

振动荷载下饱和砂土液化区的发展*

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【摘要】 本文以两相介质理论为基础, 将饱和砂土简化为一维情况, 对饱和砂土液化区的范围做了一些理论探讨。结果表明, 液化区扩展随时间而逐渐变缓, 且固相密度越大, 发展越慢, 而 dp/dn 越大, 则液化区扩展越快。

【关键词】 饱和砂土 液化区扩展

Abstract In this paper, based on the two phase theory, the saturated soil is simplified as one dimensional model. Theoretical discussions on the expansion of liquefaction volume of the saturated soil are presented. It is shown that the expansion of liquefaction volume retards with time and will slow down with the increase of density. The liquefaction volume will also develop faster with the increase of dp/dn .

Key words saturated soil, liquefaction volume expansion

我们知道, 液化范围与液化灾害大小密切相关, 但对液化区的扩展只有少数研究者进行过定性的描述^[1], 到目前为止, 还未见有深入的理论或实验研究。这里只是对这个问题作探索性研究, 通过考察扩展过程的影响因素(外力作用强度和砂土本身特性参数等)和作用机理, 以便能预测振动荷载下液化的影响范围和扩展速度。

一、数学模型

因为是探讨力学机理, 将砂土简化为平面一维情形且受到轴向荷载作用, 砂土在某一点开始液化后, 将向周围扩展, 液化区内 $e=0, \sigma=0$; 液化区外 $e > 0, \sigma > 0$ 。假设液化区半径为 $r(t)$, 这时根据两相介质理论得到基本方程如下^{[2],[3]}:

液相质量守恒方程

$$\frac{\partial n_w}{\partial t} + \frac{\partial n_w V_w}{\partial x} = 0 \quad (1)$$

固相质量守恒方程

$$\frac{\partial(1-n)_s}{\partial t} + \frac{\partial(1-n)_s V_s}{\partial x} = 0 \quad (2)$$

液相动量守恒方程

$$\frac{\partial n_w V_w}{\partial t} + \frac{\partial n_w V_w^2}{\partial x} + n \frac{\partial P}{\partial x} = -H + \rho_w g \quad (3)$$

固相动量守恒方程

$$\frac{\partial(1-n)_s V_s}{\partial t} + \frac{\partial(1-n)_s V_s^2}{\partial x} + P \frac{\partial n}{\partial x} + \frac{\partial(1-n)_t}{\partial x} = H + (1-n)(\rho_s - \rho_s)g \quad (4)$$

式(1)~(4)中, n 为孔隙率; V_w, V_s 分别为水和砂的速度; P 为孔隙压力; t 为砂粒间接触应力; ρ_w, ρ_s 分别为水和砂的质量密度; H 为相间作用力, 设为:

$$H = Kn^2(V_w - V_s) \quad (5)$$

其中 $K = \mu/k, \mu$ 为水粘性系数, k 为渗透率。

由有效应力原理知: $\sigma = \sigma_e + P$ (6)

同时又已知: $\sigma = np + (1-n)\sigma_t$ (7)

σ_e 为法向有效应力, 由式(6)、式(7)消去 σ , 然后将 σ_t 代入公式(4), 得到

$$\frac{\partial(1-n)_s V_s}{\partial t} + \frac{\partial(1-n)_s V_s^2}{\partial x} + (1-n) \frac{\partial p}{\partial x} + \frac{\partial \sigma_e}{\partial x} = H + (1-n)(\rho_s - \rho_w)g \quad (8)$$

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现假设水和砂粒密度不变, 即 w, s , 为常数, $\partial n / \partial x = 0$, 基本方程就可重写为:

$$\left\{ \begin{aligned} \frac{\partial n}{\partial t} + \frac{\partial n V_w}{\partial x} &= 0 \\ \frac{\partial n}{\partial t} + \frac{\partial(1-n)V_s}{\partial x} &= 0 \\ \frac{\partial n V_w}{\partial t} + \frac{\partial n V_w^2}{\partial x} + n \frac{\partial p}{\partial x} &= -Kn^2(V_w - V_s) + w g \\ \frac{\partial(1-n)V_s}{\partial t} + \frac{\partial(1-n)V_s^2}{\partial x} &+ (1-n) \frac{\partial p}{\partial x} + \frac{\partial \epsilon}{\partial x} \\ &= Kn^2(V_w - V_s) + (1-n)(s - w)g \end{aligned} \right. \quad (9)$$

二、解析解

将式(9)中第三式对 x 求导并将第一式对 t 求导后代入, 然后将第四式对 x 求导, 并将第二式对 t 求导的结果代入, 最后将二者相加, 并忽略其中的牵移加速度项, 即 $\frac{\partial n V_w^2}{\partial x}, \frac{\partial(1-n)V_s^2}{\partial x}$, 同时注意到在已液化的区域有效应力为零, 可得出:

$$\frac{\partial^2 n}{\partial t^2} + \frac{1}{s-w} \frac{\partial^2 p}{\partial x^2} = 0 \quad (10)$$

设 $\frac{dp}{dn} = a, \quad b^2 = -\frac{a}{s-w} \quad (11)$

则有: $\frac{\partial^2 n}{\partial t^2} - b^2 \frac{\partial^2 n}{\partial x^2} = 0 \quad (12)$

初边界条件为:

$$\left\{ \begin{aligned} n|_{x=0} &= n_1(t) & n|_{x=2} &= n_1(t) \\ n|_{t=0} &= n_t(t) & n|_{t=0} &= n_t(t) \end{aligned} \right. \quad (13)$$

设 $n = N + n_1(t)$

则: $\frac{\partial^2 N}{\partial t^2} - b^2 \frac{\partial^2 N}{\partial x^2} = 0 \quad (14)$

初边界条件为:

$$\left\{ \begin{aligned} N|_{x=0} &= n_1(t) & N|_{x=2} &= n_2(t) \\ N|_{t=0} &= n_0 & N|_{t=0} &= 0 \end{aligned} \right. \quad (15)$$

求解式(14)、(15)可以得到:

$$n = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n x}{2}\right) + B_n \sin\left(\frac{n b t}{2}\right) \sin\left(\frac{n x}{2}\right) + n_1(t) \quad (16)$$

其中 $A_n = \frac{1}{n} n_0 \sin\left(\frac{n}{2}\right) d$
 $= -\frac{2n_0}{n} [\cos(n) - 1]$

$$B_n = \frac{2}{n b_0} n_0 \sin\left(\frac{n}{2}\right) d = 0$$

因此, n 又可写为:

$$n = \sum_0^{\infty} \left[-\frac{2n_0}{n} [\cos(n) - 1] \times \cos\left(\frac{n b t}{2}\right) \right] + n_1 \quad (17)$$

这样由方程式(9)的第一式求 V_w :

$$V_w = -\frac{1}{n} \frac{\partial n}{\partial t} dx \quad (18)$$

将式(17)代入后求得:

$$V_w = \frac{1}{n} \sum_0^{\infty} \frac{-2bn_0}{n} (\cos(n) - 1) \times \cos\left(\frac{n x}{2}\right) \cos\left(\frac{n b t}{2}\right) + n_1 x + h(t) \quad (19)$$

其中 $h(t)$ 为积分常数, 要由边界条件确定, 即需满足:

$$V_w|_{x=0} = V_{w1}(t) \quad V_w|_{x=2} = V_{w2}(t) \quad (20)$$

将式(19)代入(20)得:

$$\left\{ \begin{aligned} h(t) &= -f(0, t) + n_1(t) \times V_{w1}(t) \\ f(2, t) + h(t) &= V_{w2}(t) \end{aligned} \right. \quad (21)$$

这样就得到 h 与 t 的关系。

$$n_2 V_{w2} - n_1 V_{w1} = \sum_{k=1}^{\infty} \left[\frac{8bn_0}{(2k-1)} \right] \times \sin\left(\frac{(2k-1)bt}{2}\right) + 2n_1 \quad (22)$$

从(22)可以看出, 液化半宽 h 与时间 t 是非线性的关系。

三、算例与结论

根据上面给出的 h 与 t 的关系, 给定了其中的初始参数后得到图1所示的一个计算结果, 其中 $n_1 = n_2 = 0.8 - 0.2t, V_{w1}(t) = -V_{w2}(t) = 0.001t, w = 1000.0 \text{kg/m}^3, s = 2500.0 \text{kg/m}^3, n_0 = 0.8, b = 0.1, 0.2, 0.3, 0.4$ 。

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灾后反应未见有深入的研究成果。

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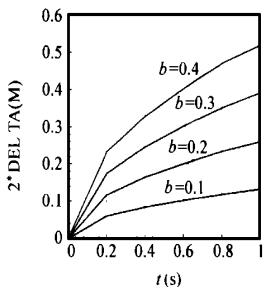


图 1 液化区宽度随时间的变化

结果表明, 液化区随时间开始增长很快, 然后逐渐变缓, 而且 b 越大, 液化区发展越快。由于该问题本身的复杂性而在本文中做了一些假设, 使得结果与实际情况有

一定差别, 但尚能够反映液化发展的趋势和液化机理。这方面的研究对于分析液化的发展过程和液化的最大影响范围是很有价值的, 但要能准确地符合实际情况, 还必须对二维、三维情况进行分析。

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