Instability from Steady and Axisymmetric to Steady and Asymmetric Floating Half Zone Convection in a Fat Liquid Bridge of Larger Prandtl Number *

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The linear instability analysis of the present paper shows that the thermocapillary convection in a half floating zone of larger Prandtl number has a steady instability mode $\omega_i = 0$ and $m = 1$ for a fat liquid bridge $V = 1.2$ with small geometrical aspect ratio $A = 0.6$. This conclusion is different from the usual idea of hydrothermal instability, and implies that the instability of the system may excite a steady and axial asymmetric state before the onset of oscillation in the case of large Prandtl number.

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The volume of liquid bridge is a sensitive critical geometrical parameter in the floating half zone convection,[1] and the marginal curves for the onset of oscillation are usually divided into two branches separated typically by a gap region as shown in Fig. 1. The results were obtained by the ground-based experiments[2-4] and the microgravity experiments performed by using the drop shaft facility.[5,6] Similar conclusions were given by the numerical simulation of two-dimensional model[7] and three-dimensional model,[8] and by the linear instability analysis.[9] The influence of liquid bridge volume on the onset of oscillation is quite different in the cases of smaller Prandtl number fluid[10] in comparison with the cases of large Prandtl numbers fluid.[9] The onset of instability in the thermocapillary convection from the steady and axi-symmetric convection has been discussed by the linear instability analysis,[9,11,12] the energy method analysis,[13] and by the unsteady and three-dimensional numerical simulation.[8,14] Many studies support the mechanism of hydrothermal instability.[15]

A liquid bridge of floating half zone between two co-axial copper rods of the same diameter $2r_0$ as shown in Fig. 2 is discussed in the present paper. The liquid bridge has a height l. There are two typical geometrical parameters: the geometrical aspect ratio $A = l/(2r_0)$ and volume ratio $V = V_l/V_0$, where V_l and V_0 are respectively the volumes of the liquid bridge and a cylinder with height l and r_0 in radius. The lower rod keeps a constant temperature T_0, and the temperature at the upper rod is $T_0 + \Delta T$, where non-negative temperature difference ΔT may be a constant or change with time. The iso-thermal case relates to $\Delta T = 0$, and the thermocapillary convection is driven by the gradient of surface tension. The surface tension σ is defined as $\sigma = \sigma_0 + (\mathrm{d}\sigma/\mathrm{d}T)(T - T^*)$, where T^* is a constant reference temperature, and $\mathrm{d}\sigma/\mathrm{d}T$ is usually negative. The instability may be excited during the increasing of the applied temperature difference.

The thermocapillary convection in the liquid bridge is controlled by the relationships of mass, momentum and energy conservation. By using the Boussinesq approximation, these relationships in the microgravity environment may be written mathematically as follows.

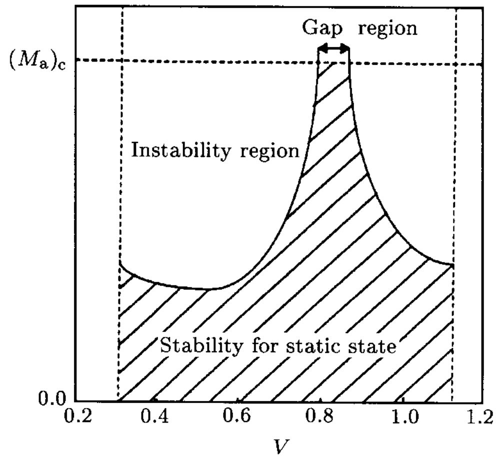

Fig. 1. Typical marginal curves for onset of oscillatory thermocapillary convection of larger Prandtl number fluid.

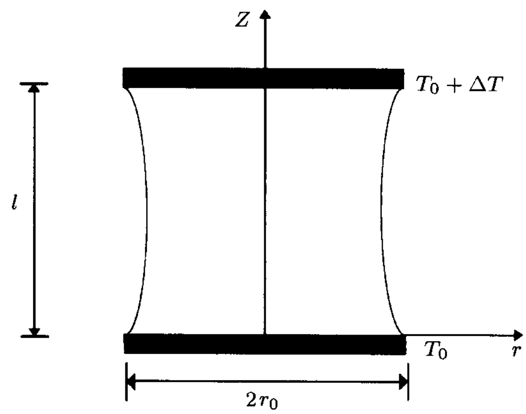

Fig. 2. Configuration of the liquid bridge in a floating half zone.

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla\left(\frac{p}{\rho}\right) + \nu \Delta \boldsymbol{v}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \nabla T = \kappa \Delta T, \tag{3}$$

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where ρ, p, and T are respectively the density, pressure, and temperature of the liquid, $\boldsymbol{v} = (u, v, w)$ is the velocity vector, Δ is the Laplace operator, ν and κ are respectively the kinematics viscosity and thermal diffusion coefficients. Equations (1)−(3) may be written in a cylindrical coordinate system as adopted in Fig. 2. The equations should be solved under the associated boundary conditions, see, for example, Chen and Hu.[9]

Non-dimensional parameters of the Reynold number, Marangoni number, and Prandtl number are respectively

$$R_e = \frac{v^* l}{\nu}, \quad M_a = \frac{v^* l}{\kappa}, \quad P_r = \frac{\nu}{\kappa}, \qquad (4)$$

where the typical velocity v^* is defined by the thermocapillary effect as $v^* = |\mathrm{d}\sigma/\mathrm{d}T|\Delta T/\rho\nu$, where $\mathrm{d}\sigma/\mathrm{d}T$ is a constant and ΔT is the constant temperature difference. Three parameters in Eq. (4) are related by $M_a = R_e P_r$. There are two geometrical parameters A and V, and the volume ratio V is defined as

$$V = \frac{1}{r_0^2 l_0} \int_0^l R^2(z) \mathrm{d}z,$$

where $r = R(z)$ is the equation of free surface. The case of a cylindrical liquid bridge relates to $V = 1$.

The linear instability analysis is applied for a liquid bridge of large Prandtl number.[9,10] At first, a steady and axisymmetric state is obtained numerically for a fixed applied temperature difference ΔT. Then, the linear perturbation theory is used to analyze the instability mode, and the perturbation quantities (u', v', w', p', T') are expanded as a sum of the spectral terms, for example

$$u'(r, \theta, z, t) = \sum \exp(\omega t + \mathrm{i}\, m\theta) u_m(r, z) + \text{c.c.},$$

c.c. denotes the complex conjugate terms, and $\omega = \omega_r + \mathrm{i}\,\omega_i$, which is dimensionless by $v^*/(2r_0)$. The instability states were obtained for the case of $P_r = 100$ and $A = 0.6$ in Table 1.

Table 1. Instability states for the case $P_r = 100$ and $A = 0.6$.

Ratio volume V	Critical M_a	Azimuthal mode m	Frequency ω_i	Related state
0.6	9595	1	2.49	Oscillatory
0.8	—	–	—	Not obtained
1.0	26949	1	4.30	Oscillatory
1.2	20860	1	0	Steady

The case $V = 0.6$ relates to a slender liquid bridge, and the instability excites directly the onset of oscillation ($\omega_i = 2.49 \neq 0$). The case $V = 0.8$ relates to the gap region as shown in Fig. 1. The case $V = 1.2$ shows that the steady and axisymmetric thermocapillary convection transits to a steady ($\omega_i = 0$) and asymmetric ($m = 1$) convection in a fat liquid bridge of larger Prandtl number. The conclusion is similar to the ones in the case of small Prandtl number fluid,[10,12,16] where the instability mechanism is not the hydrothermal.[17]

A new feature of the liquid bridge volume is given here, in addition to the ones described in Fig. 1. Until now, all the studies both in theoretical and experimental approaches suggest that, the steady and axisymmetric thermocapillary convection in a liquid bridge of large Prandtl number will transit directly to the onset of oscillatory thermocapillary convection when the applied temperature difference is larger than a critical value. Many studies believe that the onset of oscillation is due to the so-called hydrothermal instability, which requires that the perturbation state be shown as a travelling wave. The results of present paper give an example, which shows that, the onset of the instability relates to the transition to a steady and axial asymmetric state but not an oscillatory state in a fat liquid bridge of large Prandtl number. Therefore, the instability mechanism cannot be the hydrothermal model in this case. Furthermore, the direct numerical simulation of three dimensional and unsteady model shows that there may be two bifurcation transitions: firstly from steady and axisymmetric convection to the steady and asymmetrical convection, and then to the oscillatory convection in a fat liquid bridge of larger Prandtl number with small aspect ratio. The results are similar to the ones discussed in the case of small Prandtl number,[17] and will be discussed in details elsewhere. These results of present paper may shed light to the mechanism studies on the onset of oscillatory thermocapillary convection in the case of a liquid bridge of large Prandtl number.

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