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2010 Chinese Phys. Lett. 27 088104

(http://iopscience.iop.org/0256-307X/27/8/088104)

CHIN. PHYS. LETT. Vol. 27, No. 8 (2010) 088104

Critical Biot's number for Determination of the Sensitivity of Spherical Ceramics to Thermal Shock *

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(Received 17 November 2009)

A critical Biot number, which determines both the sensitivity of spherical ceramics to quenching and the durations of the temperature-wave propagation and the thermal stresses in the ceramics subjected to thermal shock, is theoretically obtained. The results prove that once the Biot number of a ceramic sphere is greater than the critical number, its thermal shock failure will be such a rapid process that the failure only occurs in the initial regime of heat conduction, whereas the thermal shock failure of the ceramic sphere is uncertain in the course of heat conduction. The presented results provide a guide to the selection of the ceramics applied in the thermostructural engineering with thermal shock.

PACS: 81.40.Np, 44.10.+i, 65.40.−b DOI: 10.1088/0256-307X/27/8/088104

Enhancing the resistances to thermal shock has been a major challenge in a lot of thermostructural applications of ceramic materials for a long time.[1−4] The previous investigations proved that besides the physical properties of ceramics and quenching media, both the geometrical characteristics of ceramics and the durations of the temperature-wave propagation and the thermal stresses imposed by thermal gradients in the ceramics play very important roles in the behavior of the ceramic materials during thermal shock.[5−7] However, the correlation between the geometrical characteristics and the durations has not been well understood yet, despite there being some literature investigating the behaviors of ceramics during thermal shock.[8]

The geometrical characteristics of ceramics sufficiently determine the symmetry of the temperature-wave propagation in the course of heat transfer and, simultaneously, stipulate the durations of the temperature-wave propagation and thermal stresses yielded during thermal shock. For example, the planar, cylindrical and spherical ceramics surrounded by different temperature environments usually possess the plane, line and point symmetries of heat conduction, respectively, which effectively impact the thermal stress levels and distributions in the interior of the ceramics.[9,10] Because the durations in thermal shock are associated intimately with the unsteady high-rapid heat conduction that is a very difficult problem in heat transfer, their main properties have still remained unclear so far.[11−15] In particular, spherical ceramics possess the very high symmetries on geometrical characteristics, heat transfer and thermal stresses in comparison with the planar and cylindrical ceramics, so that their behaviors during thermal shock have exceptive characteristics. In addition, spherical ceramics are widely applied to the thermostrctural engineering such as the spherical valves in thermal pipes, etc. These make spherical ceramics become one of the hot spots in the study of thermal shock.[6,7]

In this Letter, Biot's number, i.e. the ratio of inter-conduction and surface-convection resistances to heat transfer, is used to describe the characteristics of the geometry and heat transfer, and the Fourier number, i.e. the dimensionless time, is applied to express the durations of the temperature-wave propagation and the thermal stresses during thermal shock. By investigating the correlation between the Biot and Fourier numbers of a spherical ceramic during thermal shock, we obtain a critical Biot number that effectively determines the sensitivity of the spherical ceramic to quenching. Simultaneously, the Fourier number corresponding to the critical Biot number is proven to stipulate the durations of the temperature-wave propagation and the thermal stresses in the ceramics subjected to thermal shock.

Consider a ceramic sphere of radius R, with a uniform initial temperature T_0. At the initial time, the surface of the sphere is suddenly exposed to a convective medium with a uniform temperature T_∞, as shown in Fig. 1. The temperature field yielded in the sphere, $T = T(r, \tau)$, satisfies the equation of heat conduction

$$\frac{\partial T}{\partial \tau} = a\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT), \quad (1)$$

where $a = k/\rho c_p$ is the thermal diffusivity of the ma-

*Supported by the National Natural Science Foundations of China under Grant No 90716004.

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terial of the sphere; k, ρ and c_p are the thermal conductivity, the density and the specific heat at constant pressure for the material of the sphere, respectively.

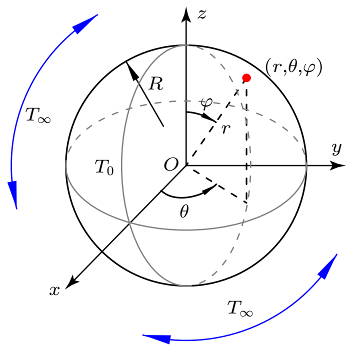

Fig. 1. A ceramic sphere of radius R and initially temperature T_0 suddenly exposed to convective medium of temperature T_∞.

The initial and boundary conditions governing Eq. (1) are written as

$$T(r,0)=T_0, \quad (2)$$

$$\frac{\partial T}{\partial r}\Big|_{r=0}=0, \quad (3)$$

$$-k\frac{\partial T}{\partial r}\Big|_{r=R}=h(T-T_\infty), \quad (4)$$

where h is the surface heat transfer coefficient. Under Eqs. (2), (3) and (4), Eq. (1) is solved by a standard separation-of-variables technique, giving[13]

$$\frac{T(r,\tau)-T_0}{T_\infty-T_0}=1-2\sum_{n=1}^{\infty}\frac{\sin(\beta_n)-\beta_n\cos(\beta_n)}{\beta_n-\sin(\beta_n)\cos(\beta_n)} \times\exp(-\beta_n^2\cdot f)\frac{\sin(\beta_n\cdot r^*)}{\beta_n\cdot r^*}, \quad (5)$$

where $f=a\tau/R^2$ is the Fourier number that describes the dimensionless time of heat conduction; $r^*=r/R$ stands for the dimensionless length; and β_n are the roots of the equation

$$1-\beta_n\cot(\beta_n)=\beta, \quad (6)$$

where $\beta=hR/k$ is Biot's number of the sphere, which is a dimensionless number and includes the material properties and the geometrical characteristics of the sphere and the ambient medium.

First of all, we use Eq. (5) to study the temperature-wave penetration time of the sphere. The temperature-wave penetration time is the duration in which the change of temperature propagates from the surface to the center of the sphere. For the convenience of computation, we take the penetration time as the duration that the change of temperature at the center of the sphere occurs and reaches 0.1% of the initial temperature, as shown in Fig. 2. The results obtained here indicate that the temperature-wave penetration time of the sphere changes with the Biot number of the sphere: the greater the Biot number is, the shorter the penetration time is.

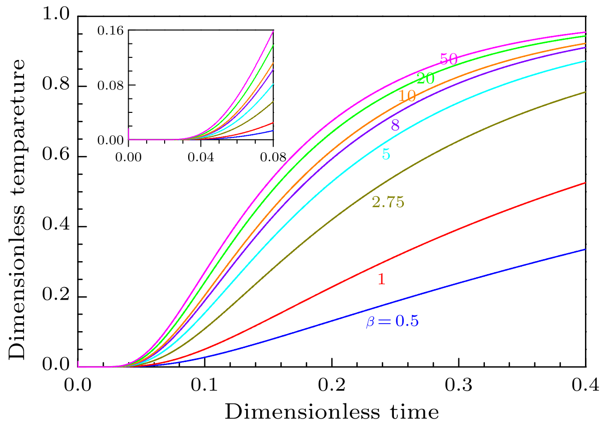

Fig. 2. The temperature profiles of the center of the sphere evolve under the condition during cold shock. Inset: the evolution of the temperature-wave penetration time.

Secondly, we investigate the thermal stress fields in the sphere and the extremum stress time, the time corresponding to the extremum stress, of the sphere. According to the theory of thermal stresses,[7] the dimensionless thermal stress of the sphere is written by

$$\sigma^*=\frac{\sigma(r,\tau)\cdot(1-\nu)}{\alpha E(T_\infty-T_0)}, \quad (7)$$

where E, ν and α are Young's modulus, Poisson's ratio and the coefficient of thermal expansion of the material of the sphere, respectively; $\sigma(r,\tau)$ is the actual thermal stress field in the sphere, which is expressed as[15]

$$\sigma_r(r,\tau)=\frac{2\alpha E}{1-\nu}\cdot\Big[\frac{1}{R^3}\int_0^R(T-T_0)r^2dr -\frac{1}{r^3}\int_0^r(T-T_0)r^2dr\Big], \quad (8)$$

$$\sigma_\theta(r,\tau) =\sigma_\varphi(r,\tau)=\frac{\alpha E}{1-\nu}\cdot\Big[\frac{1}{R^3}\int_0^R(T-T_0)r^2dr +\frac{2}{r^3}\int_0^r(T-T_0)r^2dr-(T-T_0)\Big]. \quad (9)$$

In Eqs. (8) and (9), there are two relationships between the temperature and the radius of the sphere[15]

$$\lim_{r\to 0}\frac{1}{r^3}\int_0^r(T-T_0)r^2dr=\frac{1}{3}[T(0,\tau)-T_0], \quad (10)$$

$$\lim_{r\to 0}\frac{1}{r^2}\int_0^r(T-T_0)r^2dr=0. \quad (11)$$

According to Eqs. (8)–(11), we find that at the center of the sphere, $r/R=0.0$, the stresses satisfy

$$\sigma_r=\sigma_\theta=\sigma_\varphi, \quad (12)$$

and on the surface of the sphere, $r/R = 1.0$, the stresses satisfy

$$\sigma_r = 0, \quad \sigma_\theta = \sigma_\varphi. \tag{13}$$

From Eqs. (5), (8) and (9), the thermal stress fields in the sphere are readily calculated, as shown in Fig. 3. The results indicate that the thermal stress generated on the surface is tensile during cooling and the thermal stress presented at the center is tensile during heating. Because ceramics are much weaker in tension than under compression,[7,12] failure often occurs on the surface during cooling and at the center during heating. Therefore, only the tensile thermal stresses are investigated in what follows.

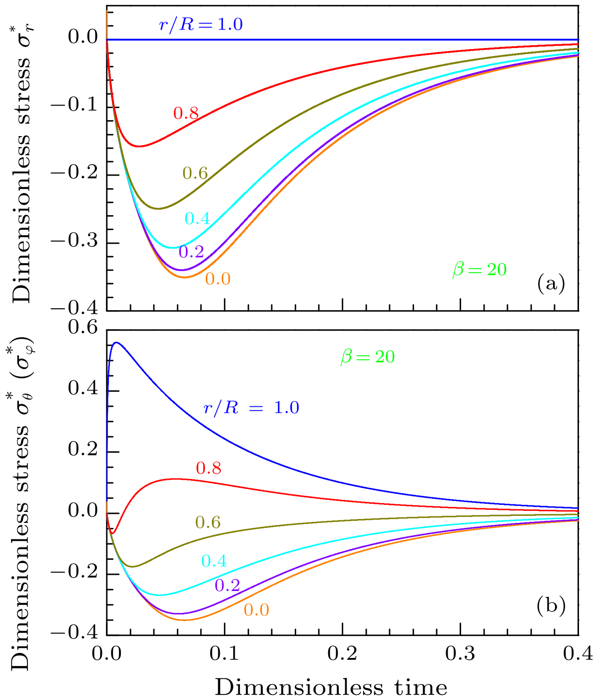

Fig. 3. The evolution of the stress fields in the sphere ($\beta = 20$) under the condition of cold shock: (a) the radial stress; (b) the tangential stress.

Further, both the surface tensile stress field during cooling and the tensile stress of the center during heating are obtained by solving Eqs. (5), (8) and (9), as shown in Fig. 4. This is an indication that the maximum thermal stress in the sphere, such as the temperature-wave penetration time, is associated intimately with the Biot number of the sphere and does not occur at the start of thermal shock except the Biot number is infinite. Under the condition of the same absolute value of temperature difference, $|T_0 - T_\infty|$, the surface tensile stress during cooling is much greater than the central tensile stress during heating. In terms of each of the different Biot's numbers of the sphere, there is a dimensionless extremum stress time, f^*, at which the surface tensile stress during cooling or the central tensile stress during heating reaches its own maximum values. Obviously, the greater the Biot number is, the shorter the extremum time is, as shown in Fig. 4. From the theory of thermal shock fracture,[7,12] as the tensile thermal stress yielded in the sphere during thermal shock is greater than the strength of the material of the sphere, the failure of the sphere occurs. Therefore, if the failure of the ceramic sphere occurs, we can confirm that the failure is bound to take place at or before the extremum stress time.

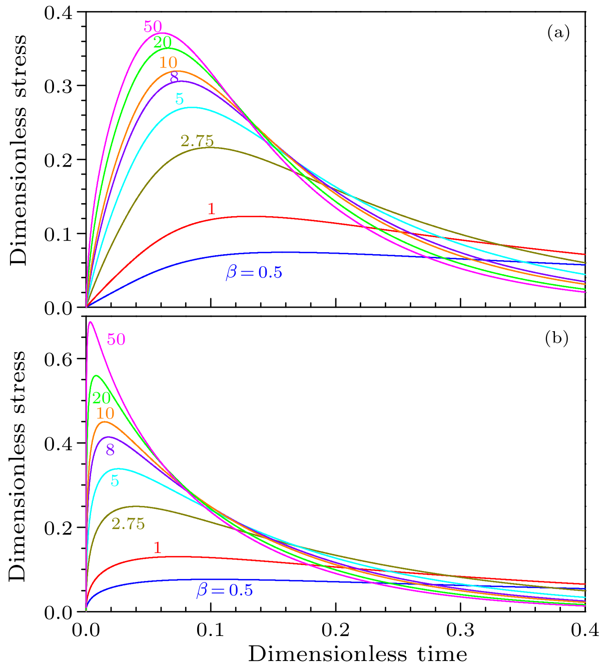

Fig. 4. The dimensionless stress fields with different Biot's numbers: (a) the tensile stresses at the center of the sphere during hot shock; (b) the tensile stresses on the surface of the sphere during cold shock.

By comparing the temperature-wave penetration time f_p, with the extremum stress time during cooling f^*, we find that, on the one hand, there is a critical value of the Biot number of the sphere, $\beta_c = 2.75$, at which the two dimensionless times are the same $f^* = f_p = 0.0413$, as shown in Fig. 5. When the Biot number of a sphere is greater than this critical value $\beta > \beta_c$, the dimensionless time satisfies that the extremum stress time is less than the temperature-wave time $f^* < f_p$ and when $\beta < \beta_c$ the dimensionless time $f^* > f_p$. This proves that if the Biot number of a ceramic sphere is greater than this critical value, the failure of the sphere during cold shock consequentially occurs in the initial regime of the heat conduction process of the materials, in which the temperature-wave propagating from the surfaces does not arrives at the center of the sphere.[13,14] Obviously, this is a very rapid failure process of the materials. This is an indication that the critical Biot number determines the main stress duration characteristic for the ceramic spheres subjected to thermal shock and, therefore can be applied to evaluate the sensitivity of the ceramic spheres to thermal shock. Ceramic spheres, for example, are defined as the sensitive to thermal shock if their Biot numbers are greater than 2.75. However, when the Biot numbers of ceramic spheres are less than the critical value, the failure of the ceramics during thermal shock can theoretically occur at any time before their extremum stress time.

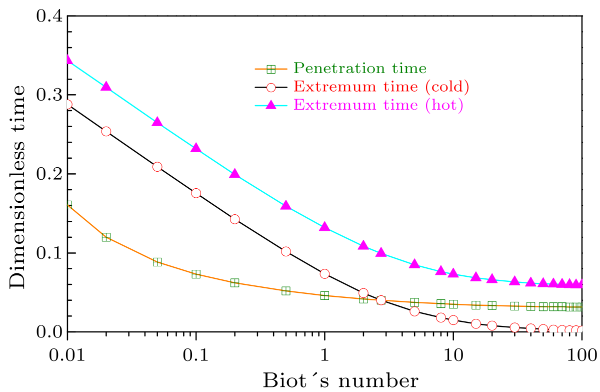

Fig. 5. The temperature-wave penetration time, the extremum stress time during cold shock and the extremum stress time during hot shock change with the Biot number of the sphere. There is an intersection of the penetration time and cold extremum stress time, where the Biot number is 2.75 and dimensionless time is 0.0413. However, there is no intersection of the penetration time and hot extremum stress time, and the hot extremum stress time is always greater than the penetration time.

On the other hand, under the condition of hot shock there is no point of intersection between the temperature-wave penetration time and the extremum stress time, as shown in Fig. 5. This is because the maximum thermal tensile stress appears just at the center of the sphere under the condition of hot shock, where the penetration time is always faster than the extremum stress time. Therefore, the failure of a ceramic sphere during hot shock can theoretically occur at any time before its extremum stress time, i.e. in the initial and regular regimes of the heat conduction of the ceramics.

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