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RESEARCH PAPER

Stability analysis of the moving interface in piston- and non-piston-like displacements

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Abstract From the macroscopic point of view, expressions involving reservoir and operational parameters are established for investigating the stability of moving interface in piston- and non-piston-like displacements. In the case of axisymmetrical piston-like displacement, the stability is related to the moving interface position and water to oil mobility ratio. The capillary effect on the stability of moving interface depends on whether or not the moving interface is already stable and correlates with the wettability of the reservoir rock. In the case of non-piston-like displacement, the stability of the front is governed by both the relative permeability and the mobility ratio.

Keywords Piston-like displacement · Non-piston-like displacement · Two-phase percolation · Moving interface · Stability analysis

1 Introduction

In the later stage of oil exploitation, less and less natural energy from the expansion of underground fluids and formation rock can be used for oil production. In order to keep oil production or enhance oil recovery, such fluids as water, gas or chemical solution with some special additives are injected into the formation to hold and improve the production capability [1]. During the process of fluid injection, a moving interface will occur between the displacing and displaced fluids. The stability of this moving interface has an important effect on oil production. If instability occurs, viscous fingering with dendritic branching will much likely take place. In this case, the displacing fluid will soon reach production well due to break-through, and thus lower the sweep area and oil recovery.

Tan et al. [2] investigated nonlinear mechanism of the viscous fingering and found that its stability depends on the mobility ratio of displacing fluid to displaced fluid, and some viscous fingering might hinder others to develop. Guo et al. [3] conducted numerous experiments on microscopic flows in porous media with chemical flooding, and revealed the onset and evolution of viscous fingering in detail. Jiang [4] and Tanveer [5] reviewed recent studies on experimental and theoretical aspects of viscous fingering. Bonn [6] studied the viscous fingering in complex fluids in which the viscosity or the interfacial tension is changed by means of surfactants or polymers. Masami [7–9] investigated the viscous fingering of HPMC solution by pushing air into radial and linear Hele–Shaw cells. The results showed that the fingering tip velocity of HPMC solution is scaled with 1.5 power of the ratio of injection pressure to viscosity. Tanuja [10] calculated the isothermal and non-isothermal viscous fingering of heavy oil in a porous formation by high temperature and pressurized water. Riaz and Meiburg [11] reported on the stability of axial and helical perturbations. In their subsequent work [12,13], they analyzed numerically three dimensional miscible displacement in homogeneous porous media with gravity override. They conducted a numerical parametric study to investigate the influence of velocity-induced dispersion

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and concentration-dependent diffusion on linear stability of radial displacement in porous media. Kong et al. [14] studied 1D piston-like displacement without considering capillary force and found that the interface stability was governed by the mobility ratio.

From the literatures, we can see that previous studies on the microscopic mechanism of the onset and propagation of viscous fingering in porous media have been mainly conducted by means of numerical and experimental approaches. In the present paper, we firstly extend Kong's work on 1D piston-like displacement to the case of axi-symmetric displacement. Then, the theory of stability analysis is adopted to study the stability of the moving interface in piston- and non-piston-like displacement and its dependence on the parameters of reservoir and operation.

2 Stability of the axi-symmetric piston-like displacement

2.1 Equation governing the moving interface

In water flooding reservoirs, the mixing zone of water and oil phases is considerably small compared with the reservoir dimension. Therefore, it can be assumed as a discontinuous interface. One side of it is filled with movable water and stagnant residual oil, and the other side with movable oil and stagnant irreducible water. This is the so-called piston-like displacement. In this section, we consider an axi-symmetric piston-like displacement with capillary effect involved. The diagrammatic sketch is shown in Fig. 1, in which R_e, r_w and r_c are the radius of the reservoir and the injection well, and the position of the moving interface, respectively.

Generally speaking, in the process of fluid injection into a large and thin reservoir, the pressure gradient is far more influential to the percolation than gravity. By neglecting the gravity and the compressibility of fluids and rock, the pressure distribution is subjected to the following equations

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r}\frac{\partial p_1}{\partial r} = 0, \quad r_w \le r \le r_c, \tag{1a}$$

$$\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r}\frac{\partial p_2}{\partial r} = 0, \quad r_c \le r \le R_e. \tag{1b}$$

With boundary conditions

$$p_1 = p_w, \quad r = r_w, \tag{1c}$$

$$p_2 = p_e, \quad r = R_e, \tag{1d}$$

$$p_1 - p_2 = p_c, \quad r = r_c = \xi(t), \tag{1e}$$

$$M\frac{\partial p_1}{\partial r} = \frac{\partial p_2}{\partial r}, \quad r = r_c = \xi(t), \tag{1f}$$

prescribed, the system is closed. Here, $M = (k_1/\mu_1)/(k_2/\mu_2)$ represents the mobility ratio, μ_1 and μ_2 are the viscosity, k_1 and k_2 are the permeability, p_1 and p_2 denote the pressure on both sides of the moving interface, p_c is the capillary force, p_e and p_w stand for the pressure at the reservoir boundary and the bottom of the injection well, respectively.

Solving Eq. (1) yields

$$p_1 = p_w + \frac{p_e + p_c - p_w}{M\ln\frac{R_e}{r_c} + \ln\frac{r_c}{r_w}}\ln\frac{r}{r_w}, \quad r_w \le r \le r_c, \tag{2}$$

$$p_2 = p_e + \frac{M(p_e + p_c - p_w)}{M\ln\frac{R_e}{r_c} + \ln\frac{r_c}{r_w}}\ln\frac{r}{R_e}, \quad r_c \le r \le R_e. \tag{3}$$

And thus the injection rate can be derived as follows

$$Q = -\frac{k_1}{\mu_1}(2\pi r_w h)\left(\frac{\partial p_1}{\partial r}\right)_{r=r_w} = 2\pi h\frac{k_1}{\mu_1}\frac{p_w - p_e - p_c}{M\ln\frac{R_e}{r_c} + \ln\frac{r_c}{r_w}}, \tag{4}$$

where h is the thickness of the reservoir.

The equation of the moving interface can be written as

$$F(r_c, t) = r_c - \xi(t) = 0. \tag{5}$$

Its derivative gives the velocity of the moving interface, namely

$$v = \frac{d\xi}{dt} = \frac{Q}{A_e} = \frac{k_1}{\Delta s_w \mu_1 \phi}\frac{p_w - p_e - p_c}{M\ln\frac{R_e}{r_c} + \ln\frac{r_c}{r_w}}\frac{1}{r_c}. \tag{6}$$

Here, $A_e = 2\pi r_c h\phi\Delta s_w$, ϕ is the porosity and $\Delta s_w = 1 - s_{wi} - s_{or}$, where s_{wi} denotes the irreducible water saturation and s_{or} the residual oil saturation.

2.2 Stability of the moving interface

On the basis of the theory of stability analysis, we presume an infinitesimal perturbation at a given point, point N for instance (Fig. 1), on the moving surface at time t. This perturbation results in an additional distance denoted by ε at

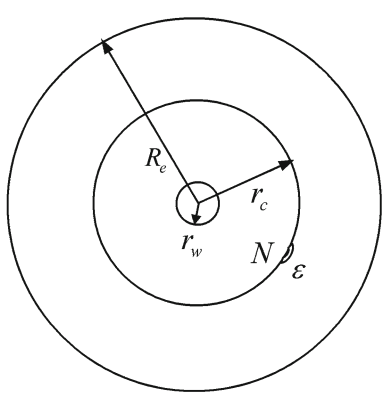

Fig. 1 Physical model of the axi-symmetric piston-like displacement

point N. According to Eq. (6), we have

$$\frac{\mathrm{d}(r_c+\varepsilon)}{\mathrm{d}t} = \frac{k_1}{\Delta s_w \mu_1 \phi} \frac{p_w - p_e - p_c}{M \ln \frac{R_e}{(r_c+\varepsilon)} + \ln \frac{(r_c+\varepsilon)}{r_w}} \frac{1}{(r_c+\varepsilon)}. \quad (7)$$

Subtracting Eq. (6) from Eq. (7), we get

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{k_1(p_w - p_e - p_c)}{\Delta s_w \mu_1 \phi} \times \left(\frac{1}{(r_c+\varepsilon)(M \ln R_e - \ln r_w + (1-M)\ln(r_c+\varepsilon))} - \frac{1}{r_c(M \ln R_e - \ln r_w + (1-M)\ln r_c)} \right). \quad (8)$$

For infinitesimal ε,

$$\ln(r_c+\varepsilon) \approx \ln r_c + \frac{\varepsilon}{r_c}.$$

Substituting this expression into Eq. (8) and omitting the second order terms, we obtain

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{k_1(p_w - p_e - p_c)}{\Delta s_w \mu_1 \phi} \frac{1}{r_c(M \ln R_e - \ln r_w + (1-M)\ln r_c)} \times \left(\frac{1}{1 + \frac{M \ln R_e - \ln r_w + (1-M)\ln r_c + (1-M)}{M \ln R_e - \ln r_w + (1-M)\ln r_c} \frac{\varepsilon}{r_c}} - 1 \right). \quad (9)$$

Because of $\varepsilon \ll r_c$, the complex fraction in the last parentheses on the right hand side of Eq. (9) can be expanded in series. By omitting the second and higher order terms of the series, Eq. (9) can be simplified as

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} \approx \frac{k_1(p_e + p_c - p_w)}{\Delta s_w \mu_1 \phi} \frac{1}{r_c^2(M \ln R_e - \ln r_w + (1-M)\ln r_c)^2} \times \left(M \ln \frac{R_e}{r_c e} + \ln \frac{r_c e}{r_w} \right) \varepsilon, \quad (10)$$

in which e is Euler constant. By judging the sign of $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}$, we may interpret whether the moving interface is stable or not.

In the whole process of fluid injection, the injection pressure p_w is larger than p_e and the expressions $|p_c| < p_w - p_e$, $M > 0$ and $r_c e > r_w$ always hold. Hence, the stability of the moving interface depends on whether the inequality $M \ln \frac{R_e}{r_c e} + \ln \frac{r_c e}{r_w} > 0$ holds or not. If it holds, the interface is stable, and vice versa. Let us see in which circumstances it holds or not. If $r_c < \frac{R_e}{e}$, we have $M \ln \frac{R_e}{r_c e} + \ln \frac{r_c e}{r_w} > 0$ and $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} < 0$. That is to say in the case of $r_c < \frac{R_e}{e}$ the infinitesimal perturbation always decays for any mobility ratio, implying that the moving interface is stable. When $r_c > \frac{R_e}{e}$, we have two situations to deal with. One is $M < \frac{\ln(r_c e/r_w)}{\ln(r_c e/R_e)}$. In this case, $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} < 0$, suggesting a stable moving interface. On the contrary, i.e. $M > \frac{\ln(r_c e/r_w)}{\ln(r_c e/R_e)}$, we have $M \ln \frac{R_e}{r_c e} + \ln \frac{r_c e}{r_w} < 0$ and $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} > 0$, suggesting an unstable moving interface. Consequently, in the case of $r_c e > R_e$, the stability of the moving interface is dependent on the value of the mobility ratio.

From the above analysis, we may conclude that the stability of moving interface in the case of axi-symmetric piston-like displacement is different from that of 1D displacement, in which the stability is governed by the mobility ratio only [14]. In the axi-symmetric piston-like displacement, the stability of moving interface is related to both the interface position and the mobility ratio.

2.3 The effect of the capillary force on the stability

If the capillary force is not considered, namely $p_c = 0$ at $r = r_c = \xi(t)$, Eq. (10) takes a simple form

$$\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t} \approx \frac{k_1(p_e - p_w)}{\Delta s_w \mu_1 \phi} \frac{1}{r_c^2(M \ln R_e - \ln r_w + (1-M)\ln r_c)^2} \times \left(M \ln \frac{R_e}{r_c e} + \ln \frac{r_c e}{r_w} \right) \varepsilon_n, \quad (11)$$

where ε_n denotes the infinitesimal perturbation when neglecting the effect of capillary force. Because $|p_c| < p_w - p_e$ and $p_w > p_e$, $\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t}$ and $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}$ have the same sign. So, we may draw the following conclusions by comparing Eq. (10) with Eq. (11).

(1) For oleophylic reservoirs, in which $p_c > 0$, we have $|\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t}| > |\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}|$. When the moving interface is stable, namely $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} < 0$, the capillary force tends to make the interface unstable. When the moving interface is already unstable, namely $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} > 0$, the capillary force tends to make the interface stable.

(2) For hydrophilic reservoirs, in which $p_c < 0$, so $|\frac{\mathrm{d}\varepsilon_n}{\mathrm{d}t}| < |\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}|$. When the moving interface is stable, the capillary force intensifies its stability. When the moving interface is unstable, the capillary force makes the interface more unstable.

3 Stability of non-piston-like displacements

3.1 1D non-piston-like displacement

The above analysis is based on the ideal assumption of piston-like displacements. But in actual field practice, the displacement is in fact non-piston-like in which the displacing and displaced fluids are separated by a two-phase zone instead of an ideal section with no thickness. In this section, we analyze the stability of non-piston-like displacement based on the Buckley–Leverett theory. The diagrammatic sketch is shown in Fig. 2. According to Zhang [15], the Buckley–Leverett equation for the front motion can be given by

$$\frac{\mathrm{d}x_f}{\mathrm{d}t} = \frac{f'_w(s_{wf})}{\phi A} Q. \quad (12)$$

For 1D displacement, the injection rate becomes

$$Q = \frac{p_w - p_e}{\frac{\mu_w}{kk_{rw}Lh} x_o + \frac{\mu_w}{kLh}\Omega + \frac{\mu_o}{kk_{ro}Lh}(x_e - x_f)}, \quad (13)$$

where

$$\Omega=\int_{x_o}^{x_f}\left[D+B\Delta s_w\left(\frac{x-x_o}{x_f-x_o}\right)^{\frac{1}{b-1}}+C\Delta s_w^2\left(\frac{x-x_o}{x_f-x_o}\right)^{\frac{2}{b-1}}\right]\mathrm{d}x,$$

in which L and h are width and thickness, respectively, A the area of cross section, x_o, x_f and x_e are the positions of the fronts of the single phase zone of displacing fluid, the mixing zone of two-phase fluid and the single phase zone of displaced fluid, respectively, as depicted in Fig. 2. B, C, D and b are constants in a given system and $f_w'(s_{wf})$ is a constant denoting the derivative of water ratio at the displacement front. Define

$$M=\frac{kk_{rw}/\mu_w}{kk_{ro}/\mu_o},$$

where k is the absolute permeability, k_{rw} and k_{ro} are relative permeability of water and oil, respectively, μ_w and μ_o are viscosity of water and oil, respectively.

$$E=\frac{Db(1+b)+B(b^2-1)\Delta s_w+C(b-1)b\Delta s_w^2}{b(b+1)}.$$

Then, the Buckley–Leverett equation becomes

$$\frac{\mathrm{d}x_f}{\mathrm{d}t}=\frac{f_w'(s_{wf})(p_w-p_e)kk_{rw}}{\phi\mu_w}\times\frac{1}{x_o+k_{rw}E(x_f-x_o)+M(x_e-x_f)}. \quad (14)$$

Similar to the analysis in Sect. 2.2, we readily come to the following expression

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\approx\frac{f_w'(s_{wf})(p_e-p_w)kk_{rw}}{\phi\mu_w}\times\frac{1}{[x_o+k_{rw}E(x_f-x_o)+M(x_e-x_f)]^2}\times(Ek_{rw}-M)\varepsilon. \quad (15)$$

The stability can be easily judged from Eq. (15). Generally speaking, p_w is larger than p_e; if $M>Ek_{rw}$, $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}>0$, suggesting the moving interface is unstable. Otherwise, $\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}<0$, thus the moving interface is stable. In 1D piston-like displacement, the stability is governed by the mobility only. When the mobility is larger than one, the interface is unstable [14]. However, in 1D non-piston-like displacement, it is controlled jointly by the mobility ratio and the relative permeability of water phase.

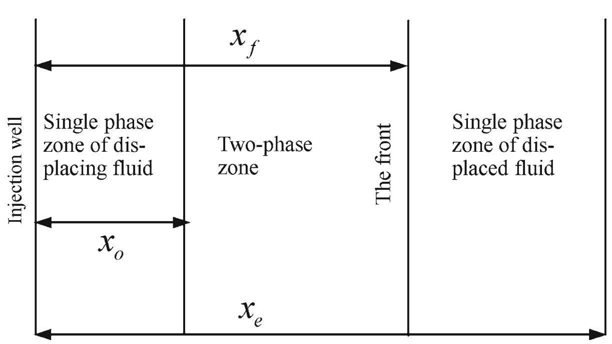

Fig. 2 The physical model of Buckley–Leverett theory

3.2 Axi-symmetric non-piston-like displacement

For axi-symmetric displacement, let us replace x, x_e, x_f and x_o in the above expressions by r, r_e, r_c and r_o, respectively, and conduct the following analysis.

$$\frac{\mathrm{d}r_c}{\mathrm{d}t}=\frac{Q}{2\pi r_c h\phi}f_w'(s_{wf}). \quad (16)$$

When $1<\dfrac{\mu_o}{\mu_w}<10$, the injection rate is

$$Q=\frac{p_w-p_e}{\frac{\mu_w}{2\pi kk_{rw}h}\Omega_1+\frac{\mu_w}{2\pi kh}\Omega_2+\frac{\mu_o}{2\pi kk_{ro}h}\Omega_3}, \quad (17)$$

where

$$\Omega_1=\ln\frac{r_o}{r_w},\quad \Omega_3=\ln\frac{r_e}{r_c},$$

$$\Omega_2=\left[D\ln\frac{r_c}{r_o}+B\Delta s_w\left(1-\frac{r_o}{\sqrt{r_c^2-r_o^2}}\arccos\frac{r_o}{r_c}\right)+C\Delta s_w^2\left(\frac{1}{2}-\frac{r_o^2}{r_c^2-r_o^2}\ln\frac{r_c}{r_o}\right)\right].$$

Substituting Eq. (17) into Eq. (16), we have

$$\frac{\mathrm{d}r_c}{\mathrm{d}t}=\frac{kk_{rw}f_w'(s_{wf})(p_w-p_e)}{\mu_w\phi r_c}\frac{1}{\Omega_1+k_{rw}\Omega_2+M\Omega_3}. \quad (18)$$

Introducing an infinitesimal perturbation ε as above, we have

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}=\frac{kk_{rw}f_w'(s_{wf})(p_w-p_e)}{\mu_w\phi r_c}\frac{1}{\Omega_1+k_{rw}\Omega_2+M\Omega_3}\times\left(\frac{r_c(\Omega_1+k_{rw}\Omega_2+M\Omega_3)}{(r_c+\varepsilon)(\Omega_1+k_{rw}\Omega_2'+M\Omega_3')}-1\right), \quad (19)$$

in which

$$\Omega_2'=\left[D\ln\frac{r_c+\varepsilon}{r_o}+B\Delta s_w\left(1-\frac{r_o}{\sqrt{(r_c+\varepsilon)^2-r_o^2}}\arccos\frac{r_o}{r_c+\varepsilon}\right)+C\Delta s_w^2\left(\frac{1}{2}-\frac{r_o^2}{(r_c+\varepsilon)^2-r_o^2}\ln\frac{r_c+\varepsilon}{r_o}\right)\right],$$

$$\Omega_3'=\ln\frac{r_e}{r_c+\varepsilon}.$$

Substitution of the series expansions of Ω_2' and Ω_3' with higher order terms omitted into Eq. (19) gives

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{kk_{rw} f_w'(s_{wf})(p_w - p_e)}{\mu_w \phi r_c} \frac{1}{\Omega_1 + k_{rw}\Omega_2 + M\Omega_3} \times \left(\frac{1}{1 + \frac{\Omega_o}{\Omega_1 + k_{rw}\Omega_2 + M\Omega_3}} - 1 \right), \tag{20}$$

where

$$\Omega_o = \frac{\varepsilon}{r_c} \left\{ \Omega_1 + k_{rw}\Omega_2 + M\left(\Omega_3 - \frac{1}{r_c}\right) + \Omega_o' \right\}.$$

Because of $\varepsilon \ll r_c$, $\Omega_o \ll \Omega_1 + k_{rw}\Omega_2 + M\Omega_3$ holds. Therefore,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} \approx \frac{kk_{rw} f_w'(s_{wf})(p_e - p_w)}{\mu w \phi r_c^2} \frac{1}{(\Omega_1 + k_{rw}\Omega_2 + M\Omega_3)^2} \times \left(\Omega_1 + k_{rw}\Omega_2 + M\left(\Omega_3 - \frac{1}{r_c}\right) + \Omega_o' \right) \varepsilon. \tag{21}$$

Here,

$$\Omega_o' = k_{rw} r_c \left[A\frac{1}{r_c} - B\Delta s_w \left(\frac{r_o^2}{r_c^2 - r_o^2} - \frac{r_o r_c \arccos(r_o/r_c)}{(r_c^2 - r_o^2)^{3/2}} \right) - C\Delta s_w^2 \left(\frac{r_o^2}{(r_c^2 - r_o^2) r_f} - \frac{2 r_c r_o^2}{(r_c^2 - r_o^2)^2} \ln\frac{r_c}{r_o} \right) \right].$$

Obviously, the moving interface is stable when $\Omega_1 + k_{rw}\Omega_2 + M\left(\Omega_3 - \frac{1}{r_c}\right) + \Omega_o' > 0$. Otherwise, it loses its stability.

4 Conclusions

By theoretical analysis of the stability of the moving interface in piston- and non-piston-like displacement, we come to the following conclusions.

(1) In the case of axi-symmetric piston-like displacement, the stability of the moving interface is related to the mobility and the position of the interface. When $r_c < \frac{R_e}{e}$ the perturbation tends to decay and the moving interface is stable for any mobility ratio. In the case of $r_c > \frac{R_e}{e}$, the moving interface is unstable when $M < \frac{\ln(r_c e/r_w)}{\ln(r_c e/R_e)}$ and vice versa.

(2) The capillary effect on the stability of the piston-like displacement depends on whether or not the moving interface is stable and correlates with the wettability. For oleophilic reservoirs, when the moving interface is stable, the capillary force tends to make it unstable. When it is already unstable, the capillary force tends to make it stable. For hydrophilic reservoirs, when the moving interface is stable, the capillary force intensifies its stability. If it is unstable, the capillary force makes it more unstable.

(3) In the case of 1D non-piston-like displacement, the stability of the interface is governed by both the mobility ratio and the relative permeability of water phase. If $M > Ek_{rw}$, the front is unstable and vice versa. In axi-symmetric non-piston-like displacements, the front is stable if $\Omega_1 + k_{rw}\Omega_2 + M(\Omega_3 - \frac{1}{r_c}) + \Omega_o' > 0$, and vice versa.

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