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Simulation of the Motion of a Bead in Shallow Water via SPH Method

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Abstract This paper numerically investigated the two-dimensional motion of a bead in shallow water down a steep rough bed via smoothed particle hydrodynamics (SPH). The motivation of this work is to study the mechanism of particle transport in shallow flows. The simulated results show agreement with the experimental data which also verifies the validity of SPH. Moreover, a series of numerical experiments are carried out to discuss the influences of bead material, slope gradient on the characteristics of bead motion and the state of shallow flow.

Key words: particle transport, shallow water, steep rough bed, smoothed particle hydrodynamics (SPH)

INTRODUCTION

Solid particle transport by water flow is the dominant process in a lot of natural and industrial flows, such as water-rock debris flow, bed load transport in rivers, particle jets, chemical reactors, etc. This kind of transport process is very different from the well-studied solid-liquid two-phase flow due to its full coupling between particle motion and flow. With the particle size, roughness and flow depth in the same dimension, not only the shallow flow controls the motion of particle but also the particle motion greatly influences the state of the shallow flow. Due to its complexity, a simplified model of a single bead motion on immovable steep rough bed is commonly used to study the mechanism of particle transport in the flow. While many experimental results have been carried out[1,2], the numerical simulation study is still raw[1].

Smoothed Particle Hydrodynamics (SPH) is a Lagrangian meshless particle method where the fluid domain is divided into a finite number of mass carrying paticles. The movement of the particles and the pressure distribution are obtained by solving N-S equation and equation of state. It shows great advantage in simulating the flows with free surface and solving the fluid-solid coupling problems. Therefore, the purpose of this research is to employ the SPH method to simulate this complex problem and study the particle motion characteristic in shallow flows.

GOVERNING EQUATION

1 Continuous phase equation

In SPH, N-S equation which describes fluid movement can be writtern as

$$\frac{d\rho_i}{dt} = \sum_j m_j v_{ij} \nabla_i W_{ij} \tag{1}$$

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^{\,2}} + \frac{P_j}{\rho_j^{\,2}} + \prod_{ij}\right)\nabla_i W_{ij} + \boldsymbol{g} \tag{2}$$

Where P_k and ρ_k are the pressure and density corresponding to particle k (evaluated at i or j), $\boldsymbol{g}$ is the gravitational acceleration, $\prod_{ij}$ is the viscosity term[3].Cubic spline is choosed as the kernel function W .

2 Bead motion equation

The bead treated as rigid body is expressed by a group of boundary particles with its shape kept. Each boundary particle *k* experiences a force per unit mass given by

$$\boldsymbol{f}_k = \sum_{i \in WPs} \boldsymbol{f}_{ki} \tag{3}$$

where *WPs* denotes water particles and $\boldsymbol{f}_{ki}$ is the force per unit mass exerted by water particle *i* on boundary particle *k*. The equations of basic rigid body dynamics are used to describe the motion of the moving bead, given by

$$M\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}=\sum_{k\in BPs} m_k \boldsymbol{f}_k,\qquad I\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t}=\sum_{k\in BPs} m_k(\boldsymbol{r}_k-\boldsymbol{R}_0)\times \boldsymbol{f}_k \tag{4}$$

where *M* is the mass of the object, *I* is the moment of inertia, $\boldsymbol{V}$ is the velocity of the object, $\boldsymbol{\Omega}$ is the rotational velocity of the object, $\boldsymbol{R}_0$ is the position of the centre of mass and *BPs* denotes boundary particles.
Each boundary particle that describes the moving body has a velocity given by

$$\boldsymbol{u}_k = \boldsymbol{V} + \boldsymbol{\Omega}\times(\boldsymbol{r}_k - \boldsymbol{R}_0) \tag{5}$$

NUMERICAL MODEL

For the sake of simplicity, the short interesting area($0.6m\times 0.2m$) is cut out in this mumerical model. As shown in figure 1, the nominal diameter $0.06m$ of the mobile bead and the bed roughness are at the same magnitude with the water depth. The bead and the bed roughness are expressed by a group of boundary particles which could interact with each other with their shape kept. Both the inlet and the outlet of the slope are set as periodic open boundary conditions in x direction. The repulsive boundary condition proposed by Monaghan[4] is adopted to simulate wall and the effect of air at the free surface is neglected. And the effect of slope is achieved by changing gravity direction. We focus on the motion characteristics after the bead and the flow fully interacting with each other. As shown in table 1, seven cases with different parameters are dealed with in the numerical experiments.

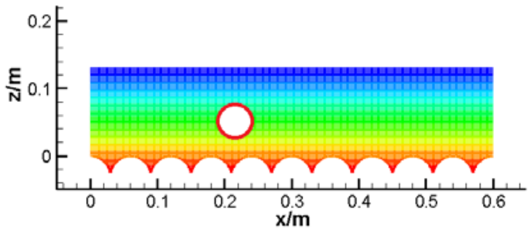


Figure 1: The numerical model

Table1 Parameters of numerical experiments

Case	Bead	Slope (tan θ)	Depth (m)
1	none	0.10	0.13
2	glass	0.05	0.13
3	steel	0.05	0.082
4	glass	0.10	0.13
5	steel	0.10	0.13
6	glass	0.15	0.13
7	glass	0.20	0.13

Annotation: ρ_{glass} = 2500 kg/m^3 ρ_{steel} = 7 750 kg/m^3

NUMERICAL RESULTES

As shown in Fig.2, the simulated velocity profile of the water stream mainly shows agreement with the experimental data. And in Fig.3, the rolling trajectory of bead center shows good agreement with the experimental observation. Thus it can be seen, SPH is valid in solving this complex problem. Then, we pay specific attention to three particular points for Fig.4. (1) The bead motion greatly influences the state of the flow.The water velocity has a significant increase near the bead, however, the free surface close to the bead is lower than when far away from the bead. (2) Comparing (c) with (f), the movement of steel bead is slower than that of glass bead which moved more another period. And, the steel bead always rolls while the glass bead saltates sometimes. (3) Observing pictures from (b) to (e), the slope has a great influence on the coupling movement of the bead and the flow.With the increase of the slope, both the bead velocity and the water velocity increase correspondingly, in addtion, the bead more easily appears saltation phenomena in the flow.

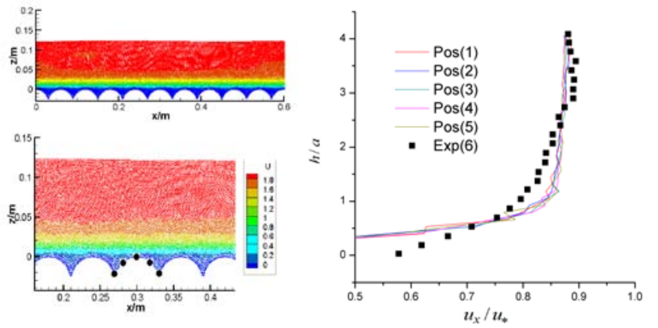


Figure 2: Velocity profile of the water stream at 5 different positions along one roughness(a) is compared with the experimental data(b) in case 1 condition.

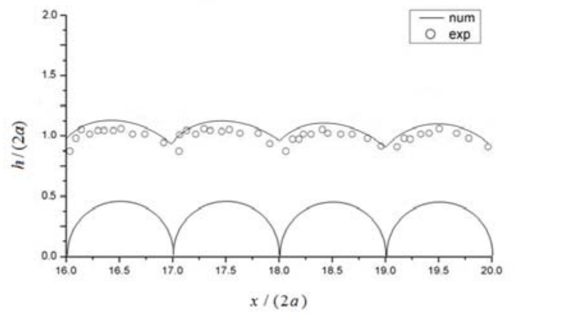
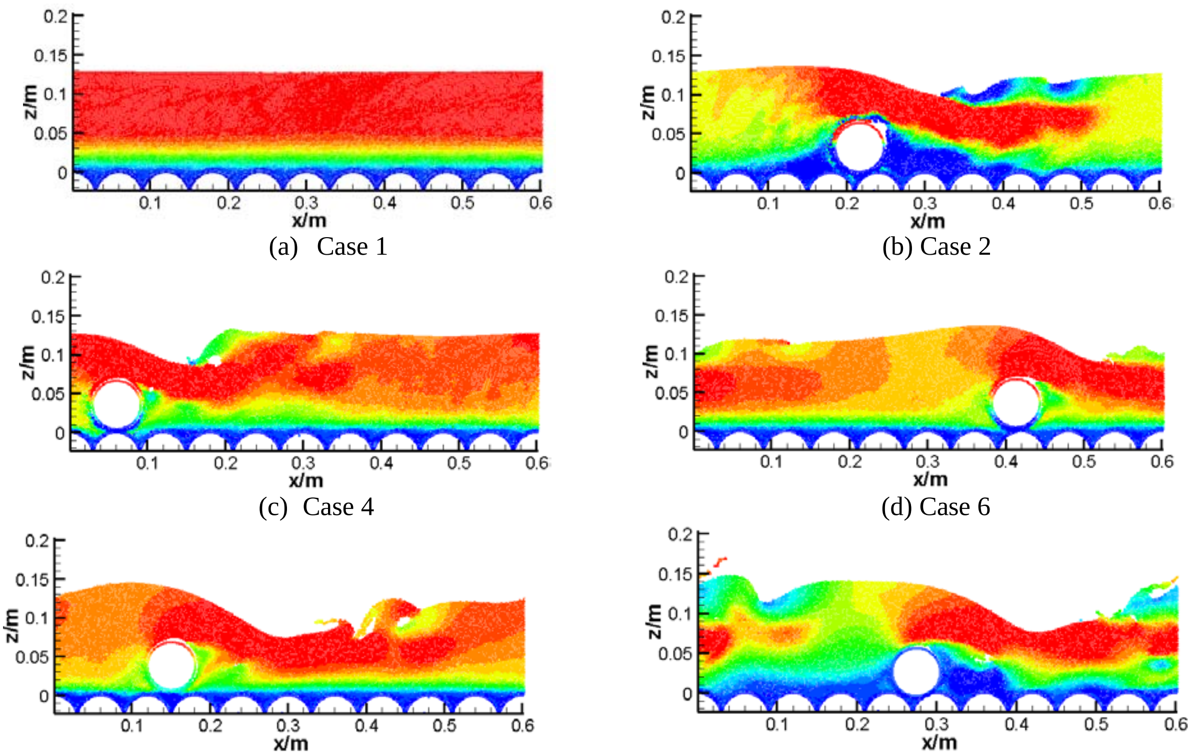


Figure 3: The rolling trajectory of bead center is compared with the experimental observation in in case 3 condition.



(a) Case 1
(b) Case 2
(c) Case 4
(d) Case 6
(e) Case 7
(f) Case 5

Figure 4: Velocity distribution of all cases without case 3 at the same time(1.4s)

CONCLUSION

This paper investigated the two-dimensional motion of a bead in shallow water down a steep rough bed by numerical method of the SPH. The main differences from previous studies are that (1) the bead, roughness and flow depth in the same dimension so that the bead and the flow greatly influence each other, (2) we have done a series of numerical experiments, and discussed the influences of bead materials, slope gradient on the characteristics of bead motion and the state of shallow flow.The numerical simulations show good agreement with the experimental data.

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