

非平整、多孔介质海底上波浪传播的复合方程¹⁾

黄 虎

(上海大学上海市应用数学和力学研究所, 上海 200072)

(中国科学院力学研究所非线性力学国家重点实验室, 北京 100080)

摘要 为了反映近岸区域实际存在的多孔介质海底效应, 并且考虑到波浪在刚性海底上传播模型的最新研究进展, 运用 Green 第二恒等式建立了波浪在非平整、多孔介质海底上传播的复合方程. 假设水深和孔隙介质海底层厚度均由两种分量组成: 慢变分量, 其水平变化的长度尺度大于表面波的波长; 快变分量, 其水平变化的长度尺度与表面波的波长等阶, 但其振幅小于表面波的振幅. 另外, 多孔介质层下部边界的快变分量比水深的快变分量小 1 个量级. 针对水体层和孔隙介质层, 选择 Green 第二恒等式方法给出了波浪传播和渗透的复合方程, 它在交接面上满足压力和垂直渗透速度的连续性条件, 可充分考虑波数变化的一般连续性, 并包含了某些著名的扩展型缓坡方程.

关键词 多孔介质, 非平整海底, 复合方程, Green 第二恒等式, 扩展型缓坡方程

引 言

近岸水域存在着广泛分布的可渗透多孔介质海底, 这将直接影响波浪传播过程中所发生的各种现象和能量分布. 以往提出的多种近岸波浪传播模型^[1~6], 尽管从形式上看越来越复杂和高级, 但是它们大都假定波浪在刚性海底上传播, 不发生渗透运动, 因而往往是不充分的. 较之于刚性海底, 多孔介质海底以两种主要方式作用于波浪场: 波浪衰减增加, 波速改变. 由于波浪和孔隙介质海底相互作用理论构造的成功是否在很大程度上有赖于所给定的经验公式及其参数的精确性, 并不单由具有一般意义上的非线性作用机制来决定, 因此, 目前关注多孔介质海底效应的主流工作还是从属于线性波的理论框架内^[6~10], 仅有不多的研究涉及非线性波与多孔介质海底的相互作用^[11,12]. 另外, 线性波理论是弱非线性水波理论的基础和出发点. 为了处理问题的简单和方便, 以往提出的模型可以假定海底是缓变、不透水的^[1]; 或虽然包含陡坡海底效应, 但海底依然保持刚性^[2~5]; 以及考虑到了海底的缓变、陡变性和多孔介质特性, 但假设可透水层位于缓变的海底^[6]. 至今, 据作者所见, 所有提出的多孔介质海底上波浪传播方程^[6~12]均忽略波数的变化, 而波数直接关于于波幅的变化(即使为常水深), 由此将引起波浪传播特性的非连续变化^[13], 所得到

的模型理论是不完善的.

非平整海底的变化呈多样性, 可以诱发波浪反射和波浪与海底的非线性共振, 人们已通过实验发现了一种典型的 Bragg 共振反射现象^[14], 这将有助于解释实际海底地形的复杂演变过程. 多孔介质海底显著地减小了波浪透射和反射的能量, 据此在近岸水域中构造一种人工多孔介质坝, 对于工程实践将是一种积极的探索和应用. 为了较完整地刻画波浪在近岸水域传播的多种现象, 体现某种综合性, 本文在典型的海底地形条件下, 考虑海底的多孔介质效应, 提出了一种更为一般、实际的波浪传播和渗透的复合方程.

1 非平整、多孔介质海底

现在考虑一列水波在非平整、多孔介质海底上的传播. 令 $\boldsymbol{x} = (x, y)$ 表示水平坐标, z 表示垂直向上坐标. $z = 0$ 为无扰动的自由表面, $z = -h(\boldsymbol{x})$ 为水深. 假设水体无黏、无旋. 水深 h 和孔隙介质层厚度 h^p 分别由 2 个和 3 个分量构成(如图 1 所示)

$$h = h_{01}(\boldsymbol{x}) - h_1(\boldsymbol{x}) \quad (1)$$

$$h^p = h_{02}(\boldsymbol{x}) + h_1(\boldsymbol{x}) - h_2(\boldsymbol{x}) \quad (2)$$

其中 h_{01} 和 h_{02} 代表慢变分量, h_1 和 h_2 代表快变分量, 它们满足下列量级关系

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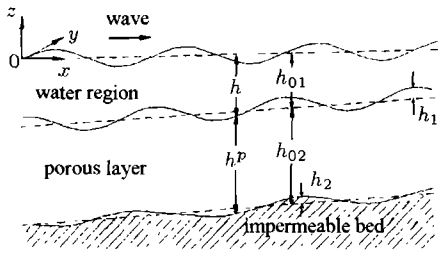


图 1 水深分量和多孔介质层分量的定义
Fig.1 Definition of components for water depth and porous layer

$$O(\beta) = O\left(\frac{\nabla h_{01}}{kh_{01}}\right) = O\left(\frac{\nabla h_{02}}{kh_{02}}\right) = \frac{\lambda}{A} \ll 1 \quad (3)$$

$$\left. \begin{aligned} O(\delta) &= O(kh_1) \\ O\left(\frac{h_1}{A}\right) &= O(\delta^2) \\ O\left(\frac{\nabla h_1}{kh_1}\right) &\approx O(1) \end{aligned} \right\} \quad (4)$$

$$\left. \begin{aligned} O(\gamma^2) &= O(kh_2) \\ O\left(\frac{h_2}{A}\right) &= O(\gamma^3) \\ O\left(\frac{\nabla h_2}{kh_2}\right) &\approx O(1) \end{aligned} \right\} \quad (5)$$

$$O(\beta) \approx O(\delta) \approx O(\gamma) \quad (6)$$

其中, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$; A 表示 h_{01} 和 h_{02} 变化的水平长度尺度; β, δ 和 γ 均表示各分量变化的调制小参数; $\lambda = 2\pi/k$ 为表面水波的波长. 因此, 式 (1) 和 (2) 可重新写为

$$h = h_{01}(\beta \mathbf{x}) - \delta h_1(\mathbf{x}) \quad (7)$$

$$h^p = h_{02}(\beta \mathbf{x}) + \delta h_1(\mathbf{x}) - \gamma^2 h_2(\mathbf{x}) \quad (8)$$

2 控制方程和边界条件

针对位于多孔介质层之上的水体层, 由不可压缩、无黏流体的无旋运动理论, 可得到关于速度势 ϕ 的控制方程和其边界条件

$$\nabla^2 \phi + \phi_{zz} = 0 \quad (-h_{01} \leq z \leq 0) \quad (9)$$

$$\phi_{tt} + g\phi_z = 0 \quad (z = 0) \quad (10)$$

$$\begin{aligned} \phi_z &= -\nabla h_{01} \cdot \nabla \phi + \delta \nabla \cdot (h_1 \nabla \phi) - \delta h_1 \nabla \phi_z \\ (\nabla h_{01} - \delta \nabla h_1) + w^w & \quad (z = -h_{01}) \quad (11) \end{aligned}$$

其中, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, 方程 (11) 是将底部边界条件在 $z = -h_{01}$ 处展开到 $O(\delta\beta, \delta^2)$ 而得到的, w^w 表示流体层与多孔介质层交界面上的垂直渗透速度, 构成整个系统解的一部分. 流体层内的压力 p^w 可表示为

$$p^w = -\rho(\phi_t + gz) \quad (-h_{01} \leq z \leq 0) \quad (12)$$

其中 ρ 为流体密度.

假定在多孔介质层内的渗流运动为无旋的, 则可引入一个渗流速度势 φ . φ 的控制方程和其边界条件如下

$$\nabla^2 \varphi + \varphi_{zz} = 0 \quad (-h_{01} - h_{02} \leq z \leq -h_{01}) \quad (13)$$

$$\begin{aligned} \varphi_z &= -\nabla h_{01} \cdot \nabla \varphi + \delta \nabla \cdot (h_1 \nabla \varphi) - h_1 \nabla \varphi_z \\ (\delta \nabla h_{01} - \delta^2 \nabla h_1) + w^p & \quad (z = -h_{01}) \quad (14) \end{aligned}$$

$$\left. \begin{aligned} \varphi_z &= -\nabla(h_{01} + h_{02}) \cdot \nabla \varphi + \gamma^2 \nabla \cdot (h_2 \nabla \varphi) \\ & \quad (z = -h_{01} - h_{02}) \end{aligned} \right\} \quad (15)$$

其中, w^p 表示上、下层交接面上的垂直渗透速度. 方程 (14) 和 (15) 是将多孔介质层的上、下部边界条件分别在 $z = -h_{01}$ 处和 $z = -h_{01} - h_{02}$ 处展开到 $O(\delta\beta, \delta^2, \gamma^2)$ 而得到的. 多孔介质层压力 p^p , 可表示为 [15]

$$p^p = -\rho \left(\frac{\tau}{n} \varphi_t + gz + f \frac{\omega}{n} \varphi \right) \quad (-h_{01} - h_{02} \leq z \leq h_{01}) \quad (16)$$

其中, τ 是惯性系数, f 是摩擦系数, n 是孔隙度, ω 是角频率.

在交接面处, 压力和渗透速度必须是连续的

$$\left. \begin{aligned} p^w &= p^p \\ w^w &= w^p \quad (z = -h_{01}) \end{aligned} \right\} \quad (17)$$

3 复合方程模型的建立

水体层和多孔介质层的速度势函数 ϕ 和 φ 可分别表示为

$$\phi(\mathbf{x}, z, t) = f^w(\mathbf{x}, z) \tilde{\phi}(\mathbf{x}, t) + \text{非传播模式} \quad (18)$$

$$\varphi(\mathbf{x}, z, t) = f^p(\mathbf{x}, z) \tilde{\varphi}(\mathbf{x}, t) + \text{非传播模式} \quad (19)$$

在水平海底 (即 $\nabla h_{01} = \nabla h_{02} = 0$) 的假定条件下, 可得到垂直分布函数 f^w, f^p 和色散关系的表达式 [7]

$$f^w = \frac{1}{A} [\cosh kh_{02} \cosh k(h_{01} + z) + \alpha \sinh kh_{02} \sinh k(h_{01} + z)] \quad (20)$$

$$f^p = \frac{1}{A} \alpha \cosh k(h_{01} + h_{02} + z) \quad (21)$$

$$\omega^2 = gk \frac{\tanh kh_{01} + \alpha \tanh kh_{02}}{1 + \alpha \tanh kh_{01} \tanh kh_{02}} \quad (22)$$

其中

$$\left. \begin{aligned} A &= \cosh kh_{02} \cosh kh_{01} \cdot \\ &\quad (1 + \alpha \tanh kh_{02} \tanh kh_{01}) \\ \alpha &= \frac{\tau}{\tau + i f} \end{aligned} \right\} \quad (23)$$

按照 Smith 和 Sprinks^[16] 采用的方法, 我们将 Green 第二恒等式分别应用于 ϕ 和 φ 的传播分量, 则得到

$$\int_{-h_{01}}^0 f^w \phi_{zz} dz - \int_{-h_{01}}^0 \phi f_{zz}^w dz = [f^w \phi_z - \phi f_z^w]_{-h_{01}}^0 \quad (24)$$

$$\int_{-h_{01}-h_{02}}^{-h_{01}} f^p \varphi_{zz} dz - \int_{-h_{01}-h_{02}}^{-h_{01}} \varphi f_{zz}^p dz = [f^p \varphi_z - \varphi f_z^p]_{-h_{01}-h_{02}}^{-h_{01}} \quad (25)$$

分别对方程 (24) 和 (25) 中的各项做积分运算, 可得

$$\begin{aligned} -\nabla \cdot \int_{-h_{01}}^0 f^w \nabla(\tilde{\phi} f^w) dz + \nabla h_{01} \cdot [f^w \nabla(\tilde{\phi} f^w)]_{z=-h_{01}} + \int_{-h_{01}}^0 [\nabla f^w \cdot \nabla(\tilde{\phi} f^w)] dz - \tilde{\phi} \int_{-h_{01}}^0 f^w f_{zz}^w dz = \\ -\frac{1}{g} \tilde{\phi}_{tt} [(f^w)^2]_{z=0} - \tilde{\phi} [f^w f_z^w]_{z=0} - [f^w]_{z=-h_{01}} \cdot \left\{ -\nabla h_{01} \cdot \nabla(\tilde{\phi} f^w) + \delta [\nabla h_{01} \cdot \nabla(\tilde{\phi} f^w) + h_1 (f^w \nabla^2 \tilde{\phi} + \right. \\ \left. 2\nabla f^w \cdot \nabla \tilde{\phi}) - h_1 \frac{\partial f^w}{\partial z} \nabla \tilde{\phi} \cdot (\delta \nabla h_{01} - \delta^2 \nabla h_1) + w^w \right\}_{z=-h_{01}} + \tilde{\phi} [f^w f_z^w]_{z=-h_{01}} + O(\delta \beta^2, \delta^2 \beta) \end{aligned} \quad (26)$$

$$\begin{aligned} -\nabla \cdot \int_{-h_{01}-h_{02}}^{-h_{01}} f^p \nabla(\tilde{\varphi} f^p) dz - \nabla h_{01} \cdot [f^p \nabla(\tilde{\varphi} f^p)]_{z=-h_{01}} + \nabla(h_{01} + h_{02}) \cdot [f^p \nabla(\tilde{\varphi} f^p)]_{z=-h_{01}-h_{02}} + \\ \int_{-h_{01}-h_{02}}^{-h_{01}} [\nabla f^p \cdot \nabla(\tilde{\varphi} f^p)] dz - \tilde{\varphi} \int_{-h_{01}-h_{02}}^{-h_{01}} f^p f_{zz}^p dz = [f^p]_{z=-h_{01}} \left\{ -\nabla h_{01} \cdot \nabla(\tilde{\varphi} f^p) + \right. \\ \left. \delta [\nabla h_1 \cdot \nabla(\tilde{\varphi} f^p) + h_1 (f^p \nabla^2 \tilde{\varphi} + 2\nabla f^p \cdot \nabla \tilde{\varphi}) - h_1 \frac{\partial f^p}{\partial z} \nabla \tilde{\varphi} \cdot (\delta \nabla h_{01} - \delta^2 \nabla h_1) + w^p \right\}_{z=-h_{01}} - \\ \tilde{\varphi} [f^p f_z^p]_{z=-h_{01}} + \{f^p [\nabla(h_{01} + h_{02}) \cdot \nabla(\tilde{\varphi} f^p) - \gamma^2 f^p \nabla \cdot (h_2 \nabla \tilde{\varphi})]\}_{z=-h_{01}-h_{02}} + \\ \tilde{\varphi} [f^p f_z^p]_{z=-h_{01}-h_{02}} + O(\delta \beta^2, \delta^2 \beta, \beta \gamma^2) \end{aligned} \quad (27)$$

其中

$$\nabla f^w = \frac{\partial f^w}{\partial h_{01}} \nabla h_{01} + \frac{\partial f^w}{\partial h_{02}} \nabla h_{02} + \frac{\partial f^w}{\partial k} \nabla k, \quad \nabla f^p = \frac{\partial f^p}{\partial h_{01}} \nabla h_{01} + \frac{\partial f^p}{\partial h_{02}} \nabla h_{02} + \frac{\partial f^p}{\partial k} \nabla k$$

由方程 (26) 和 (27), 并结合方程 (17), 可消去 w^w 和 w^p , 即为

$$\begin{aligned} [f^p]_{z=-h_{01}} \left[\frac{1}{g} \tilde{\phi}_{tt} (f^w)^2 + \tilde{\phi} f^w f_z^w \right]_{z=0} - [f^p]_{z=-h_{01}} \nabla \cdot \int_{-h_{01}}^0 f^w \nabla(\tilde{\phi} f^w) dz - \\ [f^w]_{z=-h_{01}} \nabla \cdot \int_{-h_{01}-h_{02}}^0 f^p \nabla(\tilde{\varphi} f^p) dz + \nabla h_{01} \cdot \{f^w f^p [\nabla(\tilde{\phi} f^w) - \nabla(\tilde{\varphi} f^p)]\}_{z=-h_{01}} + \\ [f^p]_{z=-h_{01}} \int_{-h_{01}}^0 [\nabla f^w \cdot \nabla(\tilde{\phi} f^w)] dz + [f^w]_{z=-h_{01}} \int_{-h_{01}-h_{02}}^0 [\nabla f^p \cdot \nabla(\tilde{\varphi} f^p)] dz - \\ \tilde{\phi} [f^p]_{z=-h_{01}} \int_{-h_{01}}^0 f^w f_{zz}^w dz - \tilde{\varphi} [f^w]_{z=-h_{01}} \int_{-h_{01}-h_{02}}^0 f^p f_{zz}^p dz + \\ [f^w]_{z=-h_{01}} \nabla(h_{01} + h_{02}) \cdot [f^p \nabla(\tilde{\varphi} f^p)]_{z=-h_{01}-h_{02}} + [f^w f^p]_{z=-h_{01}} \left\{ -\nabla h_{01} \cdot [\nabla(\tilde{\phi} f^w) - \nabla(\tilde{\varphi} f^p)] - \tilde{\phi} f_z^w + \right. \end{aligned}$$

$$\begin{aligned} & \tilde{\varphi} f_z^p - h_1(\delta \nabla h_{01} - \delta^2 \nabla h_1) \cdot \left(\frac{\partial f^w}{\partial z} \nabla \tilde{\phi} - \frac{\partial f^p}{\partial z} \nabla \tilde{\varphi} \right) + \delta [\nabla h_1 \cdot (\nabla(\tilde{\phi} f^w) - \nabla(\tilde{\varphi} f^p))] + \\ & h_1(f^w \nabla^2 \tilde{\phi} + 2 \nabla f^w \cdot \nabla \tilde{\phi} - f^p \nabla^2 \tilde{\varphi} - 2 \nabla f^p \cdot \nabla \tilde{\varphi}) \Big|_{z=-h_{01}} - [f^w]_{z=-h_{01}} \{ f^p [\nabla(h_{01} + h_{02}) \cdot \nabla(\tilde{\varphi} f^p) - \\ & \gamma^2 f^p \nabla \cdot (h_2 \nabla \tilde{\varphi})] + \tilde{\varphi} f_z^p f_z^p \Big|_{z=-h_{01}-h_{02}} = O(\delta \beta^2, \delta^2 \beta, \beta \gamma^2) \end{aligned} \quad (28)$$

方程 (26) 与 (27), 或方程 (28), 即构成了近岸水域波浪在多孔介质非平整海底上传播和渗透的复合方程, 它们可以精确刻画多孔介质海底上、下部边界慢变和快变的一般地形特征, 具有广泛的适用性. 它们包含了下述具有代表性的特殊方程 (没有考虑波数 k 的一般变化性):

(1) 如果不考虑多孔介质海底的效应, 即 $h_{02} = h_2 = 0$, 则可对方程 (26) 简化, 得到 Dingemans^[17] 提出的波浪在刚性非平整海底上传播的扩展型缓坡方程

$$\begin{aligned} & \frac{\partial^2 \tilde{\phi}}{\partial t^2} - \nabla \cdot [C C_g \nabla \tilde{\phi}] + (\omega^2 - k^2 C C_g) \tilde{\phi} + \\ & \frac{\delta}{2 \cosh^2 k h_{01}} [\tilde{\phi} \omega^2 (2k^2 h_{01} h_1 + h_{01} \nabla^2 h_1) + \\ & 2 \nabla \cdot (h_1 \nabla \tilde{\phi}) (\omega^2 h_{01} - g)] = 0 \end{aligned} \quad (29)$$

其中, $C = \frac{\omega}{k}$, $C_g = \frac{\partial \omega}{\partial k}$. Dingemans 方程 (29) 包含了比 Kirby 方程^[2] 更多的项, 可以看作是对后者的一个重要补充. 需要指出, 最初的 Dingemans 方程涉及一个关于 $\nabla^2 h_1$ 项系数的符号运算错误, 现在对此做了修正.

(2) 如果忽略多孔介质层下部边界快变的地形特征, 即 $h_2 = 0$, 则由方程 (28) 得到 (只保留部分 $O(\delta)$ 项)

$$\begin{aligned} & [f^p]_{z=-h_{01}} \left[\frac{1}{g} \tilde{\phi}_{tt} (f^w)^2 + \tilde{\phi} f^w f_z^w \right]_{z=0} - \\ & [f^p]_{z=-h_{01}} (\nabla^2 \tilde{\phi}) \int_{-h_{01}}^0 (f^w)^2 dz - \\ & [f^w]_{z=-h_{01}} (\nabla^2 \tilde{\varphi}) \int_{-h_{01}-h_{021}}^0 (f^p)^2 dz - \\ & \tilde{\phi} [f^p]_{z=-h_{01}} \int_{-h_{01}}^0 f^w f_{zz}^w dz - \\ & \tilde{\varphi} [f^w]_{z=-h_{01}} \int_{-h_{01}-h_{02}}^0 f^p f_{zz}^p dz + \\ & [f^w f^p (-\tilde{\phi} f_z^w + \tilde{\varphi} f_z^p)]_{z=-h_{01}} - \\ & \tilde{\varphi} [f^w]_{z=-h_{01}} [f^p f_z^p]_{z=-h_{01}-h_{02}} + \\ & \delta \{ f^w f^p [f^w \nabla \cdot (h_1 \nabla \tilde{\phi}) - \\ & f^p \nabla \cdot (h_1 \nabla \tilde{\varphi})] \Big|_{z=-h_{01}} = 0 \end{aligned} \quad (30)$$

从方程 (30) 可直接得到如下的广义缓坡方程^[6], 它自身可包括 4 种现有的波浪模式理论, 其中就含有著名的 Kirby 方程^[2]

$$\begin{aligned} & \frac{1}{g} (\tilde{\phi}_{tt} - \omega^2 \tilde{\phi}) - \nabla \cdot (\mu \nabla \tilde{\phi}) - \kappa^2 \mu \tilde{\phi} + \\ & \frac{\delta \cosh^2 k h_{02}}{A^2} (1 - \alpha) \nabla \cdot (h_1 \nabla \tilde{\phi}) = 0 \end{aligned} \quad (31)$$

其中, $\mu = \int_{-h_{01}}^0 (f^w)^2 dz + \frac{1}{\alpha} \int_{-h_{01}-h_{021}}^{-h_{01}} (f^p)^2 dz$. 采用变换 $\tilde{\phi} = \bar{\phi} e^{-i\omega t}$, 可将方程 (31) 化为与时间无关的缓坡方程.

目前一般是围绕着 Bragg 共振反射这一典型近岸波浪场现象, 针对某一特定的规则实验地形或解析地形进行对非平整、多孔介质海底模型的数值计算^[2,6], 以确定波浪传播的反射、透射系数和波幅的空间分布, 发现较之于刚性海底均有显著的不同. 本文给出的复合方程 (28) 提供了一个较为一般的线性波理论框架, 可刻画近岸波浪场的真实特征.

4 结果与讨论

为了在波浪传播模型中包含多孔介质海底效应, 并考虑到近岸海底地形变化的复杂性, 本文把引入的多孔介质海底的上、下部边界处理成“慢变和快变相组合”的一个较为自然的海底地形, 运用 Green 第二恒等式建立了波浪传播和渗透的复合方程模式, 可充分保证波数 k 变化的一般连续性, 并且证明它包含了某些著名的扩展型缓坡方程.

波浪传播和渗透模型的有效建立, 有赖于对波浪传播特征和在多孔介质中物质与能量输运规律认识的进一步提高. 我们将在充分反映波浪传播各种现象和机制的基础上, 例如波-流相互作用, 破碎波和耗散, 及时地吸收和总结在多孔介质中流体渗流运动的最新成果, 以期发展和完善波浪传播和渗透的复合方程模型.

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COMPOSITE EQUATIONS OF WATER WAVES OVER UNEVEN AND POROUS SEABED¹⁾

Huang Hu

(Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China)

(LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China)

Abstract The composite equations for water waves propagating over a porous uneven bottoms are derived from Green's second identity, which incorporates the effects of porous medium in the nearshore region and considers the advances in models of water waves propagation over rigid bottoms. Assuming that both water depth and thickness of the porous layer consist of two kind of components: The slowly varying component whose horizontal length scale is longer than the surface wave length, and the fast varying component with the horizontal length scale as the surface wave length. The amplitude of the fast varying component is, however, smaller than the surface wave length. In addition, the fast varying component of the lower boundary surface of the porous layer is one order of magnitude smaller than that of the water depth. By Green's second identity and satisfying the continuous conditions at the interface for the pressure and the vertical discharge velocity the composite equations are given for both water layer and porous layer, which can fully consider the general continuity of the variation of wave number and include some well-known extended mild-slope equations.

Key words porous medium, uneven bottoms, composite equations, Green's second identity, extended mild-slope equations

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