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A Simplified Model of Mat-shape Algae Horizontal Drift

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Abstract: *Wind can lead to large-scale heterogeneity in the horizontal distribution of phytoplankton in lakes. By analyzing the stress mechanism of the mat-shape algae on the surface of water, we treat the algae layer as a kind of liquid and establish a two-layer fluid model. A basic relationship between wind speed and algae moving velocity is deduced theoretically, which indicates that the drift velocity of mat-shape algae changes with wind speed in a quadratic function. Finally, the theoretical result is examined by the observation data and fits well.*

Key Words: mat-shape algae, drift velocity, wind speed, two-layer fluid model.

1. Introduction

Harmful algal blooms (HABs) refer to the event of unusual increase in biomass and phytoplanktons' rapid accumulation on the surface of water (Whiton and Pottseds, 1996; Kong and Song, 2011), which results from eutrophication in water bodies. HABs are formed when chlorophyll concentration in water is over 10mg/m^3 or the number of cyanobacteria exceeds 1500 cells/ml (Kong and Song, 2011).

Algal blooms often show great heterogeneity of distribution in space. In large scale water bodies, the horizontal heterogeneity is always related with environmental factors such as water temperature, irradiance, hydraulic conditions and nutrients (George, 1993; Marcé et al., 2007). While in small to medium scale water bodies where the water temperature and nutrient distribute relatively even, wind-driven currents, vertical mixing rates and migration speed of phytoplankton are vital for the formation of longitudinal patchiness (Javier et al., 2010). Specifically, wind plays an important role in the process. Not only can wind change the horizontal distribution of biomass either by wind-driven currents or its direct driving force, but also it can strengthen the water mixing process through inducing wave and hence influence the distribution of biomass in the vertical direction (Webster, 1990).

A lot of observations have been done about wind and its influence on the horizontal distribution of phytoplankton. Yang (1996) and Chen et al. (2003) observed that the algae distribution is greatly influenced by the prevailing wind in Taihu Lake, China. George and Edwards (1976) found that cyanobacterials commonly gather at the downwind side of a eutrophic reservoir and the degree of horizontal heterogeneity of chlorophyll *a* (*chla*) distribution depends greatly on wind speed. George and Heaney (1978) outlined that the horizontal distribution of phytoplankton density is closely related with currents, which is especially true for those with a certain vertical density profile. In order to further illustrate the problem, some models have been established. Webster (1990) discussed the effect of wind on the distribution of phytoplankton by developing a 2D advection-diffusion model, which indicated that buoyant phytoplankton would increase exponentially towards the downwind end of the lake whereas the sinking ones transported by a return flow at lower water would decrease. This result was in accord with the observations done by George and Edwards (1976). Verhagen (1994) further extended the model by adding a dispersion coefficient in the wind direction and found that the horizontal and vertical patchiness increases with the floatation velocity of phytoplankton and decreases with wind speed and the strength rate of horizontal currents to the vertical ones. Zhu and Cai (1997) built a 3D numerical model which takes into account the effects of wave and cells' buoyancy. They concluded that algae would aggregate at the downwind side when the wind speed is below the critical one, or they would disperse in the mixing water. All the models mentioned above assumed that the algae moves at the same speed with currents, which, however, is not the case in reality.

As Bai (2005) pointed out, plankton cells move at the same speed of water only when they are disperse as granules in the water, otherwise they would drift faster than water under the direct driving force of wind when accumulating like a mat on water surface. They also proposed an exponential formula between wind speed and drift velocity of mat-shape algae through experimental and in-situ observation. In order to be more accurate in simulation, Tu et al. (1990) added a drift coefficient in their model which accounted for the influence of wind to phytoplankton.

Here, by analyzing the stress mechanism of the mat-shape algae on the surface of water, we treat the algae as a kind of liquid and establish a two-layer fluid model, from which we deduce the relationship between wind speed and algae drift velocity theoretically. And theoretical results are examined by the observation data of Bai's (2005).

2. Model

Under proper weather conditions, phytoplankton cells come up from the water, gather and become a kind of viscous material covering on the water surface. Then the mat-shape algae, with driving stress from wind at the upper boundary and viscous stress from water at the lower boundary, has the same force mechanism as water surface does in a normal water body driven by wind. And the cells are small enough compared with the length scale of algae layer (e.g. the diameter for colonial microcystis is about 40~100μm). Thus, it is reasonable for the algae layer to be treated as a kind of liquid.

For mat-shape algae, the horizontal length scale is much larger than the vertical one (e.g. the ratio of horizontal length scale to vertical is smaller than 0.001 in Chaohu Lake of China), therefore the influences from horizontal boundaries can be neglected and the model can be simplified into a two-layer fluid model infinite in the horizontal.

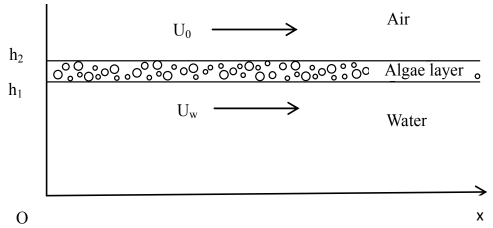

Figure1. Schematic of the Two-layer Fluid Model

We consider a two-dimensional case, in which the horizontal direction is denoted as *x* coordinate and the vertical direction is *z*. The lake bottom is $z=0$, the water surface $z=h_1$, and the surface of algae layer $z=h_2$. The wind blows at a stable speed of U_0 and the water has a basic velocity of U_w caused by other factors (e.g. gravity), both of which are parallel to the *x* coordinate, as shown in Fig.1.

In this essay, only steady state solution is considered, thus the momentum equation can be written as follows:

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_{al}}\frac{\partial p}{\partial x}+\nu_{al}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right) \tag{1}$$

In which *u* and *w* are the algae layer velocity components in the x and z directions, respectively; *p* the stress imposed in *x* direction; ρ_{al} and ν_{al} the density and the kinematic viscosity of algae layer, respectively.

And the equation of continuity in a two-dimensional case should be

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{2}$$

As mentioned above, the model is simplified into a parallel shear flow model, hence the vertical velocity should be zero, that is $w=0$. With equation (2), we get $\partial u/\partial x=0$, which can be further deduced into $\partial^2 u/\partial x^2=0$. As the motion is driven by shear stress from boundaries and no pressure gradient is imposed in the *x* direction, the term $\partial p/\partial x$ should be zero. Thus equation (1) can be simplified as:

$$\frac{\partial^2 u}{\partial z^2}=0 \tag{3}$$

Boundary conditions are expressed in terms of stress. Wind stress on the upper boundary is estimated by the quadratic drag law. With the assumption that the algae layer satisfies Newton's law of viscosity, wind stress imposed on the upper boundary can be written as:

$$\tau_a=\rho_a C_a U_0{}^2=\eta_{al}\left.\frac{\mathrm{d}u}{\mathrm{d}z}\right|_{h=h_2} \qquad (3)$$

Similarly, the lower boundary condition for algae layer, which accounts for the viscous stress between algae layer and water surface, can be expressed as follows:

$$\tau_{a1}=\rho_{a1}C_{a1}\left[u\left(h_1\right)\text{-}U_{\mathrm{w}}\right]^2=\eta_{al}\left.\frac{\mathrm{d}u}{\mathrm{d}z}\right|_{h=h_1} \qquad (4)$$

In which ρ_a ρ_{a1} are the densities of air and algae layer, respectively, C_a , the drag coefficients of wind to algae and algae to water, respectively, and η_{al} the dynamic viscosity of algae layer.

Using boundary conditions of equations (4) and (5), the solution to equation (3) is

$$u\left(z\right)=\frac{\rho_a C_a\left(z\text{-}h_1\right)}{\eta_{al}}U_0{}^2\pm\sqrt{\frac{\rho_a C_a}{\rho_{a1}C_{a1}}}U_0+U_{\mathrm{w}} \qquad (5)$$

And the mean velocity of algae layer is represented by the velocity at 1/2 thickness, which is

$$\begin{aligned}u_{al}&=u\left(\frac{h_1+h_2}{2}\right)\\&=\frac{\rho_a C_a\left(h_2-h_1\right)}{2\eta_{al}}U_0{}^2\pm\sqrt{\frac{\rho_a C_a}{\rho_{a1}C_{a1}}}U_0+U_w\end{aligned} \qquad (6)$$

3. Discussion

Equation (7) shows that the algae velocity changes with wind speed in a quadratic function. And the velocity of algae layer becomes equal to the basic velocity U_w if wind speed is 0. This is because the algae layer is driven by currents below when there is no wind, and the velocity of algae would reach the velocity of currents after steady state condition is attained.

As Bai (2005) pointed out, the velocity of algae can be influenced by some physical characters of algae layer, such as density and thickness. However, due to the difficulties in measurement, little record about physical characters of algae layer can be found, and it is hard to establish the relationship with algae velocity by observations. According to our model, the relationship between the physical factors and velocity of algae layer is deduced theoretically. As shown in equation (7), under certain wind speed, the velocity of algae increases with the thickness of the algae layer and the density of air, while decreases with the dynamic viscosity and the density of algae layer.

In our model, the algae layer is treated as a layer of liquid with no diffusion in the horizontal. But in actual fact, the algae layer is made up of water and phytoplankton colonies in μm length scale, which would disperse from the place of high density to low. Therefore, a term representing diffusion is added for better description. Considering that the rate of diffusion is positively correlated with gradient of algae density, the velocity of algae layer can be modified as following form:

$$u_{al}=\frac{\rho_a C_a\left(h_2-h_1\right)}{2\eta}U_0{}^2\pm\sqrt{\frac{\rho_a C_a}{\rho_{a1}C_{a1}}}U_0+U_w+f\left(\frac{\partial\rho_{a1}}{\partial x}\right) \qquad (7)$$

Moreover, in-situ observations (Zhu & Cai, 1997; Bai, 2005; Kong and Song, 2011) have found out that there exists a critical wind speed, above which the vertical mixing caused by wind-induced waves would prevent mat-shape algae from forming. Thus, equation (8) holds only when wind speed is lower than the critical one, that is $U_0\le U_c$.

4. Calibration

Based on our simplified model, the velocity of mat-shape algae is deduced theoretically. However, some coefficients in the equation are difficult to determined, such as the density, dynamic viscosity and thickness of algae layer, due to the lack of research into such physical factors. Bai et al.(2005) have recorded the velocity of algae under wind-driven condition in their flume experiment and in in-situ observation in Taihu Lake of China, which would be used in calibrating our theoretical result. In his observations, Bai used mean algae layer velocity and mean wind speed to describe the motions of algae and wind respectively, but he did not measure any physical characters of algae layer due to difficulties in measurement. The observation data in Bai's essay is used to fit in the quadratic relationship of algae drift velocity and wind speed in Eq.(7), and the fitting results are as follows:

for the flume experimental data:

$$u_{al} = 0.01754 U_0^2 - 0.0388 U_0 + 0.06101 \tag{8}$$

for the in-situ observation in Taihu Lake of China:

$$u_{al} = 0.04291 U_0^2 - 0.1111 U_0 + 0.09141 \tag{9}$$

Fitting curves are shown in Fig.2 and Fig.3, respectively. And both of flume experiment and in-situ observation data fit well into the quadratic function. The correlation coefficients R^2 for equation (9) and equation (10) are 0.959 and 0.947 respectively, which demonstrates that the theoretical result is rational. Note that coefficients are quite different between equation (9) and equation (10). This accords with the theoretical result that coefficients are related to algae layer thickness, dynamic viscosity and other empirical factors, which would change under different situations.

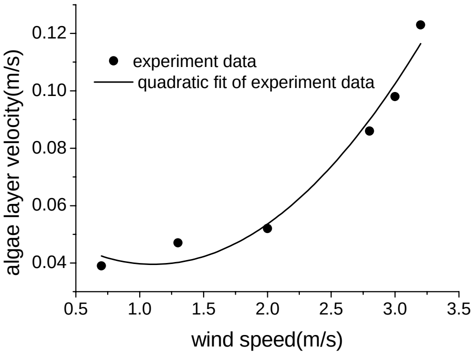

Figure 2. Variation of algae layer velocity with wind speed for flume experiment

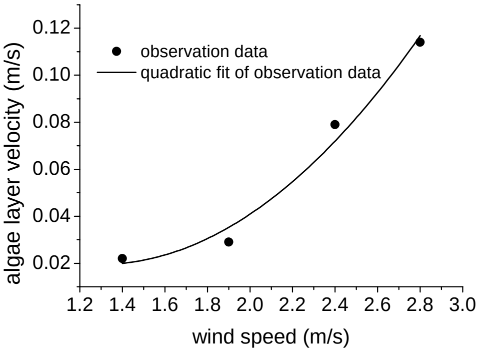

Figure 3. Variation of algae layer velocity with wind speed for observation in Taihu Lake of China

Conclusions

By analyzing the stress mechanism of the mat-shape algae on the surface of water, we treat the algae layer as a kind of liquid and establish a two-layer fluid model. A theoretical relationship between wind speed and algae drift velocity is deduced and calibrated by the observed data. And the following conclusions are drawn from this study:

(1) A theoretical expression of algae velocity is given, which indicates that the drift velocity of mat-shape algae changes with wind speed in a quadratic function.

(2) Under certain wind speed, the velocity of algae increases with the thickness of the algae layer and the density of air, while decreases with the dynamic viscosity and the density of algae.

(3) The theoretical result obtained from our model is calibrated by flume experiment and in-situ observation data, from which two sets of quantitative relationship between the velocity of algae layer and wind speed are attained. The good fitting of both observation data into the quadratic function demonstrates the rationality of our model.

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