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A BIEM OPTIMIZATION METHOD FOR FRACTURE DYNAMICS INVERSE PROBLEM*

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ABSTRACT: In the present paper, based on the theory of dynamic boundary integral equation, an optimization method for crack identification is set up in the Laplace frequency space, where the direct problem is solved by the author's new type boundary integral equations and a method for choosing the high sensitive frequency region is proposed. The results show that the method proposed is successful in using the information of boundary elastic wave and overcoming the ill-posed difficulties on solution, and helpful to improve the identification precision.

KEY WORDS: crack identification, BIEM, iterative optimization, frequency choosing

1 INTRODUCTION

From the point of modern engineering and technique, in order to estimate the safety of a structure, we not only need to determine if there exist flaws or cracks but also need to decide their exact shape and location. Technically speaking, it is a problem of nondestructive testing; theoretically speaking, it is an inverse problem of solid mechanics. Since the problem is obviously of high value in practice, several theoretical and numerical methods have been proposed[1~3]. Besides the progress of ultrasound testing techniques, the method based on theory of elastodynamics has become the focus of research[1,2]. Because the precision of flaw detection depends on both the quality of testing equipment and the level of software, to obtain a valid identification result we have to make full use of the testing information and analyze them correctly. This means that the most important task at present is to make further study on the theory of inverse problems. It should be point out that, although the solving methods for inverse problem is closely related with those for direct problem , their theories are quite different and the solution of inverse problem is usually ill-posed and highly nonlinear. Thus to solve an inverse problem is generally much more difficult than to solve a direct problem and there is not a perfect method at present. There are mainly two kinds of methods to solve elastodynamic inverse problems. The first one is based on the analytic or half analytic theories, where BORN approximation[4,5] and inverse scattering approach[6]

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are most representative. They are very difficult in application and theory. The second one is an indirect method, which is based on the solution of direct problem and uses the iterative optimization approach to identify the unknowns. Of the two kinds of methods, the second one will take a lot of CPU time of computer, but with the development of modern computer techniques, it will not be a problem. In the second kind of method, as the representative work we can quote Tanaka[7], Nishimura[8] and Chen[9], where the Chen's Pulse-Spectrum Technique (PST) has been applied with some success to identify the medium parameters[9]. Since PST can fully use the rich information of transient elastodynamic wave, it is more useful in practice than the sound wave inversion[8] and the steady wave inversion[7].

In the present paper, based on the theory of transient elastic wave, the crack identification problem is investigated. Using the similar model of PST, the inverse problem is reduced to solving an optimization problem in Laplace transform space, where the square sum of differences between the computed displacements and measured ones at selected points on outer boundary should be minimized. In the iterative process the new type boundary integral equation method[10] proposed by authors is used to solve the direct problems and is proved to be effective in reduction of the numerical error. In the choice of frequency spectrum, a method for choosing high sensitive frequency region is proposed in this paper. The results show that the method proposed can fully and reasonably use the rich information from the wide frequency region of the transient elastodynamic wave to obtain high precision identification result with less selected points and computation. The method proposed can be used in the development of nondestructive testing techniques.

2 THE MIXED-TYPE BIEM FOR DIRECT PROBLEM

Since in the optimization method, a lot of computation has to be made on the direct problem in each step of iteration, one of the keys to make successful inversion of the crack geometry parameters is to work out an accurate and fast numerical method for the direct problem. In the present paper the mixed-type boundary integral equations[10] proposed by authors are used to solve the direct problem. These equations are different from the regular boundary integral equations, where we do not need to cut along the crack and the elements can be generated automatically in each step of iteration. The method can be used to reduce the accumulated errors in iterative computation. The process is given as follows.

For the plane strain crack problem shown in Fig.1, if the crack location and length are known, to solve for the displacement and stress fields is then a direct problem, where the displacement $u_i(y,t)$ must satisfy the governing equations

$$\mu u_{i,kk} + (\lambda+\mu) u_{k,ki} = \rho \frac{\partial^2 u_i}{\partial t^2} \qquad (1)$$

and boundary and initial conditions

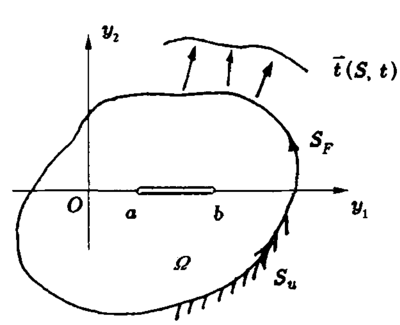

Fig.1 The dynamic plane crack problem

$$\left.\begin{array}{ll}\sigma_{ij}n_j = t_i(x,t) & x \in S_t \\ u_i = \widetilde{u}_i(x,t) & x \in S_u\end{array}\right\} t \geq 0 \tag{2}$$

$$\left.\begin{array}{l}u_i(x,t)|_{t=0} = u_i^0(x) \\ \left.\dfrac{\partial u_i(x,t)}{\partial t}\right|_{t=0} = \dot{u}_i^0(x)\end{array}\right\} x \in \Omega + S \tag{3}$$

where λ and μ are elastic moduli, ρ is density of material, Ω is the region surrounded by boundary $S + \Gamma^{\pm}$, n_i is the outward normal of boundary. Suppose the structure is static before transient loadings are applied, then we have $\dot{u}_i^0(x) = u_i^0(x) = 0$. Apply the Laplace transform about time (t) to Eqs.(1), we obtain

$$\mu\bar{u}_{i,kk} + (\lambda+\mu)\bar{u}_{k,ki} - \rho p^2\bar{u}_i = 0 \tag{4}$$

where the bar denotes the Laplace transform, p is the transform parameter. The solution in Ω can be expressed by Somigliana formulae

$$\bar{u}_k(y,p) = \int_S [\bar{t}_i(\eta,p)\bar{U}_{ik}(\eta-y,p) - \bar{u}_i(\eta,p)\bar{T}_{ik}(\eta,y,p)]\mathrm{d}S(\eta) - \int_a^b \left[\int_{\eta_1}^b \bar{T}_{ik}^{+}(\eta^*,y,p)\mathrm{d}\eta^*\right]\Delta\bar{u}_{i,1}(\eta_1,p)\mathrm{d}\eta_1 \tag{5}$$

where $\bar{U}_{ik}$ and $\bar{T}_{ik}$ are the Green fundamental solution, $\bar{T}_{ik}^{+} = \bar{T}_{ik}|_{n(0-1)}$, $\bar{t}_i$ and $\bar{u}_i$ are the traction and displacement on the outer boundary S, $\Delta\bar{u}_{i,1}$ is the dislocation density function along crack. From Eq.(5) one can find if the traction and displacement on boundary S and dislocation density along crack are known, then the solution can be obtained.

To determine the unknowns on boundary, using the techniques of Green fundamental solution and singularity analysis, the following mixed-type integral equations were derived by authors[10]

$$\frac{1}{2}\bar{u}_k(y,p) = \int_S [\bar{t}_i(\eta,p)\bar{U}_{ik}(\eta-y,p) - \bar{u}_i(\eta,p)\bar{T}_{ik}(\eta,y,p)]\mathrm{d}S(\eta) - \int_a^b \left[\int_{\eta_1}^b \bar{T}_{ik}^{+}(\eta^*,y,p)\mathrm{d}\eta^*\right]\Delta\bar{u}_{i,1}(\eta_1,p)\mathrm{d}\eta_1$$

$$k = 1,2 \qquad y \in S \tag{6a}$$

$$\int_S \left[\bar{t}_i(\eta,p)\bar{T}_{ki}^{+}(\eta,y_1,p) + \bar{K}_{ik}(\eta,y_1,p)\bar{u}_i(\eta,p)\right]\mathrm{d}S(\eta) + \frac{A}{\pi}\int_a^b \frac{\Delta\bar{u}_{k,1}(\eta_1,p)}{\eta_1 - y_1}\mathrm{d}\eta_1 + \int_a^b \bar{M}_k(\eta_1,y_1,p)\Delta\bar{u}_{k,1}(\eta_1,p)\mathrm{d}\eta_1 = \bar{q}_k(y_1,p)$$

$$k = 1,2 \qquad y_1 \in (a,b) \tag{6b}$$

where, $\bar{q}_k = \bar{\sigma}_{k2}|_{L^{\pm}}$ is the loading along crack, $A = \mu/2(1-v), \bar{K}_{ik}$ and $\bar{M}_k$ are the known integrable kernels whose expressions can be found in Ref.[10].

It is necessary to point out that the above Eqs.(6a,6b) are different from the regular integral equations, in which the traction-type BIE(6a) along the crack are Cauchy singular integral equations with dislocation density functions as unknowns. Eqs.(6a,6b) can be reduced to solving a set of linear equations by combining the numerical method of singular integral equation with the boundary element method. Since the numerical method can be easily used for general problems and has very high numerical precision, it is more suitable for the iterative calculation.

3 THE ITERATIVE OPTIMIZATION METHOD FOR CRACK IDENTIFICATION

For the problem shown in Fig.1, if the crack location and shape are unknown and need to be identified by measurement and analysis, then it is an inverse problem. In the present paper the problem of straight line crack identification is considered. Because of the high nonlinear and ill-posed nature of the solution it is impossible to solve the inverse problem directly from Eqs.(6a,6b). In order to overcome the difficulties the indirect method is used in this paper, where we suppose that the displacements at some selected boundary points are measured to be the identification information. These selected points are usually called over-prescribed boundary points whose displacement and traction conditions are known at the same time. By use of the numerical method of the direction problem and the iterative optimization, the crack identification is reduced to minimizing the square sum of differences between the computed displacements and measured ones at selected points on outer boundary. Our method is illustrated as follows.

For a two dimensional problem, only 4 parameters $\boldsymbol{\zeta}(a_0, \theta, x_0, y_0)$ are needed to determine the crack, in which a_0 is the half length of the crack, θ is the angle between the crack and the horizontal direction, (x_0, y_0) is the central point of the crack (in the coordinate system XOY). To identify these parameters, we choose the following objective function in the iterative optimization

$$W(\boldsymbol{\zeta}) = \sum_{k=1}^{p}\sum_{l=1}^{N}\sum_{i=1}^{2}[\bar{u}_i(X_l, p_k) - \bar{u}_i^*(X_l, p_k)]^2 \tag{7}$$

where P is the number of selected frequency points, p_k is the value of frequency. N is the number of selected measurement points, $\bar{u}_i(X_l, p_k)$ is the displacement at point X_l by computation, $\bar{u}_l^*(X_l, p_k)$ is the displacement at point X_l by measurement.

Now the inverse problem is reduced to determining the best parameters $\boldsymbol{\zeta} = (a_0, \theta, x_0, y_0)$ by minimizing the objective function $W(W_{\min} = 0)$. In order to choose a suitable convergence criterion for the iterative computation, the non-dimensional objective function is defined as

$$Z = W \Big/ \sum_{k=1}^{p}\sum_{l=1}^{N}\sum_{i=1}^{2}[\bar{u}_i^*(X_l, p_k)]^2 \tag{8}$$

and the parameters $\boldsymbol{\zeta} = (a_0, \theta, x_0, y_0)$ are iterated in the following manner

$$\boldsymbol{\zeta}^{n+1} = \boldsymbol{\zeta}^{(n)} + l\boldsymbol{d}^{(n)} \tag{9}$$

where l is optimal step length, to improve the convergence a method of multi-constant step is used, in which the range of residual value is divided into $\epsilon_1 > \epsilon_2 > \cdots > \epsilon_m > \epsilon^*$ and the correspondent steps are chosen as $l_1 > l_2 > \cdots > l_m$. If $\epsilon_{j+1} < Z < \epsilon_j$, then $l = l_j$. $\boldsymbol{d}^{(n)}$ is the vector of the search direction at the nth iteration and can be determined as[11]

$$\boldsymbol{d}^{(n)} = -\frac{\text{grad } W}{|\text{grad } W|} \tag{10}$$

where grad $W = (\partial W/\partial a_0, \partial W/\partial \theta, \partial W/\partial x_0, \partial W/\partial y_0)$ and the convergence criterion used in this paper is [7]

$$Z^{(n)} < \epsilon^* \qquad |Z^{(n)} - Z^{(n-1)}| < \epsilon^{**} \tag{11}$$

where ϵ^* and ϵ^{**} are small positive real number. The first inequality implies that the non-dimensional objective function is less than a given criterion and the second one states that the change in the objective function is less than a given value.

It should be point out, since a lot of computation must be carried out and the crack parameters are changed in each step of iteration, the coordinate system has to be chosen carefully. According to the numerical method in the above section, the direct computation is done in a local system along the crack while the objective function is calculated in the general system(XOY). The relation between the local system and the general system can be expressed as follows

$$\left.\begin{aligned} y_1 &= (X - x_0)\cos\theta + (Y - y_0)\sin\theta \\ y_2 &= -(X - x_0)\sin\theta + (Y - y_0)\cos\theta \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \bar{u}_1 &= \bar{\bar{u}}_1\cos\theta - \bar{\bar{u}}_2\sin\theta \\ \bar{u}_2 &= \bar{\bar{u}}_1\sin\theta + \bar{\bar{u}}_2\cos\theta \end{aligned}\right\} \tag{13}$$

where $\bar{\bar{u}}_1$ and $\bar{\bar{u}}_2$ are the displacements in local system. By use of relations (12),(13) all the information obtained in the local system can be transform into those in the general system.

The computation of the iterative optimization is carried out in following steps:

(1) Input data of the structural shape and material constants;

(2) Assume the shape and location of a straight crack $\boldsymbol{\zeta}^{(0)}$;

(3) Calculate the residual $Z^{(n)}$ (for initial step $n = 0$) and choose the optimal step length $l(l = l_m)$;

(4) Take parameters $(\boldsymbol{\zeta}^{(n)} + \Delta\boldsymbol{\zeta}_i(|\Delta\boldsymbol{\zeta}_i| << 1, i = 1, 2, 3, 4)$ and calculate the residuals $Z^{(n)} + \Delta Z_i$, and then compute the directional vector $\boldsymbol{d}^{(n)}$ approximately by use of difference method;

(5) Take the revised parameters $\boldsymbol{\zeta}^{(n+1)} = \boldsymbol{\zeta}^{(n)} + l_m\boldsymbol{d}^{(n)}$ and calculate the residual $Z^{(n+1)}$;

(6) Check convergence according to the criterion inequalities (11). If the solution converges, then stop iteration. Otherwise, substitute the parameters $Z^{(n)}$ into $Z^{(n+1)}$ and go to Step (3)

4 FREQUENCY CHOICE AND NUMERICAL EXAMPLES

From the method of the above section, the identification precision is mainly related with measurement point X_l and frequency value p_k. Although a large number of X_l and p_k will offer more information, good identification precision is not always achieved and more computation has to be done. Thus, how to use the finite information reasonably, i.e. using the most effective part of the information and achieving good result with small computation cost, is another key for a successful identification. In the present paper, a method of high sensitive frequency region choice is proposed, where the frequency values p_k are chosen only in the most sensitive frequency region of real half-axis $(0, \infty)$. The method proposed is based on the following considerations:

(1) It can use the information reasonably and effectively with less computation cost;

(2) The computation is only in real space and the complex computation is avoided.

The following examples show the method is successful and the expected aim is achieved.

In the examples the materials constants are defined as $\mu = 8 \times 10^{10}\,\text{Pa}$, $\nu = 0.29, \rho = 7\,800\,\text{kg/m}^3$. The structure is a rectangle with geometry of 2.0 m× 1.0 m and length and location of the crack are unknown. The outer boundary is divided into 30 elements. The measurement points (by numerical imitation) are selected at the central points of No.7 and No.14 elements and the impact loading is acting on the central part of the up side or right and left sides.

Example 1 The rectangle with a central horizontal crack with a half-length of 0.1m.

The real frequency spectrums at No.7 element are shown in Fig.2a, where the most sensitive frequency region is obviously between $0.25 < p/c_2 < 4.0$. The frequency values are selected as $p_k = kc_2/2, k = 1, 2, 3, 4, 5$. The spectrums obtained by identification are shown in Fig.2b which is very identical with Fig.2a. The identification process is given by Fig.3 and Table 1.

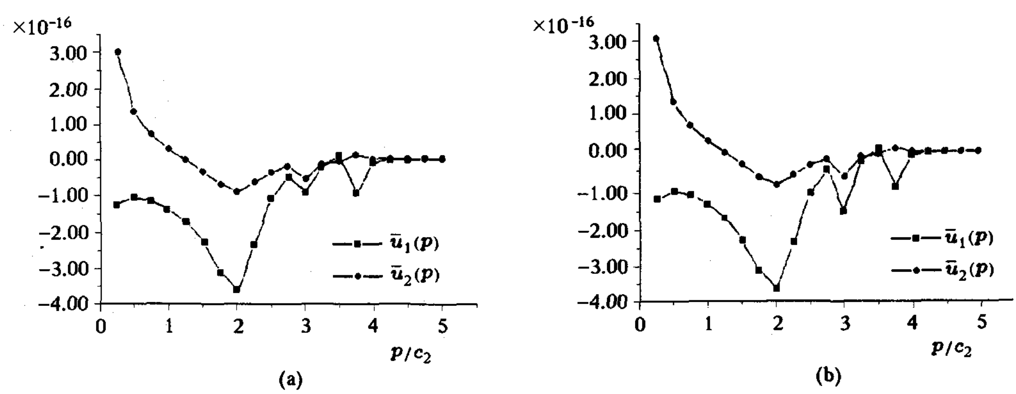

Fig.2

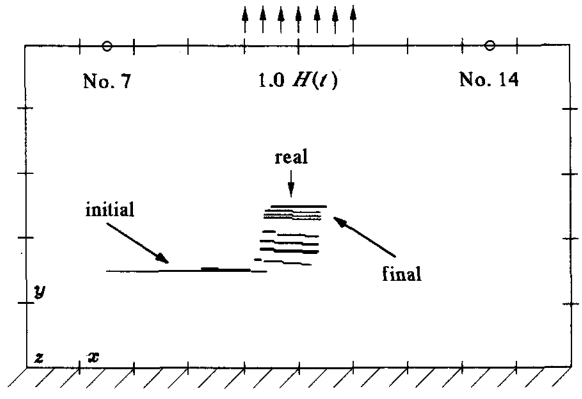

Fig.3

Table 1 The numerical results of crack identification

n	$a_0^{(n)}$	$\theta^{(n)}$	$x_0^{(n)}$	$y_0^{(n)}$	$Z^{(n)}(\%)$
0	0.200 000 0	0.000 000	0.500 000	0.300 000	5.068 914
2	0.119 545	−0.029 703	0.766 140	0.302 891	0.832 969
4	0.108 132	−0.068 823	0.942 502	0.328 535	0.121 692
6	0.106 923	−0.044 542	0.965 496	0.360 000	0.065 107
8	0.105 963	−0.040 041	0.969 309	0.367 314	0.061 392
10	0.105 488	−0.037 750	0.971 083	0.388 500	0.042 530
12	0.104 587	−0.033 909	0.974 467	0.414 461	0.026 143
14	0.102 494	−0.028 874	0.978 126	0.461 344	0.009 630
16	0.102 758	−0.023 796	0.979 136	0.469 794	0.008 426
FINAL	0.100 869	−0.020 680	0.982 905	0.483 193	0.004 219
REAL	0.100 000	0.000 000	1.000 000	0.500 000	/

Example 2 The rectangle with a central vertical crack with half-length of 0.1m.

The real frequency spectrums at No.7 element are shown in Fig.4a, where the most sensitive frequency region is also between $0.25 < p/c_2 < 4.0$. The frequency values are selected as $p_k = kc_2/2, k = 1, 2, 3, 4, 5$. The spectrums obtained by identification are shown in Fig.4b which is almost identical with Fig.4a. The identification process is give by Fig.5 and Table 2.

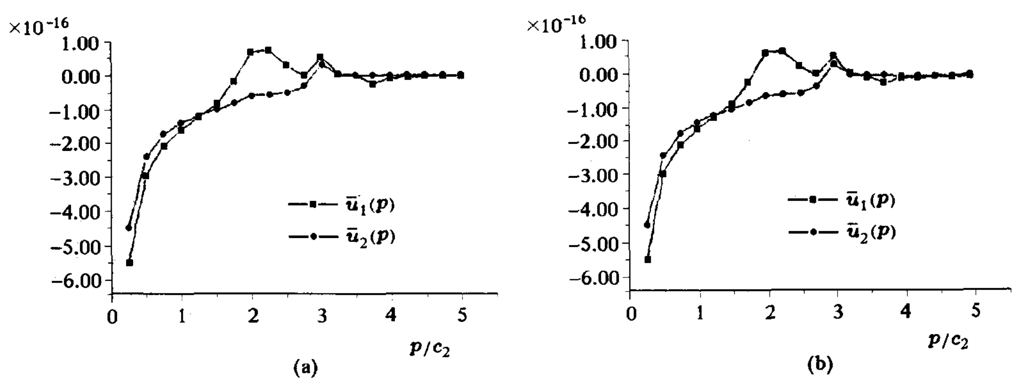

Fig.4

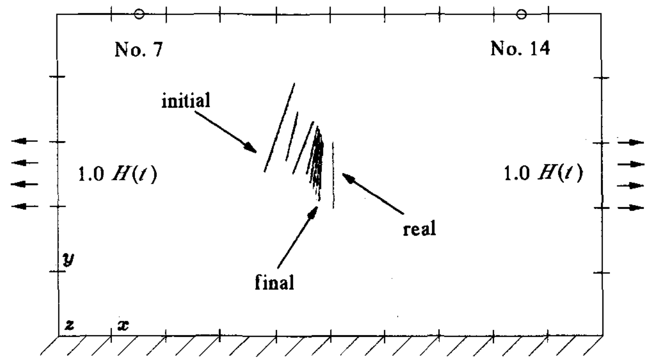

Fig.5

Table 2 The numerical results of crack identification

n	$a_0^{(n)}$	$\theta^{(n)}$	$x_0^{(n)}$	$y_0^{(n)}$	$Z^{(n)}$ (%)
0	0.150 000	1.200 000	0.800 000	0.650 000	1.639 492
3	0.076 313	1.251 795	0.846 663	0.621 141	0.025 909
6	0.073 297	1.282 943	0.868 466	0.576 054	0.022 164
9	0.085 073	1.328 672	0.900 294	0.583 952	0.012 943
12	0.085 771	1.353 090	0.922 846	0.575 439	0.011 859
15	0.087 221	1.389 264	0.928 520	0.558 410	0.006 631
18	0.090 554	1.437 291	0.934 091	0.546 452	0.004 064
21	0.093 138	1.464 812	0.935 817	0.533 296	0.002 780
24	0.094 006	1.480 332	0.943 668	0.529 591	0.002 092
FINAL	0.095 908	1.512 871	0.952 228	0.512 940	0.000 997
REAL	0.100 000	1.570 796	1.000 000	0.500 000	/

Above example results show that the method proposed in the present paper is successful to obtain good identification precision. Its advantages include: (1) The direct problem calculation reduces the errors effectively; (2) The wide frequency region is fully used and only few measurement points are needed; (3) The most sensitive information is used to reduce the computation cost and obtain good identification results by the method of high sensitive frequency region choice.

On the other hand, according to the experience obtained in this paper, the following methods can be used to further improve the identification effect and precision:

(1) Fully use the current testing techniques and experiences to improve the precision of the initial guess;

(2) When the crack is small, increase the number of measurement points and use more strict convergence criterion;

(3) Select the measurement points in such a way that they are located at the largest values of sensitivities of frequency spectrum;

(4) Select measurement information of multiple loadings as the additional conditions.

5 CONCLUSION

In the present paper a crack identification method, based on the BIEM for direct problem and iterative optimization technique and method of high sensitive frequency region choice, is proposed. The results show that the method proposed is successful in using measurement information sufficiently and obtaining good identification effect with less computation. The results obtained in this paper are better than those in references and are of importance for the nondestructive technique development. It has to be point out that there is not a unified approach for the problems of the general flaw identification and much investigation is expected in the future.

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