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两种弹性损伤模型的基本方程与色散关系讨论*

邓守春** 刘金兴 张 晶 梁乃刚
(中国科学院力学研究所非线性力学国家重点实验室,中国科学院研究生院,北京,100080)

摘 要 分析了传统连续介质损伤理论的控制方程在损伤过程中的变化特点,方程类型的改变导致错误预测波不能在损伤区域传播. 二阶隐式应变梯度损伤理论是对传统理论的一种改进,严格证明了其控制方程在损伤过程中类型不变,这意味着损伤区域能传播波,同时有助于克服病态的网格相关性. 色散分析结果表明:传统连续介质损伤理论不能反映色散现象,隐式梯度模型可以并对波长有上限截断作用.

关键词 损伤, 控制方程, 色散, 应变梯度, 应变局部化

0 引言

材料由各类原子组成,这些原子通过电磁场的相互作用形成的键联结在一起. 原子键的性质决定了材料的宏观性质如机械和热性质等,而材料中的势能就是这些粒子间相互作用势能的总和[1-4]. 在外部作用下,材料产生相应的抵抗变形,原子间的相对位移改变了其键合力的大小,原子间势能变化的和与材料的(宏观)应变能相对应. 随着载荷的逐渐增加,材料的变形和原子间的相对位移不断增大,当载荷增加到原子间的结合键开始破坏时,便开始了损伤过程. 损伤对材料的弹性性能有直接的影响[5-18],这是由于与弹性有关的原子键的数目随着损伤的增大而减少. 损伤的出现和演化将改变材料的声传播特性,同时也会改变材料的其他如电磁和热等性质. 原子间的相互作用势的大小随着原子间的位置变化而变化,因此,单个原子的状态不仅和其本身相关,而且与其周围临近的原子相关,即材料本身具有某种"非局部"的特性.

材料从根本上是一离散多体系统[19-22],即使在微观尺度上,材料因具有如晶粒,微孔洞和微裂纹等微观结构而不能被视作均匀的连续介质. 由于这种内禀的非均匀性,材料内部的变形也表现出非均匀的特点. 变形越大,非均匀性愈明显,特别是出现损伤或塑性软化以后,材料中的变形出现高度的"应变局部化"现象[23-30]. 实验观察指出,此时的变形多发生在一个有限厚度的区域内,而材料的其它部分变形较小或减小. "应变局部化"通常是材料灾难性失效的前兆.

自 Kachanov[31] 于 1958 年提出"连续度"的概念至今,几十年的时间内损伤力学取得了重要的进展[5-18]. 经典连续介质中的应力只与应变相关,它的一个缺点在于描述损伤和塑性软化时不稳定,体现在数值模拟上就是病态的网格相关性[23,24,31-37],即在数值计算中,随着网格越来越密,局部化区域体积趋零,耗散能也趋于零. 数学分析表明,由于损伤的出现和演化,其控制方程会改变类型[18,23,24,37],如静力问题中椭圆型方程因损伤而转变为双曲型方程和动力问题中的双曲型方程转变为椭圆型方程. 向损伤及软化本构中引入高阶项可以得到物理上真实的,数学上有意义的结果[38]. 高阶项既可以是积分形式[33,39],也可以是梯度形式[40-44]. 梯度形式可以由非局部模型得到[30,45,46],显式梯度模型与隐式梯度模型之间也存在很大差别[30,43,46].

本文以传统连续介质损伤和二阶隐式梯度损伤理论为代表,分析了各自控制方程在损伤过程中的特点,如方程类型变化,首次严格证明了隐式梯度控制方程的静力保椭圆型和动力保双曲型,而这被认为是物理客观的基本要求. 损伤的出现和演化改变了材料的弹性性质,这也直接改变了声波在材料中的传播特性,本文将以所选两模型为代表,对各自声

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** 通讯作者. Tel: 010-82544010, E-mail: dengsc@imech.ac.cn.

波的色散作了分析. 针对“损伤充分发展的材料(刚度为零或为负)能否有波传播”的问题本文也作了相应的探讨.

1 损伤控制方程分析

1.1 传统连续介质损伤理论的控制方程及其特点

损伤改变了材料的刚度,而刚度的改变会改变材料控制方程的类型. 为简单记,考虑各向同性损伤,D 为损伤因子. 当损伤持续演化时,有:

$$\sigma_{ij} = (1-D)[\lambda u_{k,k}\delta_{ij} + \mu(u_{i,j}+u_{j,i})] = \frac{1-D}{2}[\lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm})](u_{m,n}+u_{n,m}) \tag{1}$$

其率形式为:

$$\dot{\sigma}_{ij} = (1-D)[\lambda\dot{u}_{k,k}\delta_{ij} + \mu(\dot{u}_{i,j}+\dot{u}_{j,i})] - \frac{\sigma_{ij}}{2(1-D)}\frac{\partial D}{\partial\varepsilon_{mn}}(\dot{u}_{m,n}+\dot{u}_{n,m}) \tag{2}$$

即弹性损伤切线张量为:

$$C_{ijmn} = (1-D)[\lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm})] - \frac{\sigma_{ij}}{1-D}\frac{\partial D}{\partial\varepsilon_{mn}} \tag{3}$$

由上式,损伤发展到一定程度后,损伤率所诱导的刚度变化 $\sigma_{ij}/(1-D)\cdot\partial D/\partial\varepsilon_{mn}$ 可使得材料的切线刚度矩阵由正定向不定或负定转变,材料失稳. 损伤演化时的控制方程为(不计重力):

$$\sigma_{ij,j} = \frac{1}{2}\left\{(1-D)[\lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm})] - \frac{\sigma_{ij}}{1-D}\frac{\partial D}{\partial\varepsilon_{mn}}\right\}(u_{m,nj}+u_{n,mj}) = \rho\ddot{u}_i \tag{4}$$

当切线刚度矩阵正定时,上面的方程是双曲型方程(静力问题 $\ddot{u}_i = 0$ 时,为椭圆型方程);由于损伤的作用,当切线刚度矩阵变为不定或负定时,上述方程向椭圆型方程转变(静力问题向双曲型转变). ρ 为质量密度.

为直观起见,下面以单调拉伸等截面均匀长杆的弹性损伤过程为例说明. 杆的弹性损伤应力-应变曲线见图 1.

ε_0 是损伤临界应变,ε_c 完全损伤时应变,E 杨氏模量. 杆的应力-应变总可表达为:

$$\sigma = (1-D)E\varepsilon = (1-D)Eu_{,x} \tag{5}$$

其控制方程如下:

$$\sigma_{,x} = \left[(1-D) - \varepsilon\frac{\mathrm{d}D}{\mathrm{d}\varepsilon}\right]Eu_{,xx} = \rho u_{,tt} \tag{6}$$

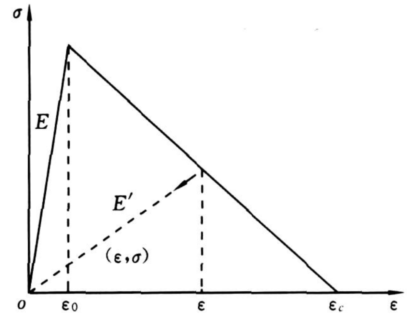

图 1 应力-应变关系曲线

Fig. 1 The stress vs. strain curve

对于双折线本构关系(图 1),持续损伤时,有

$$-\frac{\varepsilon_0}{\rho(\varepsilon_c-\varepsilon_0)}Eu_{,xx} = u_{,tt} \tag{7}$$

由上式可以看出:一旦进入损伤,$-\varepsilon_0/[\rho(\varepsilon_c-\varepsilon_0)]<0$,该方程不再是双曲型而成为椭圆型方程.

1.2 二阶隐式梯度损伤模型的控制方程及其特点

为克服传统损伤模型的缺点,一些材料模型,如积分模型,隐式梯度模型和显式梯度模型被相继用于对损伤的描述. 在二阶隐式梯度模型中,非局部应变与传统的局部应变有如下关系:

$$\bar{\varepsilon}_{ij} - c\Delta\bar{\varepsilon}_{ij} = \varepsilon_{ij} \tag{8}$$

其中,c 是材料内部尺度参数,一般,$c = l^2/2$,l 是与材料相关的内禀长度;Δ 为 Laplace 算子. 方程(8)系一非均匀 Helmholtz 方程,针对通常采用的附加边界条件($\partial\bar{\varepsilon}_n/\partial n = 0$),用 Green 函数方法求解,则有:

$$\bar{\varepsilon}(x) = \int_\Omega G(y;x)\varepsilon(y)\,\mathrm{d}\Omega \tag{9}$$

其中,$G(y;x)$ 是 Green 函数,并且有:

$$\int_\Omega G(y;x)\,\mathrm{d}\Omega = \int_\Omega \delta(x-y)\,\mathrm{d}\Omega - \bar{c}\int_\Omega \Delta G\,\mathrm{d}\Omega = 1 \tag{10}$$

由式(9)可以看出,隐式梯度模型实际上是一种特殊的积分模型,如果将其中的权函数换作 Green 函数的话.

针对各向同性损伤情形,通常认为“损伤因子是非局部应变 $\bar{\varepsilon}_{ij}$ 的函数”:

$$D = D(\bar{\varepsilon}_{ij}) \tag{11}$$

此时的控制方程为:

$$\sigma_{ij,j} = \frac{1}{2}\left\{(1-D)[\lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn}+\delta_{in}\delta_{jm})] - \frac{\sigma_{ij}}{1-D}\frac{\partial D}{\partial\bar{\varepsilon}_{kl}}\frac{\partial\bar{\varepsilon}_{kl}}{\partial\varepsilon_{mn}}\right\}(u_{m,nj}+u_{n,mj}) = \rho\ddot{u}_i \tag{12}$$

对应地,其切线刚度张量有如下形式:

$$C_{ijmn} = (1-D)[\lambda\delta_{ij}\delta_{mn} + \mu(\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm})] - \frac{\sigma_{ij}}{1-D}\frac{\partial D}{\partial\bar{\varepsilon}_{kl}}\frac{\partial\bar{\varepsilon}_{kl}}{\partial\varepsilon_{mn}} \tag{13}$$

由上式,如有$\frac{\sigma_{ij}}{1-D}\frac{\partial D}{\partial\bar{\varepsilon}_{kl}}\frac{\partial\bar{\varepsilon}_{kl}}{\partial\varepsilon_{mn}} \leq 0$恒成立,则控制方程在整个损伤过程中将保持类型不变(动力双曲型和静力椭圆型). 为直观计,仍以前述的单调加载等截面均匀长杆为例. 有:

$$\begin{cases} \sigma_{,x} = \rho u_{,tt} \\ \sigma = (1-D)E\varepsilon \\ \bar{\varepsilon} - c\Delta\bar{\varepsilon} = \varepsilon = u_{,x} \\ D = D(\bar{\varepsilon}) \end{cases} \tag{14}$$

将方程组(14)的第二和第四式代入第一式,得到其控制方程:

$$\left[(1-D)E - \frac{\sigma}{1-D}\frac{\mathrm{d}D}{\mathrm{d}\bar{\varepsilon}}\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon}\right]u_{,xx} = \rho u_{,tt} \tag{15}$$

应变的梯度项实际上是一个平均化的过程,使得在应变变化剧烈的区域通过梯度作用使得应变变化平缓一些. 因恒有 $\sigma/(1-D)\cdot\mathrm{d}D/\mathrm{d}\bar{\varepsilon} \geq 0$ 成立,于是,若有:

$$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon} \leq 0 \tag{16}$$

恒成立,则式(15)保持其双曲型不变(对静力问题将保持椭圆型). 针对常用的边界条件: $\partial\bar{\varepsilon}/\partial n = \partial\bar{\varepsilon}_{ij}/\partial x_j \cdot n_i = 0$,下面将给出数学证明. 记:

$$\phi = \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon} \tag{17}$$

由式(8),有:

$$\phi - c\phi_{,xx} = 1 \tag{18}$$

其解为:

$$\phi = \alpha e^{\sqrt{1/c}\,x} + \beta e^{-\sqrt{1/c}\,x} + 1, \quad 0 \leq x \leq L \tag{19}$$

而$\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}x} = \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}x}$,当杆中出现损伤区时,杆中变形不均匀, $\mathrm{d}\varepsilon/\mathrm{d}x \neq 0$. 由附加边界条件 $\bar{\varepsilon}_{,x}|_{x=0} = \bar{\varepsilon}_{,x}|_{x=L} = 0$,得:

$$\left.\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon}\right|_{x=0} = \left.\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon}\right|_{x=L} = 0 \tag{20}$$

进一步,杆中有:

$$\phi = \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}\varepsilon} = -2\frac{1}{e^{L/2\sqrt{1/c}} + e^{-L/2\sqrt{1/c}}} + 1 \leq 0 \tag{21}$$

式(16)获证. 这里需要指出的是,显式梯度模型并不能保证方程类型不变.

2　传统和隐式梯度损伤模型的色散分析

从数学物理上,不同类型的方程描述了不同的物理现象. 损伤的出现和演化改变着材料的刚度,也改变着其波动特性,如纵波波速减小等等. 前面的分析表明,损伤会改变传统连续介质理论控制方程的类型. 梯度项的引入使得控制方程的类型在损伤过程中保持不变,已有的分析和数值模拟表明,隐式梯度模型能很好解决病态的“网格相关性”. 下面将讨论这两个模型的波动特性.

为方便计,考虑一两端无限延伸的均匀等截面长杆,其受持续增加的拉伸载荷,应力-应变曲线仍如图 1 所示. 一维的运动方程为:

$$\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial\sigma}{\partial x} \tag{22}$$

分别代入传统和二阶隐式梯度模型的本构关系,有

$$\rho\frac{\partial^2 u}{\partial t^2} = \left[(1-D) - \varepsilon\frac{\mathrm{d}D}{\mathrm{d}\varepsilon}\right]E\frac{\partial^2 u}{\partial x^2} \tag{23}$$

$$\rho\frac{\partial^2 u}{\partial t^2} = (1-D)E\frac{\partial^2 u}{\partial x^2} - E\varepsilon\frac{\mathrm{d}D}{\mathrm{d}\bar{\varepsilon}}\frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}x} \tag{24}$$

设位移受小的简谐扰动 $\delta u = \hat{u}\exp[\mathrm{i}k(x-ct)]$,其中 $\hat{u}$ 为扰动幅度, k 和 c 分别是波数和相速度. 代入传统损伤模型的运动方程(23),有:

$$\frac{c^2}{c_e^2} = (1-D) - \varepsilon\frac{\mathrm{d}D}{\mathrm{d}\varepsilon} \tag{25}$$

其中, $c_e = \sqrt{E/\rho}$是杆的弹性纵波速. 由式(23)可以看出,随着损伤的出现和演化,不同波数波的波速做同样大小的改变,即波无色散;并且,当损伤到$(1-D) - \varepsilon\cdot\mathrm{d}D/\mathrm{d}\varepsilon \leq 0$ 时,波速为零或为虚,此时,预示该损伤区域不能传播纵波. 实际上,它依然可以传播横波:由于此时刚度为零或负,杆可被视为柔弦,设其张力为 T,则横波波速为 $c_t = \sqrt{T/\rho}$. 推广到高维损伤情形十分复杂,还有待于做进一步研究. 实际上,对图 1 所示的双折线本构关系,一旦进入损伤状态,则有:

$$\frac{c^2}{c_e^2} = (1-D) - \varepsilon\frac{\mathrm{d}D}{\mathrm{d}\varepsilon} = -\frac{\varepsilon_0}{\varepsilon_c - \varepsilon_0} < 0 \tag{26}$$

这意味着由传统损伤理论所预测的持续损伤区纵波速为虚.

为考察二阶隐式梯度模型,进一步假设存在一个同步的非局部应变扰动 $\delta\bar{\varepsilon} = \hat{\varepsilon}\exp[\mathrm{i}k(x-ct)]$. 不考虑卸载,在初始变形均匀时,有 $\bar{\varepsilon} = \varepsilon$. 将 δu 和 $\delta\bar{\varepsilon}$ 代入 $\bar{\varepsilon} - c\Delta\bar{\varepsilon} = \varepsilon = u_{,x}$,得到了 $\hat{u}$ 和 $\hat{\varepsilon}$ 的关系:

$$\mathrm{i}k\dot{u} = \varepsilon(1 + ck^2) \tag{27}$$

将式(27)代入隐式梯度模型的运动方程(24)并化简,有:

$$\frac{c^2}{c_e^2} = (1 - D) - \bar{\varepsilon}\frac{\mathrm{d}D}{\mathrm{d}\bar{\varepsilon}}\frac{1}{1 + l^2k^2/2} \tag{28}$$

上式表明,相速度与波数非线性相关,不同波数的波的速度不同因而会在传播过程中分离,即有色散现象. 针对双折线本构曲线(图 1),并取 $\varepsilon_0 = 0.001$, $\varepsilon_c = 0.1$ 和材料的内禀尺度 $l = 1$. 图 2 显示了不同应变水平时相速度与波数的关系,图 3 对应着不同波数时相速度与应变的关系曲线. 由图 2 和 3,应变水平越大,损伤愈甚,材料中的波速下降越多;另外,在给定应变水平时,波速随波数的增加而趋于一个常数.

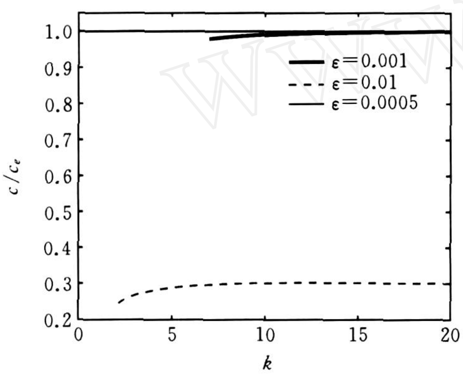

图 2 不同应变水平时的相速度-波数曲线

Fig. 2 The curves of phase velocity vs. wave number under different strain levels

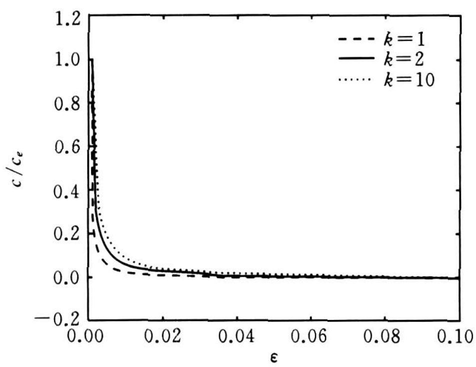

图 3 不同波数波的波速-应变曲线

Fig. 3 The wave velocity vs. strain curves under different wave number

与相速度为零时所对应的临界波数 k_{crit} 和临界波长 λ_{crit} 分别由下面二式给出:

$$k_{\mathrm{crit}} = \frac{1}{l}\sqrt{\frac{2[\bar{\varepsilon}\cdot \mathrm{d}D/\mathrm{d}\bar{\varepsilon} + D - 1]}{1 - D}} \tag{29}$$

$$\lambda_{\mathrm{crit}} = \frac{2\pi}{k_{\mathrm{crit}}} = \pi l\sqrt{\frac{2(1 - D)}{\bar{\varepsilon}\cdot \mathrm{d}D/\mathrm{d}\bar{\varepsilon} + D - 1}} \tag{30}$$

图 4 和图 5 分别是取双折线本构时的临界波数-应变和临界波长-应变曲线,图中的阴影部分对应着波速非零区域. 由图可知,损伤中的材料类似一"高通滤波器",并且临界波数随着应变的增大而增大,即,可传播波的最小波数(临界波数)随着损伤演化而逐渐增大,对应其波长则越短.

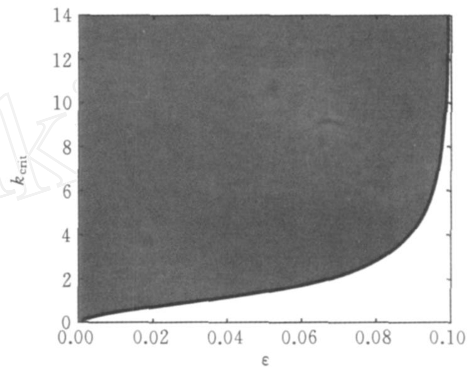

图 4 临界波数-应变演化曲线

Fig. 4 The evolution curve of critical wave number vs. strain

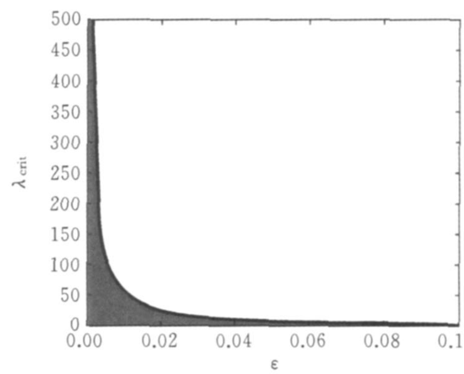

图 5 临界波长-应变演化曲线

Fig. 5 The evolution curve of critical wavelength vs. strain

3 结论

本文首先简要介绍了损伤的物理基础,着重分析了损伤过程的数学特点——传统连续介质损伤理论的控制方程类型随着损伤变化. 针对不同的改进方法,本文以二阶隐式应变梯度模型为代表,严格证

明了梯度项的引入确能保证其控制方程在损伤过程中类型不变,这将利于数值稳定.

损伤不仅改变了材料的刚度,同时也改变着材料的波动性质,因此,本文做了两个不同损伤模型(传统连续介质损伤模型和二阶隐式应变梯度模型)的色散分析.结果表明,传统损伤模型不能反映波的色散,而且,当损伤发展到一定程度时波不能在局部化带内传播;与之对应,二阶隐式梯度模型能反映波的色散现象,它对波有截断作用——大于临界波长的波不能在损伤区内传播.材料中的色散十分复杂,深入了解可参阅有关文献[20].

本文关于损伤控制方程的研究有助于考察其它损伤模型的性质,并分析各自的能力.进一步,对发展新的损伤模型也有帮助.

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ON THE GOVERNING EQUATIONS AND DISPERSION RELATIONS OF TWO ELASTIC DAMAGE MODELS

Shouchun Deng Jinxing Liu Jing Zhang Naigang Liang

(State Key Laboratory of Nonlinear Mechanics (LNM), Institute of Mechanics, Chinese Academy of Sciences (CAS), Graduate School of CAS, Beijing, 100080)

Abstract The governing equations of the conventional continuum damage models have been investigated in this paper. It is found from the analysis that the types of governing equations have been changed from elliptical to hyperbolic during the damage process, which may result in the false appearance of waves impossibly propagating in damaging zone. As an alternative model, the implicit second-order strain gradient theory is powerful in studying damage. It has been rigorously proved that the equation-type of the governing equations of the damaged media is unchangable during the whole damage process. Therefore the false appearance of no wave existing inside the damaged zone has been left. Moreover, the introduction of gradient terms conduces to overcome the pathological mesh-dependence in numerical computation. It is well known that dispersion is always in existence in solids. Further analysis indicates that the implicit second-order strain gradient theory can effectively deal with dispersion which cannot be solved by the continuum damage model. It is also observed that the more serious damage will result in decrease in the critical wave length.

Key words damage, governing equation, dispersion, strain gradient, strain localization