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Relationship between load/unload response ratio and damage variable and its application

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SUMMARY

The physics-based parameter: load/unload response ratio (LURR) was proposed to measure the proximity of a strong earthquake, which achieved good results in earthquake prediction. As LURR can be used to describe the damage degree of the focal media qualitatively, there must be a relationship between LURR and damage variable (D) which describes damaged materials quantitatively in damage mechanics. Hence, based on damage mechanics and LURR theory, taking Weibull distribution as the probability distribution function, the relationship between LURR and D is set up and analyzed. This relationship directs LURR applied in damage analysis of materials quantitatively from being qualitative earlier, which not only provides the LURR method with a more solid basis in physics, but may also give a new approach to the damage evaluation of big scale structures and prediction of engineering catastrophic failure.

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KEY WORDS: damage variable (D); load/unload response ratio (LURR); earthquake prediction; damage evaluation

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BRIEF INTRODUCTION OF LURR

From the macroscopic viewpoint, the stress–strain curve is a comprehensive description of the mechanical behaviors of materials. A typical stress–strain curve for focal media (rock) is shown in Figure 1. If the load acting on the material increases monotonously, the material will experience the regimes of elastic, damage process and failure or destabilization. The most essential characteristic of the elastic regime is its reversibility, namely that the positive process and the contrary process are reversible. In other words, the loading and the unloading modulus are equal to each other. Contrary to the elastic regime, the damage regime is irreversible and the unloading response is different from the loading one, that is, the loading modulus is different from the unloading one. This difference indicates the deterioration of the material due to damage, which also means that the parameter LURR can describe the damage degree of materials qualitatively.

Based on earthquake mechanics, fracture mechanics, damage mechanics, and nonlinear sciences, Yin Xiangchu has proposed an approach called LURR to earthquake prediction and after several years' practice, some success in earthquake prediction has been achieved [1,2].

In order to measure the difference between load response and unload one quantitatively, two parameters are defined as follows.

The first one is the response rate X defined as

$$X = \lim_{\Delta P \to 0} \frac{\Delta R}{\Delta P} \tag{1}$$

where ΔP and ΔR denote the increments of load P and response R, respectively.

The second one is load/unload response ratio (LURR) (denoted as Y) defined as

$$Y = X_+ / X_- \tag{2}$$

where X_+ and X_- refer to response rate under loading and unloading condition, respectively.

If we take the strain as the response to the loaded or unloaded stress on the rock material, from Figure 1, it is clear that $Y = 1$ when the material is in the elastic regime since $X_+ = X_-$. In the damaged regime where $X_+ > X_-$, $Y > 1$. The more severely the material is damaged, the larger the

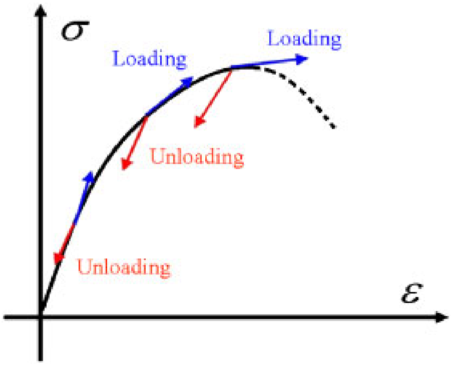

Figure 1. The stress–strain curve of rock.

Y value will be. As the media approaches failure, the Y value becomes larger and larger. Therefore, the LURR value (Y) could measure the damage of the seismogenic region qualitatively, describe the proximity to failure and also act as a precursor for the earthquake prediction/forecasting.

In actual earthquake prediction, the LURR value Y is defined directly through the ratio of released seismic energy in the loading and unloading periods as follows:

$$Y = \frac{\left(\sum_{i=1}^{N_+} E_i^m\right)_+}{\left(\sum_{i=1}^{N_-} E_i^m\right)_-} \tag{3}$$

where E denotes released seismic energy, the '$+$' sign means loading and '$-$' sign means unloading, $m=0$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$ or 1. When $m=1$, E^m is exactly the energy itself; $m=\frac{1}{2}$, E^m denotes the Benioff strain; $m=\frac{1}{3}$ and $\frac{2}{3}$, E^m represents the linear scale and area scale of the seismogenic zone, respectively; $m=0$, Y is equal to N_+/N_-, where N_+ and N_- denote the number of earthquakes which occurred during the loading and unloading duration, respectively. In this paper m is adopted as $\frac{1}{2}$, which means that Y is determined by Benioff strain during the loading duration over the unloading one.

The retrospective inspections of hundreds of earthquake cases have validated the LURR [3–6]. For more than 80% of the examined cases the Y value fluctuates around 1 during the early stage of the seismogenic process and it rises when the region approaches the onset of a strong earthquake. Then Y reaches its maximum (significantly larger than 1), but decreases sharply on the eve of the main shock.

After several years of practice in earthquake prediction, the forecasting effect of LURR has been improved obviously and applied in China mainland, West America, Japan, Australia, Italy, Iran, etc. [7,8]. At the same time, the LURR method has also been deeply studied in physics, laboratory studies, and numerical simulations [9–12]. Besides earthquake prediction, this method has been used in other fields such as slopes stability [13] and landslides [14,15].

RELATIONSHIP BETWEEN LURR AND *D*

Damage mechanics is a branch of solid mechanics. Kachanov [16] proposed to describe the collective effect of the deterioration by a field variable continuously, which is called damage variable (D). As we know, the LURR value can be used to measure the damage degree of the seismogenic region qualitatively. Hence, there must be a relationship between LURR and D. It will be the main topic discussed in this part. In this part, the relationship between LURR and D is set up and analyzed with the Weibull distribution as the probability distribution function and then the acoustic emission (AE) experiment is presented and the analysis of the experiment results validates the relationship.

A new definition of LURR

In the case of uniaxial tension or compression, it can be proved that a close relationship between LURR and D exists.

First, let us introduce the actual stress σ_a [16,17]

$$\sigma_n = \sigma_a(1-D) \tag{4}$$

where σ_n is the nominal stress, σ_a is the actual stress, D is the damage variable. Then the total differential of σ_n is

$$d\sigma_n = d\sigma_a(1-D) - \sigma_a dD \tag{5}$$

We assume $dD = 0$ when the material is under unloading state; hence,

$$\begin{aligned} d\sigma_{n(+)} &= d\sigma_{a(+)}(1-D) - \sigma_a dD \\ d\sigma_{n(-)} &= d\sigma_{a(-)}(1-D) \end{aligned} \tag{6}$$

where the sign '+' means loading and the sign '−' means unloading.

According to the Hooke law

$$\begin{aligned} d\sigma_{a(+)} &= E_0 d\varepsilon_{(+)} \\ d\sigma_{a(-)} &= E_0 d\varepsilon_{(-)} \end{aligned} \tag{7}$$

where E_0 is the initial Young's modulus.

From Equations (6) and (7), the loading and the unloading responses can be expressed as below, respectively,

$$\begin{aligned} X_+ &= \frac{d\varepsilon_{(+)}}{d\sigma_{n(+)}} = \left(E_0(1-D) - \frac{\sigma_a dD}{d\varepsilon_{(+)}}\right)^{-1} \\ X_- &= \frac{d\varepsilon_{(-)}}{d\sigma_{n(-)}} = (E_0(1-D))^{-1} \end{aligned} \tag{8}$$

From the definition of the LURR (Equation (2)), it can be calculated as

$$Y_E = \frac{X_+}{X_-} = \frac{1}{1 - \dfrac{\varepsilon}{(1-D)} \dfrac{dD}{d\varepsilon_{(+)}}} \tag{9}$$

which is the new definition of LURR with damage variable (D) and strain as response.

Taking strain as input, based on the damage model proposed by Lyakhovsky [18,19], the damage evolution in case of one-dimensional deformation can be calculated in terms of the following equation [20]:

$$\frac{dD}{dt} = C_1 E_0(\varepsilon^2 - \varepsilon_{cr}^2) \tag{10}$$

where C_1 is a positive parameter in the model to be material property and describes the rate of damage evolution for a given deformation; E_0 is the initial Young's modulus and ε_{cr} is the critical strain that corresponds to a neutral state between healing and degradation of the material, but the healing phenomena will not be considered in this text. If the damage and strain are both given, the new LURR value can be calculated by using Equation (9).

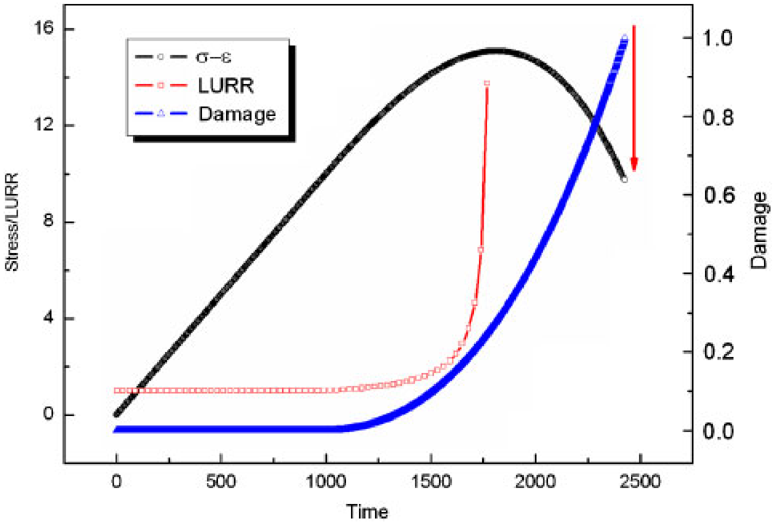

Figure 2. The damage and the new LURR evolution curves, stress–strain curve when taking linearly increasing strain as input data.

Taking the linearly increasing strain as input data, the damage and the new LURR evolution, the nominal stress–strain curves are shown in Figure 2. It is clear that the LURR value reaches the peak value when the nominal stress attains the maximum. Hence, the LURR peak value could be a good predictor for the catastrophic failure of the material.

Relationship between LURR and *D*

Based on statistical mesoscopic damage mechanics, a statistical model of heterogeneous elastic-brittle materials was proposed [21,22]. It is assumed that such a sample consists of linear elastic but brittle units, namely that all units have the same elastic modulus but different breaking stress threshold.

Suppose that the material follows a probability distribution function $h(\varepsilon_c)$ in the mesoscopic level, such as the Weibull distribution [23,24]:

$$h(\varepsilon_c) = m\varepsilon_c^{m-1}\exp(-\varepsilon_c^m) \tag{11}$$

where m is the Weibull modulus. Hence, the damage function about strain is:

$$D(\varepsilon) = \int_0^{\varepsilon} h(\varepsilon_c)\,d\varepsilon_c = 1 - e^{-\varepsilon^m} \tag{12}$$

Substituting Equation (12) into Equation (9),

$$Y_E = \frac{1}{1 - m\varepsilon^m} = \frac{1}{m(\varepsilon_F^m - \varepsilon^m)} \tag{13}$$

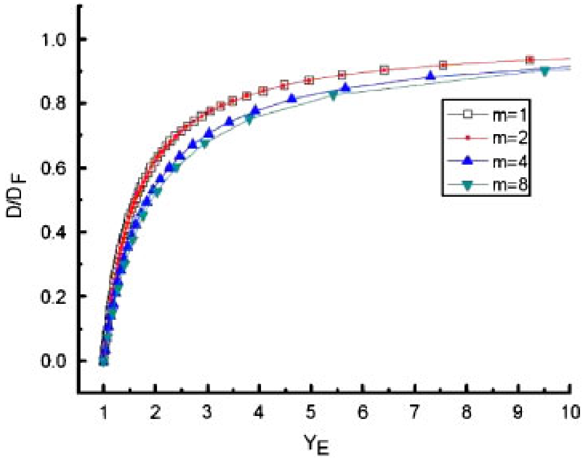

Figure 3. The curves of the relationship between D/D_{F} and Y_E, where $D_{\mathrm{F}} = 1 - \mathrm{e}^{-(1/m)}$ is the damage value when the material is failed.

is obtained. Here $\varepsilon_{\mathrm{F}} = (1/m)^{1/m}$ is the strain at the failure point. Substituting $\varepsilon_{\mathrm{F}} = (1/m)^{1/m}$ to Equation (12), the damage degree at the failure point is $D_{\mathrm{F}} = 1 - e^{-1/m}$. From Equations (12) and (13), we can get

$$Y_E = \frac{1}{1-m\varepsilon^m} = \frac{1}{1+m\ln(1-D(\varepsilon))} \tag{14}$$

as the relationship between damage variable (D) and LURR (Y). When the Weibull modulus $m = 1, 2, 4, 8$, the relationships between D/D_{F} and Y_E are shown in Figure 3.

Experimental analysis

Our group has carried out AE experiments for medium scale rock specimens by means of international cooperation [10,25–28]. The dimensions of the specimens are 300 mm × 360 mm × 25 mm. The experiments were conducted using MTS-100 servo-control experimental equipment in the Institute of Geophysics, China Seismological Bureau. The maximum load for this facility is 100 ton in the axial direction and 10 ton in the lateral direction. Boundary-displacement control is used to load the system until final failure. The samples are subjected to both axial stress σ_1 and lateral load σ_2 simultaneously and another principle stress σ_3 is zero. The greatest, intermediate and least principle stress are σ_1, σ_2, σ_3, respectively, and $\sigma_1 \neq \sigma_2 \neq \sigma_3$. Therefore, the stress state is a tri-axial stress state (in Figure 4). In our experiments the lateral stress σ_2 keeps constant until the samples fracture, and the axial stress σ_1 consists of two parts: the constant loading rate of tectonic stress build-up and a sinusoidal stress perturbation, which simulates the periodic loading and unloading cycles induced by tidal forces. The size of the rock specimens and the arrangement of the AE sensor are shown in Figure 4, while the loading process and AE event rate and energy rate versus time for the specimen are shown in Figure 5. The evolution of LURR during the rock fracture experiment has been analyzed in terms of Equation (3), AER (Accelerating Energy Release) before macro-fracture

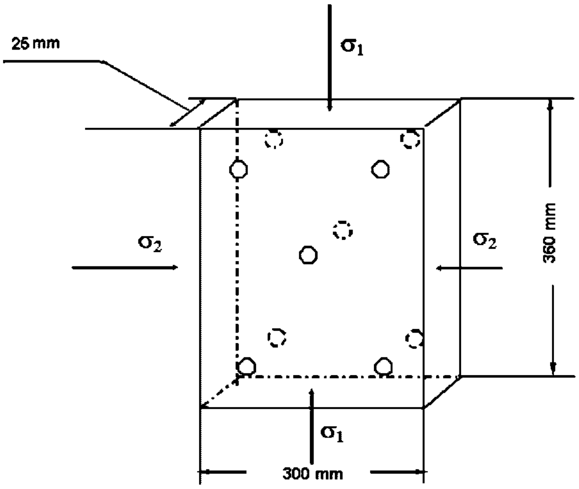

Figure 4. The Geometry of the specimens, the loading conditions, and the arrangement of AE sensors (circles).

of the samples and the correlationship between the AE and the load are also studied by Zhang [25–28]. Figure 6 shows the curve of LURR value versus time during rock fracture experiment for the specimen in terms of Equation (3).

Now, let us analyze the experimental results of the aforementioned system. Early in the 1960s, a kind of statistical description of micro cracks (i.e. number density of micro cracks) was proposed for the evolution of damage [29–31]. We will also take the statistical method to describe the damage degree of the material. Suppose the damage degree is D_{F} when the media gets failed, then the damage degree at time t can be calculated as

$$\frac{D(t)}{D_{\mathrm{F}}}=\frac{\int_0^t [E(t)]^m \,\mathrm{d}t}{\int_0^T [E(t)]^m \,\mathrm{d}t} \tag{15}$$

where $D(t)$ is the damage degree at time t, T is the total experimental time, $[E(t)]^m$ denotes the AE rate at time t, when $m=0$, $[E(t)]^m$ means the AE event rate, when $m=1$, $[E(t)]^m$ means the AE energy rate, and when $m=\frac{1}{2}$, $[E(t)]^m$ means the square root of the AE energy, that is, Benioff strain rate. Hence, the damage evolution of the rock specimen can be investigated in terms of Equation (15). The curves of the damage evolution and stress–strain are shown in Figures 7 and 8, respectively.

Zhang has analyzed the experimental data in terms of Equation (3) with Benioff strain as response rate by using the LURR method (Figure 6) [27,28]. We have got a new LURR formula with damage and strain as response in Equation (9) and the data of damage and strain of the experiment are given in Figures 7 and 8 respectively, hence, the new LURR value can be calculated and shown in Figure 9.

In Figure 9, when $t=12\,000\,\mathrm{s}$, the LURR value attained the maximum (much greater than 1), and then decreased sharply on the eve of the fracture. The dashed line visually indicates the trend of

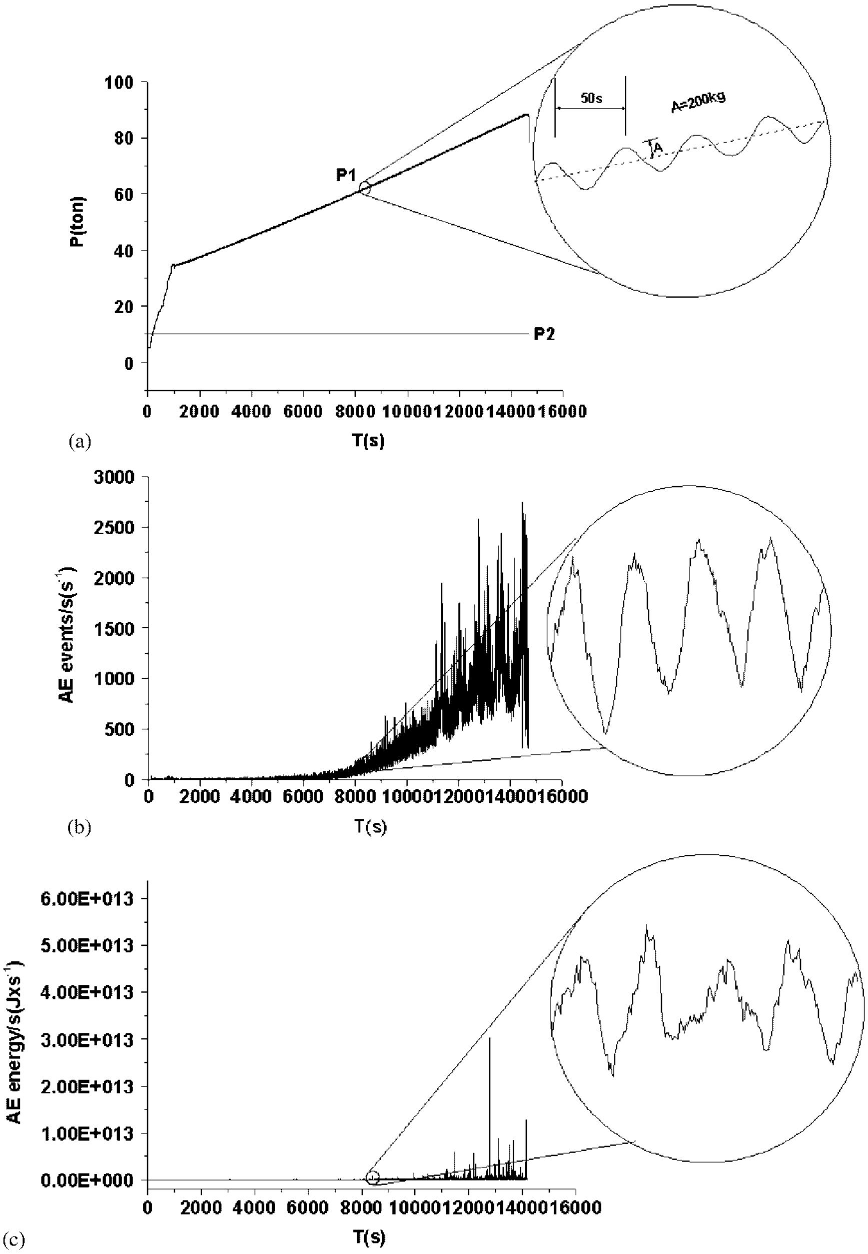

Figure 5. The loading history and the corresponding experimental results for the specimen. (a) The loading history in the experiment. P1 is the axial load and P2 is the lateral load; and (b) the AE event rate versus time for the specimen; and (c) the AE energy rate versus time for the specimen.

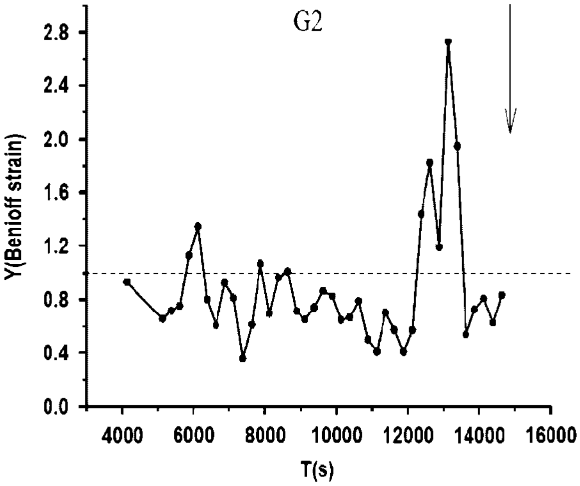

Figure 6. The curve of LURR during the rock fracture experiment of rock specimen with Benioff strain as response.

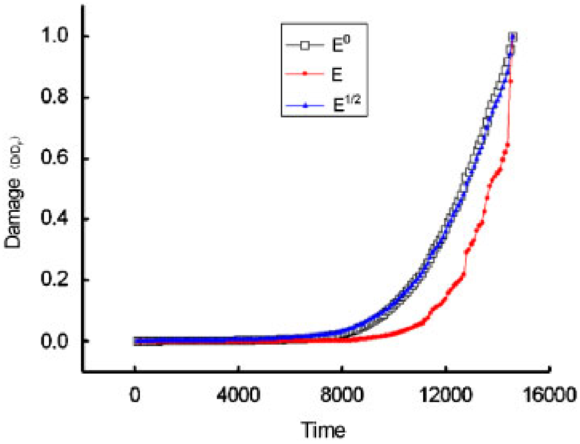

Figure 7. The evolution of damage defined with AE rate.

LURR evolution during the total experiment process, which can also reflect the damage evolution in the rock specimen to some extent. In Figure 6, when $t = 13\,120$ s, the LURR value attained the maximum. Comparing Figure 6 with Figure 9, both LURR values experienced the process that abnormity appeared, rose to the peak value with relatively slow speed, then decreased sharply and failed or destabilized quickly, which indicates that the evolution of LURR defined with damage and strain is very similar to the one defined with Benioff strain as response rate.

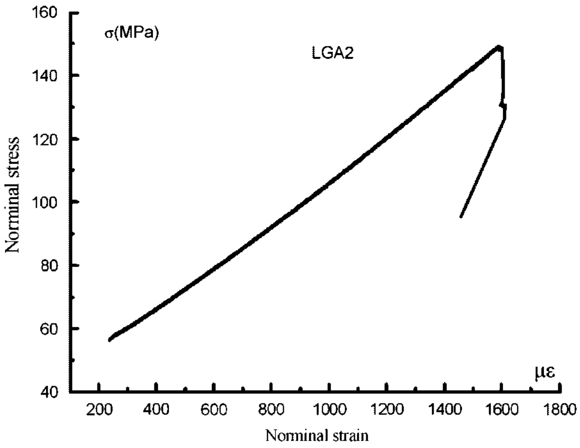

Figure 8. The curve of stress–strain in the experiment.

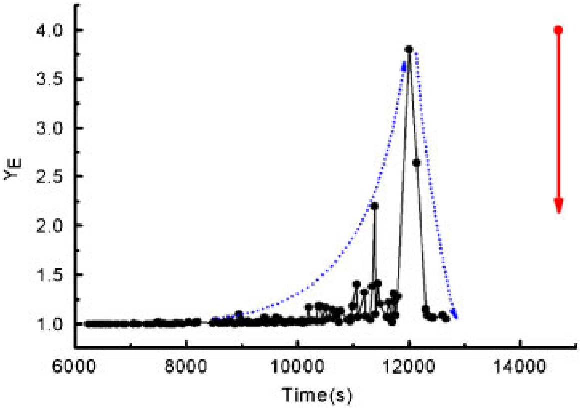

Figure 9. The curve of the new LURR versus time of the specimen (the arrow means the specimen got failed).

On the other hand, Figure 10 is the variation of LURR for the Loma Prieta earthquake which occurred in California on October 17, 1989, which is calculated with Benioff strain as the response rate by using seismic data in actual earthquake prediction. The tendency of LURR evolution in Figure 10 seems to be consistent with the ones shown in Figures 6 and 9, which may show the rationality of the LURR defined with Benioff strain as the response rate in the actual earthquake prediction.

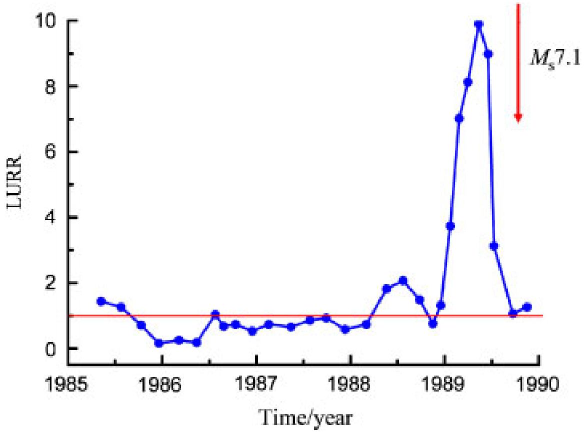

Figure 10. The variation of LURR for Loma Prieta earthquake occurred in California on October 17, 1989.

DISCUSSION FOR APPLICATIONS OF THE RELATIONSHIP

Earthquake prediction

The establishment of the relationship between LURR and D is very significant, which provides the LURR theory with a more solid basis in physics and makes the LURR applied in damage analysis of materials from being qualitative to being quantitative. Hence, the relationship can instruct LURR to be used in earthquake prediction more precisely.

In order to investigate the seismicity in the China mainland, the tempo-spatial scanning of LURR from November 1, 2002 to June 30, 2008 has been conducted with 12, 15, 18, 24, 30, 36, 42 months as time window, 1 month as time step and the corresponding $R = 70$, 100, 200, 300, 400, 500, 600 km as scanning radius, respectively. Owing to the multi time windows and space windows, the tempo-spatial scanning needs plenty of computing power. Hence, adopting the technology of domain decomposition and parallelizing using MPI, a new parallel tempo-spatial scanning program was used in the computing [32], which was carried out by Super Computer Deep-Comp 6800 of the Super Computing Center of the Chinese Academy of Science. Limited by the paper length, we only show one result with $R = 300$ km as scanning radius in every 4 months, Figure 11 is the evolution of the anomaly LURR regions in time order. The notes for every result like 2002.11.1–2004.10.31, 0–5.0, 0.25, R300, 95, 1.0, where 2002.11.1–2004.10.31 means the begin date and end date of the time window, 0–5.0 means the magnitude threshold, 0.25 means the moving step length in both the latitudinal and longitudinal directions, R300 means the radius of circle region is 300 km, 95 means the confidence is 0.95, and 1.0 means the critical LURR value which corresponds to a neutral state between ‘safety’ and ‘danger’ of the seismicity.

From Figure 11, we can find that the anomaly LURR region appears along the Longmen Shan belt in the time window with October 31, 2004 as end date, and the anomaly area gets larger, about

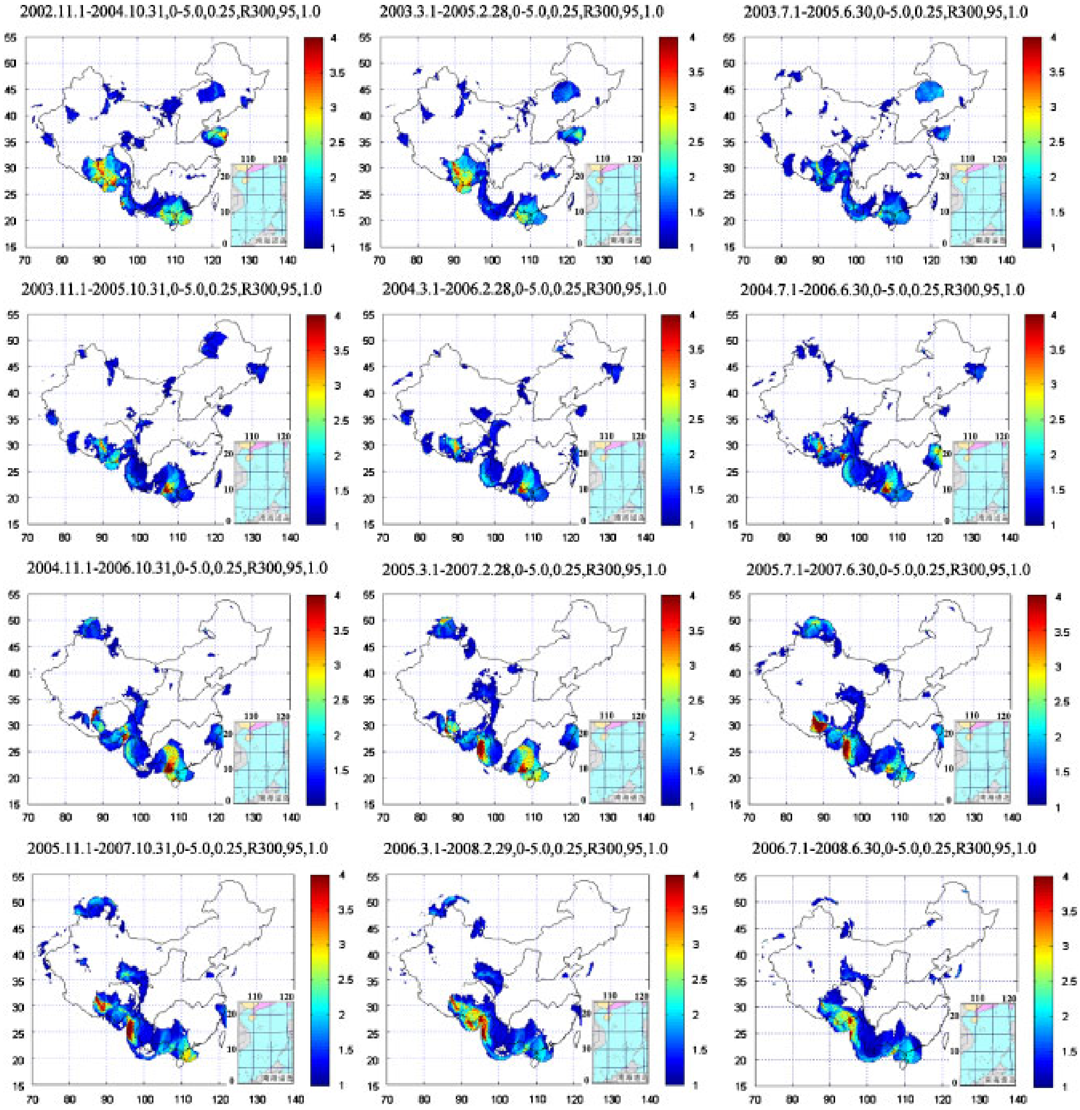

Figure 11. The temporal–spatial evolution of the anomaly LURR regions in China mainland.

up to June 30, 2006 when the area reaches the maximum, finally becoming smaller and smaller versus time, by the time of February 29, 2008, when the anomaly disappears. On May 12, 2008, a magnitude 8.0 earthquake, namely the Wenchuan earthquake occurred in the Longmen Shan belt. The evolution of the LURR in the Longmen Shan belt describes the seismogenious process of the Wenchuan earthquake clearly and completely.

Damage evaluation

The relationship between LURR and D not only provides the LURR theory with a more solid basis in physics, but may also give a new approach to the health assessment of large-scale structures and prediction of engineering catastrophic failure [33]. Some equations are indispensable in solving traditional mechanics problems such as governing equations, boundary, and initial conditions, but there is very little information about these equations to deal with disasters like earthquakes or to do health assessments of large-scale structures and ancient buildings. Fortunately, we can get the response of the structure by loading and unloading experiments, taking bridges, for example, loading and unloading can be achieved whether there are vehicles on the bridge or not. If the LURR value is obtained by experiments, the damage value can be calculated in terms of Equation (14). Furthermore, we can assess the health of the structure according to the damage value.

The process is introduced in detail as follows:

$$Y_{\mathrm{exp}} = \left(\frac{\Delta R}{\Delta P}\right)_{+} \Big/ \left(\frac{\Delta R}{\Delta P}\right)_{-}$$

where ΔP is the increment of the load in the experiment, ΔR is the corresponding response of the ΔP, R can be the displacement or strain etc. Y_{exp} is the LURR value in the experiment, according to the formula in Equation (14), the damage value can be conducted as $D = 1 - \exp(m(1 - Y_{\mathrm{exp}})/Y_{\mathrm{exp}})$, while the damage value can reflect the health extent of the structure. Hence, LURR method may be used for health assessment of large-scale structures. An intensive study of this application will be performed by means of international cooperation with researchers from Naples University, Italy in the future.

CONCLUSION

The LURR method is being proposed for more than 20 years and some success in earthquake prediction has been achieved. Earlier, the larger LURR value meant the more severely the material is damaged, namely that LURR describes damaged materials qualitatively. Now, the relationship between LURR and D is set up, which makes LURR to be applied in damage analysis for materials quantitatively. From the comparison of the LURR curves, we find that the LURR curves defined with Benioff strain as the response rate and with damage and strain as responses are very similar in trend and also validate the LURR defined with Benioff strain as response rate in the actual earthquake prediction is rational. Hence, the establishment of the relationship between LURR and D is very significant, which not only provides a more solid basis to study the damage evolution of materials utilizing LURR method, but may also give a new approach to the health assessment of large-scale structures and prediction of engineering catastrophic failure.

The applications of the relationship between LURR and D in earthquake prediction and damage evaluation or 'healthy-diagnoses' of large-scale engineering structures will be new projects for us to study in the future.

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