ACTA MECHANICA SINICA, Vol. 4. No. 4, November, 1988 ISSN 0567—7718
Science Press, Beijing, China
Allerton Press, INC., New York, U. S. A.

A NUMERICAL EXPLORATION OF SECONDARY SEPARATION IN STARTING FLOW AROUND A FLAT PLATE BY THE DISCRETE VORTEX MODEL

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ABSTRACT: In the present work, a further numerical simulation of the starting flow around a flat plate normal to the direction of motion in a uniform fluid has been made by means of the discrete vortex method. The secondary separation occurring at rear surface of the plate is explored, and predicted approximately using Thwait's method. The calculated results show that in the early stages of the flow secondary separation does occur. The evolution of flow field, the vortex growing process and the characteristics of secondary vortices have been described. The time dependent drag coefficients, the vorticity shed from the edges and rear surface, and the separation positions are calculated as well as the distributions of velocity and pressure on the plate. In the case of flow normal to the plate, the calculated secondary vortices are weak. Their existence will change the local velocity distributions and affect pressure distributions. However, the effect on drag coefficient is negligible.

KEY WORDS: starting flow, flat plate, secondary separation, discrete vortex model, numerical exploration.

I. INTRODUCTION

In the past several years, a number of research works on unsteady separated flow past a flat plate, accompanied by vortex shedding have been carried out by using the discrete vortex model, e.g. [2, 4, 5, 6, 7, 12]. In those works, much progress in calculation of vortex shedding and in description of the kinematical characteristics of vortex street has been made. Some thorough study on determining the nascent vortices' strength and positions and their initial convection in the flow field can also be found in those works. However, the calculated normal force coefficicnts are still 20—25% larger than those given by experiments. The information published on flow field, such as temporal pressure and velocity distributions along the plate seems to be incomplete, though some instructive results have been obtained[2, 4]. Gathering some results from previous works (e.g. [2], [4]) it is seen that in the early stages of the starting flow around a flat plate the recirculating flow along the rear surface will undergo an adverse pressure gradient and becomes retarded. It would be interesting to know what is the real behaviour of the recirculating flow for this situation. Some results on starting flow past a circular cylinder have shown that the secondary vortex generated by rear shear layer separatopn has an effect both on primary vortex's motion and forces prediction[1,9,10,14]. It is desirable to investigate whether the secondary separation would occur in

Received 9 Oct. 1987.

the near wake of a flat plate, and if so, what is the effect on the flow field and forces prediction. The limitation of discrete representation of vorticity prevents us from predicting accurately the flow field near the point vortices, however, as a first approximation to study the whole processes of vortex evolution and the kinematical characteristics of the flow, the model would still be valid. Moreover, the discrete vortex model will give a good approximation to the velocity field far from each element of the vortex sheet. In view of the above consideration, the present work attempts to make a further simulation of the actual flow in the early stages by the discrete vortex model, accounting for the essential features of a very complicated process, including the separating at plate edges and rear shear layers. Suppose the flow considered is laminar and Reynolds number is in the region of 10^4—10^5 and the coming flow is normal to the flat plate. The primary nascent vortices are set at a fixed distance from edges. The secondary separation is predicted approximately by Thwait's method and the flow field and forces are calculated.

II. METHOD

Using a conformal mapping to transfrom the flat plate with length $4R$ in physical (z) plane into a circle with radius R in transformed (ζ) plane. The dimensionless transformation used and the complex potential in ζ plane can be written as

$$z = i\left(\zeta - \frac{1}{\zeta}\right)$$

$$W(\zeta) = \zeta + \frac{1}{\zeta} - \sum_{n=1}^{N} i\Gamma_n \ln\left[(\zeta - \zeta_n)\left(\zeta - \frac{1}{\bar{\zeta}_n}\right)^{-1}\right] \tag{1}$$

where $W = W'/U_\infty R$, $\zeta = \zeta'/R$, $z = z'/R$, $\Gamma_n = \Gamma'_n/2\pi U_\infty R$, Γ'_n is the strength of the nth point vortex, the prime represents dimensional quantity. The velocity in z plane can be easily obtained from $\frac{dW}{dz} = \frac{dW}{d\zeta}\frac{d\zeta}{dz}$. The singularity of velocity at the two edges is removed by putting nascent point vortices near the separation points to satisfy Kutta condition at the edges in every time step. In this way, which is referred to as the MFP method[6], setting a proper initial distance of nascent vortex from edge from preliminary calculations, the strength of the vortices can be determined. In symmetric flow situation, we have [15]

$$|\Gamma| = |v_\theta|\frac{2\delta + \delta^2}{4(1+\delta)} \tag{2}$$

here δ is the nascent vortex distance downstream of and along the edges of the plate, v_θ represents the velocity at $\zeta = \exp\left(i\frac{\pi}{2}\right)$ in absence of the nascent vortices.

Noting that the velocities of any point vortex in z plane and in ζ plane are not equal, the velocity transformation for each point vortex can be shown as[15]

$$U_{n_z} - iV_{n_z} = \left(U_{n_\zeta} - iV_{n_\zeta}\right)\frac{\zeta_n^2 e^{-\frac{\pi}{2}i}}{\zeta_n^2 - e^{-\pi i}} + i\Gamma_n\frac{\zeta_n e^{-\frac{3}{2}\pi i}}{\left(\zeta_n^2 - e^{-\pi i}\right)^2} \tag{3}$$

In the present work a viscous vortex core is used for each vortex to remove the singularity of induced velocity produced by potential solution. The detial has been given in Ref. [10].

From unsteady Bernoulli's equation the pressure coefficient on the plate is determined, but the term $\frac{\partial \varphi}{\partial t}$ is calculated by two approaches in the present work for comparison. The first approach is taking real part of $W(\zeta)$, we have $\frac{\partial \varphi}{\partial t} = -\sum_{n=1}^{N} \Gamma_n \frac{\partial}{\partial t}\{\mathrm{tg}^{-1}[(r_n \sin\theta - \sin\theta_n)(r_n \cos\theta - \cos\theta_n)^{-1}] - \mathrm{tg}^{-1}[(\sin\theta - r_n \sin\theta_n)(\cos\theta - r_n\cos\theta_n)^{-1}]\}$[15]. The second is from the defination of the velocity potential, i.e. $\varphi = \int \boldsymbol{v}\cdot d\boldsymbol{s}$. Thus, in irrotational flow region $\varphi(x, t) = \varphi_{\text{start}} + \int_{x_{\text{start}}}^{x} U_z(x, t)\,dx$. Across separation region, the velocity potential has a jump, which should be equal to the sum of circulation of all point vortices shed from the edge from starting of the flow to the moment t. Then the φ along the plate can be calculated from one point to another. The forces acting the plate are calculated by the generized Blasius theorem, thus we have[15]

$$C_D = 4\pi \sum_{n=1}^{N} \Gamma_n U_{n_z} + 8\pi \sum_{n=1}^{N} \Gamma_n \frac{\partial}{\partial t}\left(\frac{\sin\theta_n}{r_n}\right) \tag{4}$$

The force along the plate is equal to zero. The drag force is also calculated by the integration of the pressure coefficient along the plate for comparison.

The rear shear layer separation on the rear surface of the plate is explored by the boundary layer theory. For doing this some approximate assumptions should be made since up to now we have very limited knowledge about the nature of the rear shear layer in the starting flow. According to our preliminary calculation of the evolution of velocity and pressure distributions on the plate, it is postulated that after the two recirculating flow regions extend to the rear stagnation point, two rather steady symmetric rear shear layers are formed; at high Reynolds number, in the early stages of the flow, the nature of the rear shear layer is similar to that of laminar boundary layer, and tends to guasi-steady state with time development. Under these assumptions, the Thwaits method is used to prediet the separations of the rear shear layer. At the positions of separation, the following relations would be satisfied[13].

$$\begin{aligned} \lambda &= \theta_*^2 \frac{dU_z}{dx} \Big/ \nu = -0.09 \\ \theta_*^2 &= 0.45\nu / U_z^6 \int_0^x U_z^5\, dx \end{aligned} \tag{5}$$

The integration is starting from rear stagnation point $x = 0$. Using velocity transformation, the numerical solutions of the above solutions can be solved in ζ plane.

In the present work we start to calculate the separation for $t \geqslant 1.0$(here $t = t' U_\infty / R$, t'—— real time of the flow). The domain of the roots of the Equations (5) is $-1.93 \leqslant \times \leqslant -1.15$. Also, we calculate the second derivative of velocity $\frac{d^2(U_z)}{dx^2}$ at the position of maximum value of recirculating flow to decide qualitatively whether the separation occurs. On upper half of the plate, if $U_z'' < 0$, in general, the separation will occur[13].

The non-dimensional strength of secondary vortex s is calculated by $\frac{\partial \Gamma_s}{\partial t} = \frac{1}{2\pi}\int_0^{\delta'} U_z \frac{\partial U_z}{\partial y} dy$

$= \frac{1}{4\pi} V_s^2$. Here the V_s represents the potential velocity at the separation in z plane. The normal distance form the plate of nascent secondary vortex can be determined by no-slip condition at the separation at the instance of introducing two symmetric new vortices, or from $\left(\frac{dW}{d\zeta}\right)_{\zeta_s} = 0$ to determine the radial distance of nascent secondary vortex in ζ plane, here ζ_s is a corresponding point of separation in ζ plane. The normal distances determined by no-slip condition are very small when time step $\Delta t = 0.1$ is used. The calculated secondary vortices line up in the vicinity of rear surface. However, when a smaller time step, say, $\Delta t = 0.05$ is used, the normal distances are increasing. The secondary vortex cluster is formed obviously. Later we found that if we put the nascent vortices at the outside of rear shear layer, the secondary vortex cluster formed is similar to those resulting from small time step. Moreover, the computer time can be saved. The magnitude of thickness of rear shear layer at the separation can be roughly evaluated by $\delta'(x_s) \sim 1/\sqrt{Re_{x_s}}$, in which the magnitude of characteristic velocity is nearly half of coming flow. However, it is difficult to make an accurate calculation of the thickness owing to the limitation of the knowledge on the rear shear layer. In view of this, we prefer to consider the nascent normal distance as a parameter, and determine the value by some preliminary calculations. In the present work, the distance $\delta_2 = 0.06078, 0.03$ are used. The vortex cluster pattern calculated using $\delta_2 = 0.03$ is very similar to that corresponding to no-slip condition and $\Delta t = 0.05$. Therefore the value $\delta_2 = 0.03$ is used for primary calculations.

The other parameters used in the calculations are $\delta = 0.1, 0.15, 0.02$; $\Delta t = 0.05, 0.1, 0.2$; $\delta_2 = 0.03, 0.06708$. In most of the calculations, $\delta = 0.02$, $\Delta t = 0.1$, $\delta_2 = 0.03$ are employed. The difference of the results corresponding to different parameters are discussed in the paper. Each vortex outside the other vortices' cores is convected with velocity of local flow field, its advancement is calculated in z plane by Eulerian simple scheme, i.e. $X_n = X_{on} + \frac{dX}{dt}\Delta t$. The velocity distributions are calculated by real number formula and complex number formula. The velocity at the positions very close to the edges is predicted approximately by linear extrapolation method.

III. RESULTS AND DISCUSSION

The computer program provides, at any specified time, the positions of all vortices, the rate of vorticity shed into shear layers from the two edges and secondary separation points, the drag coefficients, the distributions of velocity and pressure along the plate as well as the secondary separation positions.

The vortex clusters and secondary vortices are shown in Fig. 1. The detail of the secondary vortex evolution is shown in Fig. 2. With the evolution of time, the secondary vortices have a tendency to roll up, but eventually it is torn by the influence of primary vortices. Most of the secondary vortices are merged into the primary vortex sheets and then go downstream. The calculated results using $\delta_2 = 0.03$ are similar to those using no-slip condition with $\Delta t = 0.05$. The comparisons of those results with and without secondary vortices show that the effect of secondary vortices on the motion of primary vortices is not pronounced.

The vorticity shed from edges and the secondary separation is shown in Fig. 3. The calculated results of primary vorticity are in agreement with Fink and Soh's calculations. The magnitude of secondary vorticity shed from the separation of rear shear layer on each side of the plate is $O(10^{-2})$.

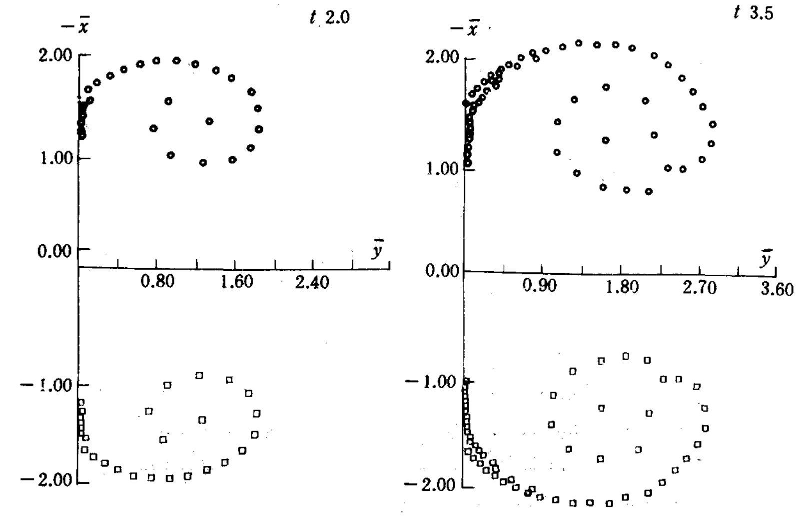

Fig. 1

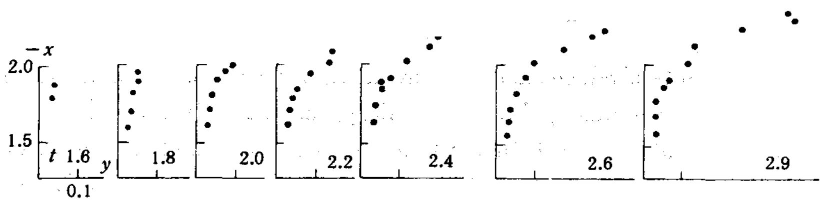

Fig. 2 The evolution of secondary vortex ($\delta_2 = 0.03$, $\Delta t = 0.1$)

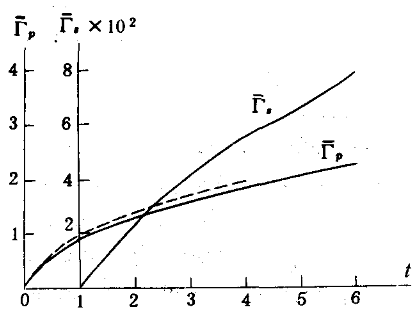

Fig. 3 The initial growth of total vorticity

--- Fink and Soh's result

Therefore it will hardly affect the prediction of the flow characteristics.

Fig. 4 gives the time dependent drag coefficients. The results from different methods are not the same. However, these differences rapidly decrease. At $t = 6$ the differences between four calculated results are within 14% and the value approaches to 2.0 which is lower than some of the previous results calculated using discrete vortex method but close to real flow situation. The calculated results are sensitive to time increment used. The smaller Δt is used the higher value of C_D is obtained in our preliminary calculation for $t \leqslant 2.0$ (with $\Delta t = 0.1, 0.15, 0.20$), but the parameter δ has no significant effect on the C_D value when $\delta = 0.01, 0.015, 0.02$ are used. The secondary vortex will decrease the value of drag coefficient, but the effect is negligible.

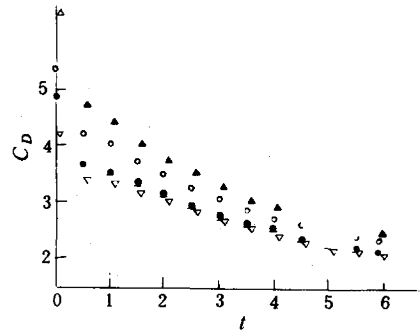

Fig. 4 The varation of drag coefficients

• • • formula (4), with forward time differential scheme

▽ ▽ ▽ formula (4), with backward time differential scheme

▲ ▲ ▲ $\int Cpdx$, $\partial\varphi/\partial t$ comes from second approach

○ ○ ○ $\int Cpdx$, $\partial\varphi/\partial t$ comes from first approach

—— calculated with secondary separation

The calculated results of velocity and pressure distribution without secondary separation show that near $t = 0.5$ a small recirculating flow region will appear. At $t = 0.8$, the region has reached the rear stagnation point. After $t = 1.0$, the recirculating flow does undergo an adverse pressure gradient. At $t = 2.0$, the maximum value of the recirculating flow velocity is near 70% of the coming flow. Later on, at $t = 5,6$, the distributions tend to a steady state and the maximum value is near 0.5 U_∞, the position is near $|X| = 1.5$. After passing the maximum value, the flow along the plate becomes retarded approaching the edges. The pressure distributions at $t = 5, 6$ become rather flat and also tend to a steady state, the values of pressure coefficient at the front and rear stagnation point are nearly 1.18 and 1.36 respectively. In the early stage of the starting flow the pressure coefficients coming from second approach (φ defination) are larger than those obtained from the first approach (mentioned in paragraph II), while on wetted surface, both results are almost the same. When $t \sim 5$, 6, however, results from diffenent methods are fairly in agreement with each other. Fig. (5) and Fig. (6) show the evolution of velocity and pressure distributions on the plate when the secondary separation is considered. Some comparisons of results calculated with and without the secondary separation are also shown in the figures in dot symbols and in dash line respectively. The secondary vortices make local velocity and pressure distributions change, decreasing the velocity of recirculating flow.

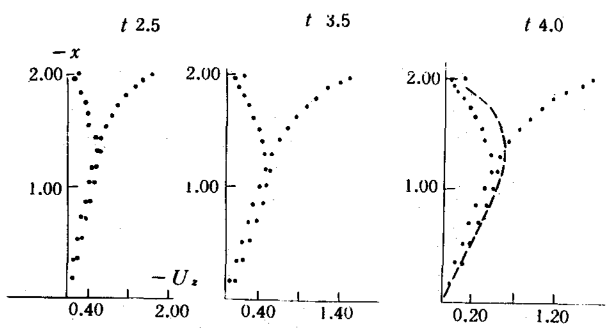

Fig. 5

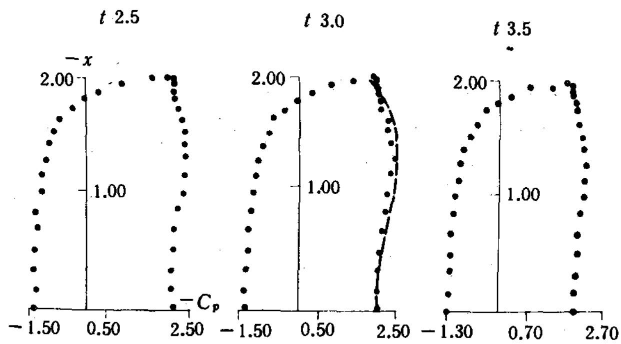

Fig. 6

The time dependent secondary separations are shown in fig. 7, the positions of maximum velocity value are also given in the diagram. With the evolution of time, the separations seem to be fluctuating. During $t = 1.5$—2.5 the positions are near $x = \pm 1.46$, while $t = 2.5$—3.5, they move upstream and then fluctuate near $x = \pm 1.22$. In that case, the secondary vortices are very weak. The secondary separation is located downstream of and near the positions with maximum velocity.

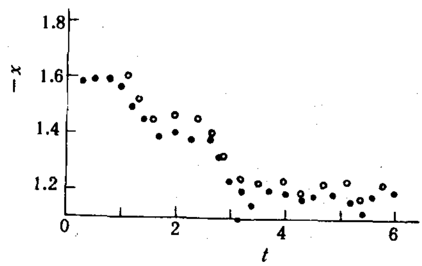

Fig. 7 The variation of secondary separation points

.... positions with maximum velocity values of recirculating flow

IV. CONCLUSION

In the present work the evolution of flow field and the vortices growing process in the early stages of a starting flow, in which a flat plate is normal to the oncoming flow, have been described in detail including the secondary separation at the rear surface of the plate. The separation positions, the characteristics of the secondary vortices and their effects have been calculated. The calculated primary vorticity shed from edges agrees with previous investigators' results, but with a little lower value, the drag coefficients are close to the experimental results and considerably lower than some of the previous results given by discrete vortex method. The results of velocity and pressure distributions without secondary separation are similar to and close to previous results. In the case of flow normal to the plate, the secondary separation does occur in the early stages. The strengths of secondary vortices are weak, in the order of 10^{-2}. Most of secondary vortices are merged into primary vortex sheets. The secondary vortices will change local distributions of velocity and affect the pressure distributions locally, but the effect on drag coefficient is negligible. The accuracy of all the present results may be evaluated partly based on the fact that the calculated results such as drag, velocity, pressure coefficients, are in fair agreement with each other when different methods are used, especially in a rather long period of time. However, the comparisons wth experiments and accurate numerical solutions are highly desirable.

Acknowledgements One of the authors (G. C. Ling) would like to express his gratitude to the Chinese Government and the Royal Society for supporting his study at Imperial College. He would also like to thank Professor P. Bradshaw and many members of the Aeronautical Department for their help in every respect.

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