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STATISTICAL ANALYSIS OF MICROSCALE GAS FLOWS AND STOCK PRICE CHANGES

Jing Fan

Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China

Abstract: First, recent studies on the information preservation (IP) method, a particle approach for low-speed micro-scale gas flows, are reviewed. The IP method was validated for benchmark issues such as Couette, Poiseuille and Rayleigh flows, compared well with measured data for typical internal flows through micro-channels and external flows past micro flat plates, and combined with the Navier-Stokes equations to be a hybrid scheme for subsonic, rarefied gas flows. Second, the focus is moved to the microscopic characteristic of China stock market, particularly the price correlation between stock deals. A very interesting phenomenon was found that showed a reverse transition behaviour between two neighbouring price changes. This behaviour significantly differs from the transition rules for atomic and molecular energy levels, and it is very helpful to understand the essential difference between stock markets and nature.

Keywords: micro-scale gas flow, information preservation method, stock price change, inverse transition

1. INTRODUCTION

There are essentially two approaches to model natural and social phenomena. One is macroscopic and deterministic, such as the Navier-Stokes equations derived from the conservation laws and the Newtonian constitutive relation, and another is microscopic and probabilistic, such as the Boltzmann description of the chaotic motion of molecules. Micro-scale gas flows and stock price changes are this paper's theme. They are apparently irrelevant: the interest in the former mainly arises from micro-electro-mechanical systems (MEMS), whereas the latter traditionally belongs to the economic kingdom. In our studies, however, they share the same microscopic and statistical approach. For the micro-scale gas flows the molecular trajectories and collisions are tracked time step by step, and macroscopic physical quantities such as the density, pressure, and velocity of a flow field, as well as the surface pressure and shear stress, are obtained through statistical average. For stock price changes every deal is analyzed based on the high frequency trade data available, and the statistical characteristics of a sequence consisting of the price changes between two neighboring deals are drawn when the number of deals is large enough.

2. ADVANCEMENT IN THE INFORMATION PRESERVATION METHOD

There is a great interest to understand in detail the aerodynamics of MEMS. The characteristic length scale of MEMS is often comparable to the mean free path of molecules, and the Knudsen number, $Kn = \lambda/L$, is no longer small enough to be negligible, where λ is the mean free path of molecules, and L

is the characteristic length of flow. Molecular-based numerical schemes, such as the direct simulation Monte Carlo (DSMC) method [1], are more physically appropriate for this kind of gas flows where non-continuum, rarefied gas effect becomes important. In DSMC, a relatively small number of model molecules that is stored in a computer are used to represent the large number of molecules in real gas flows. The molecules move through physical space and undergo collisions appropriate to the local flow conditions. Macroscopic observable quantities, such as density, velocity, temperature, and surface fluxes, are obtained by statistically averaging molecular properties. The results are therefore inherently accompanied with statistical noise due to finite sampling. A signal-to-noise ratio may be written as [2]

$$\phi \equiv \frac{u}{\delta u} = Ma\sqrt{\gamma N} \tag{1}$$

where u is the characteristic velocity of flow, δu is the statistical fluctuation, M is the Mach number, γ is the specific heat ratio, and N is the sample size. Micro-devices often operate at low M, e.g. a typical flow velocity in micro-channel experiments [3] being about 0.2 m/s that corresponds to Ma of 10^{-4}. Relation (1) indicates that for $\phi = 10$, N has to be 10^{10}. Such an enormous sample size is extremely time-consuming and beyond the capabilities of current computers.

There is a consideration[2] that for compressibility effects to be negligible at low Mach numbers, relatively small statistical errors may be obtained by performing simulations at an increased Ma to a level where compressibility are still negligible. However, this is infeasible for rarefied gas flows interested in MEMS, where the similarity parameters, Ma and Kn, must be satisfied simultaneously. A clear demonstration of this fact was provided by Millikan's measurements of drag of a small sphere over the entire flow regime [4]. The fitting formula of the measured data is dependent on two similarity parameters, Re and Kn. They are equivalent to Kn and Ma because of the relation [5]

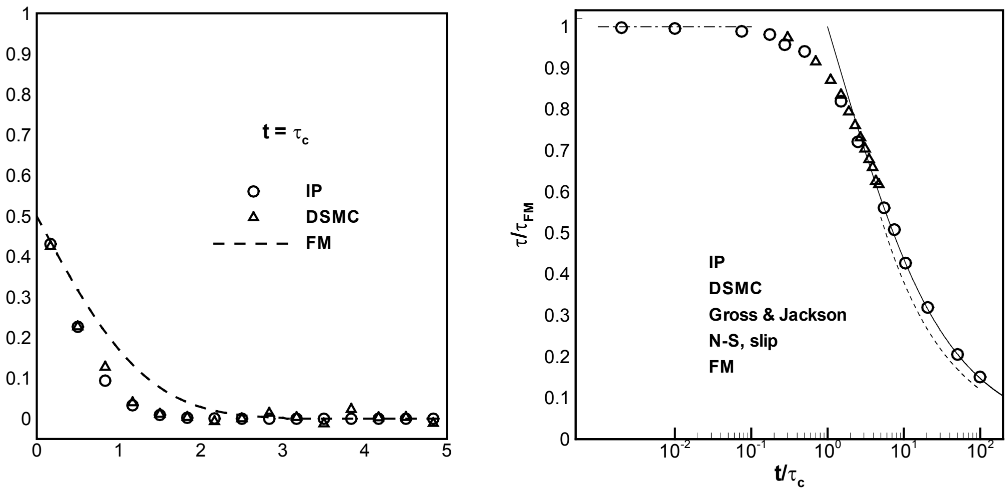
$$Kn \sim Ma / Re \tag{2}$$

An information preservation (IP) technique has been proposed to overcome the statistical noise in DSMC[6]. IP assigns each simulated molecule not only the thermal velocity, but also the physical information about the density, velocity and temperature. The thermal velocity is used to compute molecular motion following the same steps as DSMC [1], while the physical information is transported directly through the real molecular motion. Such a transport manner prevails over the entire Knudsen regime, in contrast with the linear constitutive relations used in the Navier-Stokes equations, which are accurate only in the continuum regime. Meanwhile, macroscopic quantities computed from the physical information remove the statistical noise inherent in DSMC owing to the randomness of the thermal velocity.

The IP method was firstly validated in benchmark issues, namely Couette, Poiseuille, and Rayleigh flows[6,7], which showed very good agreement with the exact solutions at the continuum and free molecular limits, and with the experimental, DSMC, linearized Boltzmann results in the transition regime. In the Rayleigh flow, the impulsive motion of the plate induced an unsteady gas flow. For a time comparable with the mean collision time, the DSMC method was employed to give a benchmark solution. To reduce the DSMC statistical scatter to a smaller level than the characteristic velocity u_W of 1m/s, an enormous sample size of 2×10^8 was used. It took about 180 CPU hours on a DEC α server 1 000A, about 3×10^4 times that required by the IP method. Comparison of the IP and DSMC velocity profiles at

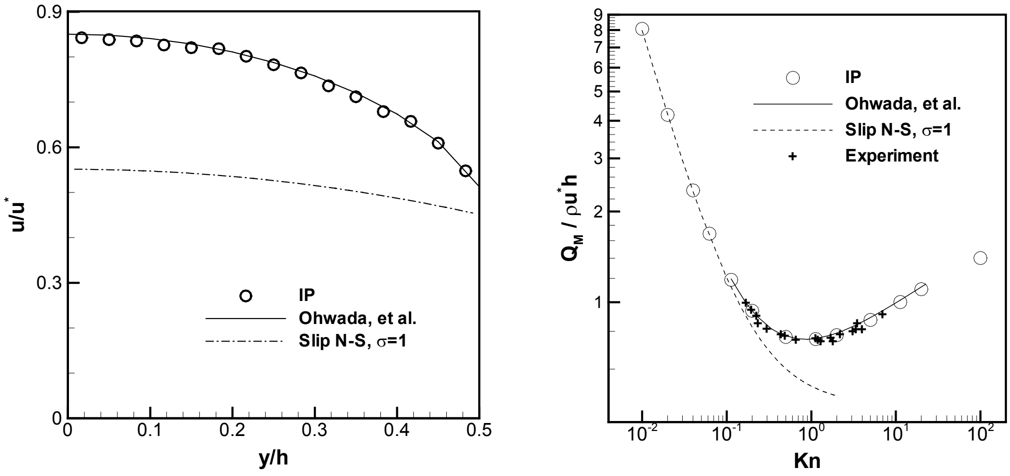
$t = \tau_c$ (τ_c : mean collision time) is shown in Fig. 1, with the collisionless solution denoted as FM for reference. A satisfactory agreement was obtained between the IP and DSMC results, though some statistical fluctuation is still seen in the latter. Figure 1 also shows the relation of the normalized surface shear stress versus time given by the IP and other methods. The IP results agree quite well with the collisionless solution at $t \ll \tau_c$ with the DSMC results at $t \sim \tau_c$, and with the slip N-S solution at $t > 5\tau_c$.

Figures 2 shows the velocity profiles obtained by the IP method, the linearized Boltzmann equation [8], and the slip N-S equation at Knudsen number of $2/\sqrt{\pi}$ based on the channel height. The IP profiles compare well with the numerical Boltzmann solution, whereas the slip N-S solutions far deviates from the



(a) velocity profiles at $t = \tau_c$ (b) drag versus time

Fig.1 Comparison of the IP and DSMC velocity and drag profiles for the Rayleigh flow [6,7]



(a) velocity profiles at $Kn = 2/\sqrt{\pi}$ (b) mass flux versus Kn

Fig.2 Comparison of the IP velocity and mass flux profiles, with the measured data of Dong, the linearized Boltzmann solutions of Ohwada et al., and the slip N-S solutions for the Poiseuille flow [5,6] two others as Kn increases. Figure 2 also shows the normalized relations of the mass flux Q_M with the Knudsen number given by the IP method, the linearized Boltzmann equation [8], experiment [9], and the N-S equation with the slip boundary condition, respectively. There is a minimum mass flux at some intermediate Knudsen number that is the famous Knudsen minimum or the Knudsen paradox. The confirmation by the present IP calculation of the Knudsen minimum and its excellent agreement with the numerical solution of the linearized Boltzmann equation and experimental data near this minimum shows clearly the fitness of the IP technique in predicting fine flow characteristics in the transition regime.

In micro-channel experiments performed by Pong et al.[10], Shih et al. [11], Arkilic et al.[12] and Arkilic[13], respectively, the channel width was much larger than the height. This made the span-wise influence neglected, and the flows were simplified as two-dimensional. The Knudsen number at the outlet Kn_0 indicated that the experimental conditions[10~13] were in the slip and transition regimes, respectively. Stream-wise pressure distributions and mass fluxes through micro-channels given by the IP method [14] agreed well with experimental data measured by Pong et al.[10], Shih et al. [11], Arkilic et al.[12] and Arkilic[13], respectively. Comparison of the IP results with the measured data of Shih et al. [14] is shown in Fig.3.

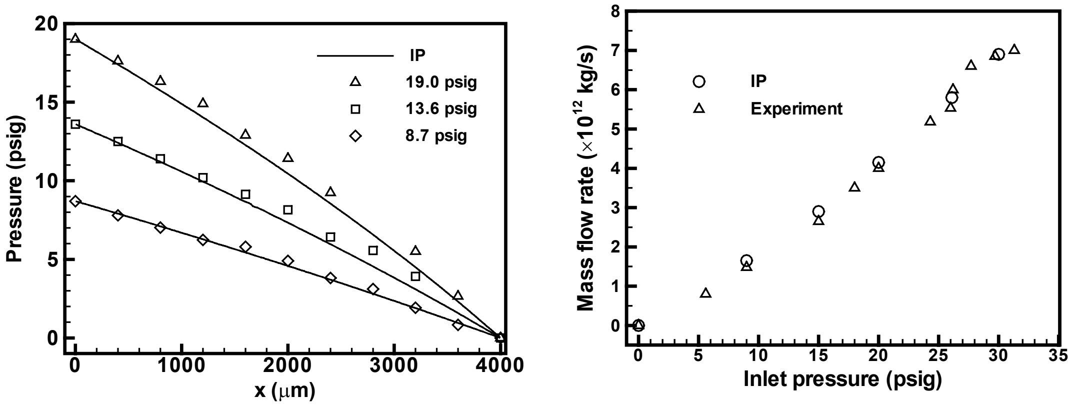


Fig.3 Comparison of the IP and measured pressure distributions in the streamwise direction and mass flow rate versus the inlet pressure for micro-channel flows [14]

Sun, Boyd & Candler [15] compared the drag coefficient of flat plates given by IP and experiments [16,17] that agreed well each other. They also developed an additional energy transfer model to describe the energy flux in IP calculations, which was verified for thermal Couette flows [18]. Recently, they combined the IP method with the N-S equations [19] to investigate airfoil aerodynamics at a Mach number of 0.2 and a Reynolds number between 1 and 200 [20]. Their studies showed that a flat plate having a thickness ratio of 5% had a better aerodynamic performance than conventional streamlined airfoils, and that there was a minimum lift slope for the plate airfoil at the Reynolds number of 10 [20].

3. REVERSE TRANSITION BEHAVIOR OF PRICE CHANGE IN CHINA STOCK MARKETS

In a modern society quite a lot of fortune is deposited in stock markets. It is the most interested for investors to understand the statistics of price changes that determine losses or gains. Since high-frequency intraday data available and easy to access, there are more and more so-called rocket scientist coming into this area. Previous studies revealed many very interesting phenomena such as the scaling behavior of S&P 500 index [21], the turbulent cascades for US dollars-German mark exchange rates [22], and an explicit Fokker-Planck equation which modeled precisely the empirical evolution of US dollars-German mark exchange rates [23]. Because these studies were based on the time-averaged or stock-averaged data, they may not reflect the microscopic characteristic for a stock deal, however.

In a microscopic view, stock markets are a quantum system, for stock price changes are usually integer times of the smallest currency unit of the country where a stock market locates. This resembles the transition style of atomic or molecular energy levels. It is very interesting to know whether the transition rules between stock prices and particle energy levels are similar or not.

Consider a sequence $\{\Delta p_i\}$ consisting of the price change between neighboring deals at Shanghai and Shenzhen stock markets [24], where $\Delta p_i = p_i - p_{i-1}$, and p_i denotes the transaction price of the ith deal of a stock. Figure 4 shows the conditional probability density function $f(\Delta p_i | \Delta p_{i+1})$ and $f(\Delta p_i | \Delta p_{i+2})$ at different values of Δp_i (the unit is fen, the smallest unit of China currency.) for the Shenzhen Airport Stock from November 6, 2001 to August 29, 2003. They display the dependence of the probability of Δp_{i+k} on the event Δp_i occurred previously. Two very interesting phenomena may be seen clearly. One is the significantly larger probability for Δp_{i+1} to exactly rebound from Δp_i than other choices except $\Delta p_{i+1} = 0$. Such a reverse transition behavior is in striking contrast with the selection rules of atomic and molecular energy levels that may alter only +1 or –1, i.e. around the neighboring levels [25]. Another is the weak dependence of $f(\Delta p_i | \Delta p_{i+2})$ on Δp_i, in comparison with the strong correlation between Δp_i and Δp_{i+1}. Further calculation of $f(\Delta p_i | \Delta p_{i+5})$ exhibits that Δp_i and Δp_{i+5} are almost uncorrelated. These microscopic characteristics provide foundations for further understanding the difference between stock markets and nature, and for statistical simulation of stock market [26].

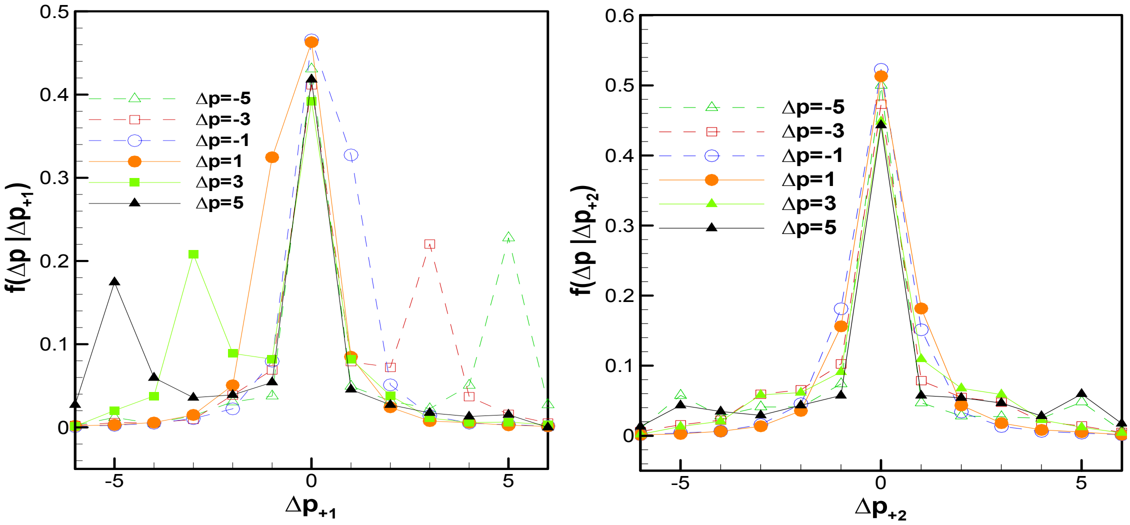


Fig.4 The conditional probability density functions of price changes between neighboring deals of the Shenzhen Airport Stock from November 6, 2001 to August 29, 2003

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