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Computation of Turbulence-Generated Noise by Large-Eddy Simulation

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Abstract The hybrid method of large eddy simulation (LES) and the Lighthill analogy is being developed to compute the sound radiated from turbulent flows. The results obtained from the hybrid method are often contaminated by the absence of small scales in LES, since the energy level of sound is much smaller than that of turbulent flows. Previous researches investigate the effects of subgrid sacle (SGS) eddies on the frequency spectra of sound radiated by isotropic turbulence and suggest a SGS noise model to represent the SGS contributions to the frequency spectra. Their investigations are conducted in physical space and are unavoidably influenced by boundary conditions. In this paper, we propose to perform such calculations in Fourier space so that the effects of boundary conditions can be correctly treated. Posteriori tests are carried out to investigate the SGS contribution to the sound. The results obtained recover the $-7/2$ law within certain wave-number ranges, but under-estimate the amplitudes of the frequency spectra. The reason for the underestimation is also discussed.

Key words: sound radiation, isotropic turbulence, large eddy simulation, lighthill analogy.

INTRODUCTION

Sound radiated from turbulence is difficult to predict via numerical simulations. Due to its relevance to small scale motions, it can be easily contaminated by the artificial noise stemming from numerical simulations. In 1952, Lighthill [1] developed a theory to calculate the sound generated by the fluctuation of fluids. The so-called Lighthill equation was derived from the Navier-Stokes equation. Proudman [2] used the Lighthill's analogy to estimate the noise radiated by the homogeneous and isotropic turbulence and obtained the $\omega^{-7/2}$ law of frequency spectrum. Kraichnan [3] later considered the scattering of sound wave from turbulent sheared flow. In that work, he decomposed the velocity field of the shear flow into three parts: the transverse part, the longitude part and their interaction. Then he analyzed the angular and frequency distribution of the noise in terms of the four-dimensional Fourier transformations of these three parts. Crighton [4] also investigated the Lighthill theory in spectral space using the four-dimensional Fourier transformation. The method of using Fourier transformation to estimate the power spectrum of noise is widely used (e.g., Ffowcs-Williams [5], Lilley [6, 7], Rubinstein & Zhou [8], Hu & Morfey & Sandham [9]).
Based on the Lighthill theory, the hybrid numerical method is developed to first calculate the acoustic source using direct numerical simulation (DNS) or LES, and then the sound power spectra at far fields. One of the great challenges in the hybrid LES/Lighthill method to predict noise radiation from turbulence is that the sub-grid scale contribution to sound generation is difficult to identified (Colonius & Lele[10], Wang & Freund & Lele[11]). As to the homogeneous and isotropic turbulence, the Lighthill analogy indicates that the acoustic pressure can be directly determined by the Lighthill tensor $T_{ij} \approx u_i u_j$. This approximation is related to some assumptions such as high-Reynolds number, low-Mach number, isentropy condition etc. It implies that as a source term, the velocity fluctuation at each scale does contribute to sound generation. DNS is considered to compute the turbulence without missing any scale. LES only resolves large-scale eddies while the small scale effects are represented by SGS model. The SGS models take no account of the sound radiation from the subgrid scales in turbulence. Thus, the investigation of such effect in the hybrid LES/Lighthill method is

needed. Sarkar and Hussaini[12] adopted the hybrid DNS/Lighthill method to obtain the frequency spectrum of the noise emitted from the isotropic decaying turbulence. They found that the dominant frequency of noise is higher than that of the energy containing eddies and that the acoustic power is in consistence with the result from Proudman. Witkowska, Juvé and Brasseur [13] applied this hybrid method to decaying isotropic turbulence. Their results are in agreement with those referred above. In their research, a comparison between LES and DNS shows that the noise is generated primarily from turbulence scales between the energy-containing and dissipative scale. Seror, Sagaut, Bailly and Juvé[14, 15] investigated the same case through the hybrid method with DNS and LES respectively to find the contribution of different scales to the far field noise. They test the efficiency of Smagorinsky's model, Bardina's model and Liu's model in computing turbulence-generated noise.

All of those researches are concerned with the computation of the turbulent source in physical space. In order to avoid the effects of boundary conditions, we calculate the turbulent source term in spectral space. Both DNS and LES are used in the computation. Then, we analyze the contribution of subgrid scales in the forced isotropic turbulence to the sound generation. The results obtained from LES are compared with those from DNS.

BASIC FORMULAE

According to Lighthill's acoustic analogy, the far-field pressure fluctuation due to sound wave propagation is

$$p(\vec{x},t) = \frac{1}{4\pi c_0^2}\frac{x_i x_j}{x^3}\int_V \frac{\partial^2}{\partial t^2}T_{ij}(\vec{y},t-\frac{|\vec{x}-\vec{y}|}{c_0})\mathrm{d}\vec{y} \tag{1}$$

$$T_{ij}(\vec{y},t) = u_i(\vec{y},t)u_j(\vec{y},t) \tag{2}$$

where $T_{ij}(\vec{y},t)$ is the Lighthill stress tensor. After non-dimensionalization and a four-dimensional Fourier transformation, a corresponding pressure fluctuation in the frequency domain is given by,

$$p(\vec{x},\omega) = 2\pi^2 M^2\omega^2\frac{x_i x_j}{x^3}\hat{\hat{T}}_{ij}(\vec{k}_0,\omega)\exp(\mathrm{i}\omega M x) \tag{3}$$

$$\vec{k}_0 = -\omega M\vec{x}/x \tag{4}$$

where $x = |\vec{x}|$, and $M$is the Mach-number. The hat denotes Fourier transform. Eq. (3) indicates that the frequency of the acoustic wave is related to the wave-number of the flow. Namely, a certain scale of the turbulence only generates noise of a corresponding frequency. This equation is then used to calculate the frequency spectra of the acoustic pressure.

$$J(\vec{x},\omega) = 2\pi\left\langle p(\vec{x},\omega)p(\vec{x},-\omega)\right\rangle/\rho_0 c_0 \tag{5}$$

$$I(\omega) = x^2 J(\vec{x},\omega) = 8\pi^5 M^4\omega^4\frac{x_i x_j x_k x_l}{x^4}\hat{\hat{T}}_{ij}(\vec{k}_0,\omega)\hat{\hat{T}}^*_{ij}(\vec{k}_0,\omega) \tag{6}$$

where the asterisk denotes conjugate. From Eq. (6) we know that $I(\omega)$ is independent of x for a fixed angle$arcos\,(x_i/x)$. The sound pressure level is defined as,

$$SPL = 10\log[I(\vec{x},\omega)] \tag{7}$$

For the purpose of studying the Lighthill tensors$\hat{T}_{ij}(\vec{k},\omega)$after the Fourier transformation, we introduce

$$S(\vec{k},\omega) = \left\langle \hat{T}_{ij}(\vec{k},\omega)\hat{T}^*_{ij}(\vec{k},\omega)\right\rangle \tag{8}$$

where the angular bracket denotes ensemble average and $\vec{k}$is independent of ω. Blackman window and Hanning window is used in data windowing when taking the time Fourier transformation of $\hat{T}_{ij}(\vec{k},t)$ to get $\hat{T}_{ij}(\vec{k},\omega)$.

NUMERICAL RESULTS

The forced isotropic turbulence is simulated using the spectral method. The simulation is performed on a $N^3 = 128^3$ uniform spatial grid in a cubic domain where periodic boundary condition is applied. The Adams - Bashford scheme is used for the time advancing. To obtain statistically stationary turbulence, the energy of the first two wave-number shells is artificially kept in constant value to satisfy $-5/3$ power law. The spectral eddy-viscosity model is used in LES. The parameters of DNS and LES for the forced isotropic turbulence are shown in Table 1. Figure 1 presents the energy spectra of DNS and LES where the $-5/3$ power law is recovered in both cases.

In order to compute the statistic characteristics of the far-field acoustic pressure, 1008 different observation positions are chosen outside the computation domain. When observation position for the cubic source is chosen in the far-field, there are the other 47 corresponding ones point-symmetric to it. Because the turbulence is isotropic, these 48 points have the same statistical characteristics. Hence, 1 008 points are divided to 21 groups each contains 48 points. According to the physical meaning inferred from Eq. (6), computation of frequency spectra in these points only requires the fixed angels deduced from their positions. Figure 2 shows the frequency spectra of $S(\vec{k}, \omega)$ in DNS at $k = 9, 24$ with the Blackman window and the Hanning window. The fluctuations, numerical noise, are observed in the high frequency range where S falls below 10^{-21}. Figure 3 compares the acoustic frequency spectra of DNS with Proudman's model and Lilley's model. The results are in agreement with theoretical models in certain range. As shown in Figure 3, the peak frequency is located at the lowest wave-number. This observation is different from that of the previous calculations, Seror, Sagaut, Bailly and Juvé[14, 15], and still needs to be justified in our future research. Figure 4 shows that the computational result from LES is inconsistent with that by DNS in amplitude although the two results have the same trend in decaying (both recover the $\omega^{-7/2}$ power-law in certain ranges of frequency).

Table 1 Parameters of the forced isotropic turbulence

Case	N			Δt	Ma
DNS	128	81	0.0062	0.004	0.3
LES	64	92.72	0.0062	0.004	0.3

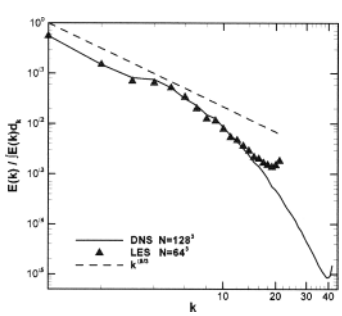

Figure 1: Comparison of energy spectra computed from DNS and LES

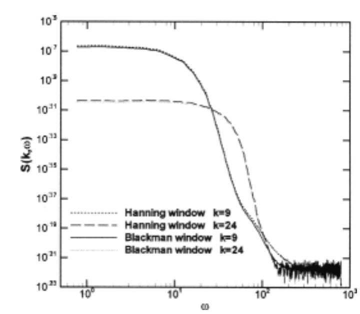

Figure 2: The frequency spectra of $S(\vec{k}, \omega)$ at $k = 9, 24$ with Blackman window and Hanning window (DNS)

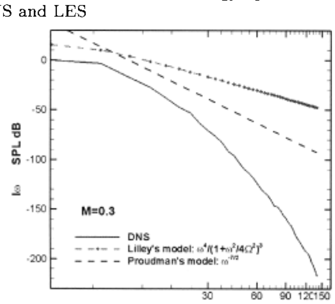

Figure 3: The acoustic frequency spectra of DNS compared with Lilley's model and Proudman's model

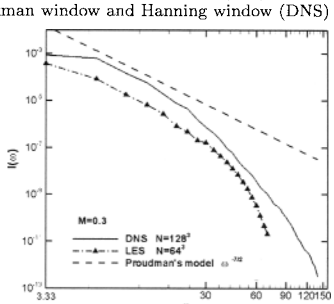

Figure 4: The acoustic frequency spectra of DNS and LES compared with Proudman's model

CONCLUSIONS

The sound radiated from the forced isotropic turbulence is calculated using the hybrid methods of DNS/ LES with the Lighthill's acoustic analogy. The spectral eddy-viscosity model is used in LES. From the formula for pressure fluctuation in the frequency domain, we know that a certain scale of the turbulence only generates the noise of a corresponding frequency. Numerical results show that all scales of the turbulent flow make contributions to the sound power spectra. It is also shown that the acoustic frequency spectrum in DNS follows Lilley's model for the frequencies for $\omega \leq 6$, and deviate from it with a faster decaying for $\omega > 6$. The spectrum recovers Proudman's result of $\omega^{-7/2}$ law for $6 \leq \omega \leq 25$, but also falls fast for $\omega > 25$. Therefore, the results are in agreement with theoretical models at certain ranges: Lilley's model at lower frequencies and Proundman's model at comparatively higher ones. However, all of the models cannot predict the correct rate of decaying at the high frequencies. Besides these, it is found that the results from LES are lower than that from DNS in amplitude. It is suggested that a novel SGS model needs to be developed in order to accurately predict the sound in the hybrid LES/Lighthill approach.

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