case with residual stress and thermal softening effect but without thermal expansion

The total observable displacements, developed during the thermal loading, mapped to the radial surface path originated at the symmetry axis are shown in Fig. 5 (a). One can see that the radial displacement increase along the radial direction due to the cumulating effect while the maximum of the normal displacement appears at the center of the heated region. The displacement increments resulted from the partially relaxation of the initial residual stress are shown in Fig. 5 (b). It is indicated that the magnitudes of the displacement increments are on the order of sub-micrometer and the great radial and normal displacement increments arise only around the heated region.

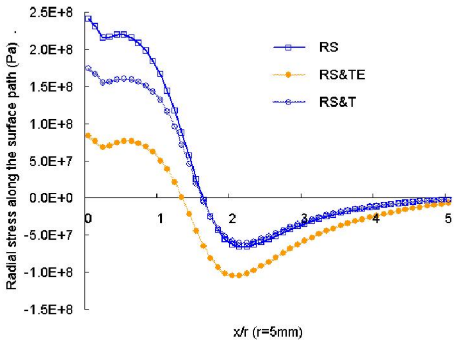


(a) (b)

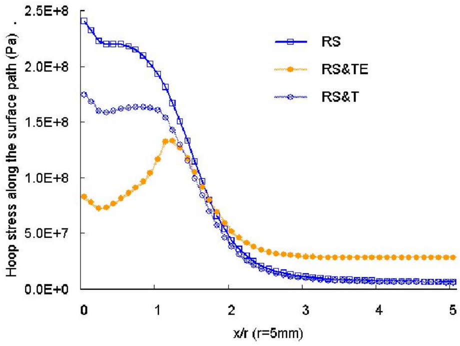
Fig.5 (a) Observable displacements upon the thermal loading and (b) displacement increments induced by the partially thermal relaxation of initial stress along the surface radial path with the symbol *r* being the circular-heated-region radius (U: displacement, X tangent direction, Y normal direction)

Finally, the thermal relaxation patterns of the residual stresses are shown in Figs. 6 (a) and (b), of which Fig. 6 (a) presents the radial stress mapped onto the radial surface path and Fig. 6 (b) corresponding to the hoop stress. In the results graphed in Figs. 6 (a) and (b), one can see that the initial (residual) stress will be released partially due to the elastic modulus reduction under elevated temperature. To be noted that the stress would be relaxed more obvious if the thermal expansion effect is taken into account, because the initial stress is tension thereby the thermal expansion would release it further. Of course, if the initial stress is compressive, the opposite effect of thermal expansion should be observed.

In sum, the initial residual stress would be relaxed partially once the elastic modulus is reduced due to thermal softening of the material. If no more plastic deformation arises during the heating experiment, such relaxation of residual stress would be restored after the thermal loading is removed and the material temperature decreases to identical to the ambient temperature. Such kind of temporarily thermal relaxation may be used for residual stress assessment once the relationship between the displacement increment and the initial stress is established, which should be conducted according to the characteristics of the object under consideration.



(a)



(b)

Fig.6 Thermal relaxation of the (a) radial stress and (b) hoop stress with the symbols 'RS' represents only the action of residual stress is included; 'RS&TE' corresponding to the case that the actions of residual stress, thermal expansion and thermal softening are all included; 'RS&T' means that only the residual stress and thermal softening are included, i.e. the thermal expansion effect is eliminated

CONCLUSION

The non-uniform thermal softening of the material would lead to partial relaxation of the pre-existed stress in an initially stressed solid subject to non-uniform heating. The total observable displacements arise during the thermal loading experiment is determined by the initial (residual) stress, the thermal softening effect and the thermal expansion effect. The incremental displacements rely on the partially released residual stress and the temperature-dependent elastic constants. Therefore, such concept of temporarily thermal relaxation may be an alternative for residual stress assessment once the relationship between the displacement increment and the initial stress is established, which should be carried through according to the characteristics of the object under consideration.

ACKNOWLEDGMENTS

The author gratefully acknowledges the funding by National Natural Science Foundation of China, under grant 11002145.

REFERENCES

Fernandes R, Stouffer DC. A general theory for elastic solids with temperature dependent mechanical properties. Nuclear engineering and design, 1973, 25, p. 301-308.

Landau LD, Lifshitz EM. Theory of elasticity (Third edition). Reed Educational and Professional Publishing Ltd., 1986, p. 16.

Pitarresi G, Patterson EA. A review of the general theory of thermoelastic stress analysis. The Journal of Strain Analysis for Engineering Design, 2003, 38, p. 405-417.

Schajer G S. Relaxation Methods for Measuring Residual Stresses: Techniques and Opportunities. Experimental Mechanics, 2010, 50, p. 1117-1127.

Withers PJ, Bhadeshia HKDH. Overview Residual stress Part 1– Measurement techniques. Materials Science and Technology, 2001 a, 17, p. 355-365.

Withers PJ, Bhadeshia HKDH. Overview Residual stress Part 2– Nature and origins. Materials Science and Technology, 2001 b, 17, p. 366-375.