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A CRACK PERPENDICULAR TO AND TERMINATING AT A BIMATERIAL INTERFACE*

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ABSTRACT: Using dislocation simulation approach, the basic equation for a finite crack perpendicular to and terminating at a bimaterial interface is formulated. A novel expansion method is proposed for solving the problem. The complete solution to the problem, including the explicit formulae for the T stresses ahead of the crack tip and the stress intensity factors are presented. The stress field characteristics are analysed in detail. It is found that normal stresses σ_x and σ_y ahead of the crack tip, are characterised by Q fields if the crack is within a stiff material and the parameters $|p_T|$ and $|q_T|$ are very small, where Q is a generalised stress intensity factor for a crack normal to and terminating at the interface. If the crack is within a weak material, the normal stresses σ_x and σ_y are dominated by the Q field plus T stress.

KEY WORDS: interface, perpendicular crack, gerneralized stress intensity factor, T stress

1 INTRODUCTION

Many modern devices, tools and engineering structures are made of advanced materials, such as fiber or particle reinforced composites, metal/ceramics interfaces, laminated ceramics, adhesive joints etc. Interface failures are common features in the advanced materials and thin films. The design process of these components requires a better understanding of the failure mechanisms of these components. An important task is to study in detail the fracture characteristics of flaws along or perpendicular to the interface.

A crack perpendicular to a bimaterial interface has attracted the attention of many investigators. Zak and Williams[1] used the eigenfunction expansion method to analyse the stress singularity ahead of a crack tip, which is perpendicular to and terminating at the interface. Cook and Erdogan[2] used the Mellin transform method to derive the governing equation of a finite crack perpendicular to the interface and obtained the stress intensity factors. Erdogan and Biricikoglu[3] solved the problem of two bounded half planes with a crack going through the interface. Bogy[4] investigated the stress singularity of an infinite crack terminated at the interface with an arbitrary angle. Wang and Chen[5] used photoelasticity to determine the stress distribution and the stress intensity factors of a crack

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perpendicular to the interface. Lin and Mar[6], Ahmad[7] and Meguid et al.[8] used finite element to analyse cracks perpendicular to bimaterial in finite elastic body. Chen[9] used the body force method to determine the stress intensity factors for a crack normal to and terminated at a bimaterial interface. Ståhle et al.[10,11] investigated a crack growing towards a bimaterial interface. Their results showed that the crack can be deflected and to follow a smooth curved path.

2 FORMULATION OF THE CRACK PROBLEM

Figure 1 shows a finite crack perpendicular to and terminating at a bimaterial interface. A Cartesian coordinate system oxy is attached to the interface. The x axis is along the interface and the y axis is normal to the interface and coincident with the crack elongation direction. Both materials are isotropic and homogenous. The material I occupies the upper half plane S_1 and the material II occupies the lower half plane S_2.

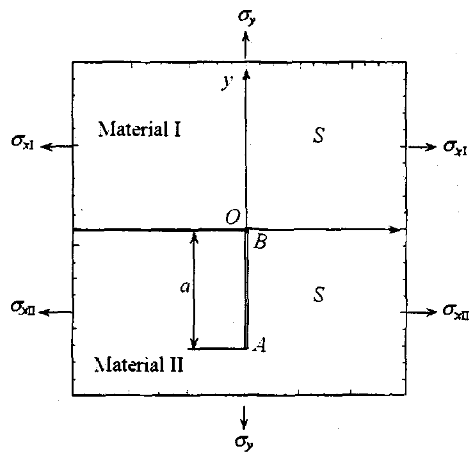

Fig.1 A finite crack perpendicular to and terminating at a bimaterial interface

2.1 Complex Potential

Stress and displacement in an elastic solid can be represented by two Muskhelishivili's potentials

$$\left.\begin{aligned} \sigma_x + \sigma_y &= 4\mathrm{Re}(\Phi(z)) \\ \sigma_y - \mathrm{i}\tau_{xy} &= \Phi(z) + \Omega(\bar{z}) + (z-\bar{z})\overline{\Phi'(z)} \\ 2\mu(u_x + \mathrm{i}u_y) &= \kappa\phi(z) - \omega(\bar{z}) - (z-\bar{z})\overline{\Phi(z)} \end{aligned}\right\} \tag{1}$$

The complex potentials for an edge dislocation at $z = s$ in an infinite elastic solid can be expressed as

$$\left.\begin{aligned} \Phi_0(z) &= \frac{B}{z-s} \\ \Omega_0(z) &= \frac{B}{z-\bar{s}} + \bar{B}\frac{s-\bar{s}}{(z-\bar{s})^2} \\ B &= \frac{\mu}{\pi\mathrm{i}(\kappa+1)}(b_x + \mathrm{i}b_y) \end{aligned}\right\} \tag{2}$$

where b_x and b_y are the x- and y-components of the edge dislocation.

The interaction problem of an edge dislocation with a bimaterial interface was studied by Dundurs[12] and Suo[13]. The complex potentials are (see Suo[13])

$$\left.\begin{aligned} \Phi(z) &= \begin{cases} (1+\Lambda_1)\Phi_0(z) & z \in S_1 \\ \Phi_0(z) + \Lambda_2\Omega_0(z) & z \in S_2 \end{cases} \\ \Omega(z) &= \begin{cases} \Omega_0(z) + \Lambda_1\Phi_0(z) & z \in S_1 \\ (1+\Lambda_2)\Omega_0(z) & z \in S_2 \end{cases} \end{aligned}\right\} \tag{3}$$

$$\Lambda_1 = \frac{\alpha+\beta}{1-\beta} \qquad \Lambda_2 = \frac{\alpha-\beta}{1+\beta}$$

where α and β are two Dundurs parameters.

The crack can be considered as a continuous distribution of dislocations. Hence, we have

$$\left.\begin{aligned} \Phi_0(z) &= \frac{\mu_2}{\pi\mathrm{i}(1+\kappa_2)}\int_0^a \frac{(b_x+\mathrm{i}b_y)}{z+\mathrm{i}t}\mathrm{d}t \\ \Omega_0(z) &= \frac{\mu_2}{\pi\mathrm{i}(1+\kappa_2)}\int_0^a \frac{(b_x+\mathrm{i}b_y)}{z-\mathrm{i}t}\mathrm{d}t + \frac{2\mu_2}{\pi(1+\kappa_2)}\int_0^a \frac{t(b_x-\mathrm{i}b_y)}{(z-\mathrm{i}t)^2}\mathrm{d}t \end{aligned}\right\} \tag{4}$$

where a is the crack length, i is a pure imaginary.

Introduce a new complex variable z^* and a new function $I(z^*)$

$$z^* = \mathrm{i}z \qquad I(z^*) = \frac{1}{\pi}\int_0^a \frac{b_x+\mathrm{i}b_y}{z^*-t}\mathrm{d}t \tag{5}$$

The function $I(z^*)$ is a holomorphic function in the whole complex plane z^* outside the cut $(0,a)$. Using the following variable transformations

$$z^* = \frac{a}{2}(1+\zeta) \qquad t = \frac{a}{2}(1+\xi) \tag{6}$$

the function $I(z^*)$ can be represented as

$$I(z^*) = \frac{1}{\pi}\int_{-1}^1 \frac{b_x+\mathrm{i}b_y}{\zeta-\xi}\mathrm{d}\xi \tag{7}$$

Assume that the dislocation density can be expanded as a series of the first Chebyshev polynomial plus a special term, which creates a desired stress singularity at crack tip B

$$b_x+\mathrm{i}b_y = \frac{1}{\sqrt{1-\xi^2}}\sum_{m=0}^{\infty}\alpha_m T_m(\xi) + \beta_0\Big(\frac{1-\xi}{1+\xi}\Big)^{\lambda_0} \tag{8}$$

where $T_m(\xi)$ is the first Chebyshev polynomial

$$T_m(\xi) = \cos m\theta \qquad \xi = \cos\theta$$

and $\lambda_0 = |\lambda_{\min}|$, $\lambda_{\min}$ is the smallest real eigenvalue of the present problem in the region $-1 < \lambda < 0$. The opening displacement on the crack surface can be expressed as

$$\delta_x + \mathrm{i}\delta_y = -\int_0^t (b_x+\mathrm{i}b_y)\mathrm{d}t = -\int_{-1}^{\xi}\Big[\frac{1}{\sqrt{1-\xi^2}}\sum_{m=0}^{\infty}\alpha_m T(\xi) + \beta_0\Big(\frac{1-\xi}{1+\xi}\Big)^{\lambda_0}\Big]\mathrm{d}\xi a_0 =$$

$$a_0\alpha_0(\pi-\theta) + a_0\sum_{m=1}^{\infty}\alpha_m\frac{\sin m\theta}{m} + \beta_0\frac{2\pi\lambda_0}{\sin\pi\lambda_0}a_0$$

where a_0 is half of the crack length. We have

$$a_0 = a/2 \qquad \xi = \cos\theta = (t-a_0)/a_0$$

The opening displacement at point A should be zero. It means that when $t=a, \xi=1$ and $\theta=0$, the opening displacement vanishes. It leads to

$$\alpha_0 + \beta_0\frac{2\lambda_0}{\sin\pi\lambda_0} = 0 \tag{9}$$

Substituting Eq.(8) into Eq.(7) and using the following equation (see Gladwell[14])

$$\frac{1}{\pi}\int_{-1}^{1}\frac{T_m(x)}{\sqrt{1-x^2}(z-x)}\mathrm{d}x=\frac{1}{\sqrt{z^2-1}}\left[z-\sqrt{z^2-1}\right]^m$$

we obtain

$$I(z^*)=\frac{1}{\sqrt{\zeta^2-1}}\sum_{m=0,1}^{\infty}\alpha_m\left[\zeta-\sqrt{\zeta^2-1}\right]^m-\frac{\beta_0}{\sin\pi\lambda_0}\left[\left(\frac{\zeta-1}{\zeta+1}\right)^{\lambda_0}-1\right]$$

$$\zeta=\frac{(z^*-a_0)}{a_0}$$

2.2 Stress Jump Across Interface

The displacement u_x and u_y should be continuous across the interface. Hence the strain ε_x should be continuous across the interface. It follows that

$$(\varepsilon_x)_{\mathrm{I}}=(\varepsilon_x)_{\mathrm{II}}$$

For plane strain problem, from this equation, we obtain directly

$$(\sigma_x)_{\mathrm{I}}=\frac{\mu_1(1-\nu_2)}{\mu_2(1-\nu_1)}(\sigma_x)_{\mathrm{II}}+\frac{\sigma_y}{1-\nu_1}\left(\nu_1-\frac{\mu_1}{\mu_2}\nu_2\right)$$

The above equation can be represented as

$$(\sigma_x)_{\mathrm{I}}=\frac{(1+\alpha)}{(1-\alpha)}(\sigma_x)_{\mathrm{II}}+\frac{2\sigma_y}{(1-\alpha)}(2\beta-\alpha)\tag{10}$$

Of course this equation is also valid for plane stress problem. We should emphasise that the stress jumping equation (10) is important for understanding the interface problem. At infinity we have

$$(\sigma_x^{\infty})_{\mathrm{I}}=\frac{(1+\alpha)}{(1-\alpha)}(\sigma_x^{\infty})_{\mathrm{II}}+\frac{2\sigma_y^{\infty}}{(1-\alpha)}(2\beta-\alpha)\tag{11}$$

2.3 Superposition Scheme

On the crack surfaces the traction free condition should be satisfied.

According to the superposition scheme, we need two solutions. The first solution is that for the bimaterial subject to uniform remote loading. We only consider the symmetric problem. The skew-symmetric problem can be treated in a similar way. The first solution for the symmetrical problem is

$$\begin{aligned}&\sigma_x=(\sigma_x^{\infty})_{\mathrm{I}}\qquad \sigma_y=\sigma_y^{\infty}\qquad \tau_{xy}=0\quad \text{in material I}\\&\sigma_x=(\sigma_x^{\infty})_{\mathrm{II}}\qquad \sigma_y=\sigma_y^{\infty}\qquad \tau_{xy}=0\quad \text{in material II}\end{aligned}\tag{12}$$

The load parameter σ_y^{∞} only produces the uniform stress fields and has no effect on the stress σ_x in material II if the load parameter $(\sigma_x^{\infty})_{\mathrm{II}}$ remains constant. Hence without loss of generality, we only study the problem with $\sigma_y^{\infty}=0$.

The second solution is that for crack perpendicular to the interface with the uniform traction prescribed on the crack faces. Thus we have

$$\sigma_x+\mathrm{i}\tau_{xy}=\Phi(z)+2\overline{\Phi(z)}-\Omega(\bar{z})-(z-\bar{z})\overline{\Phi(z)}=-\sigma$$

$$\text{at}\quad z=\pm0+\mathrm{i}y\qquad -a<y<0\tag{13}$$

where $\sigma = (\sigma_x^\infty)_{\mathrm{II}}$.

Using the function $I(z^*)$ and Eq.(4), for symmetrical problem ($b_y = 0$) one obtains

$$\Phi_0(z) = \frac{\mu_2}{(\kappa_2+1)} I(\mathrm{i}z) \tag{14}$$

$$\Omega_0(z) = \frac{\mu_2}{(\kappa_2+1)} I(-\mathrm{i}z) - \frac{2\mu_2}{(\kappa_2+1)} \mathrm{i}zI'(-\mathrm{i}z) \tag{15}$$

Substitute Eqs.(3), (14) and (15) into Eq.(13), one obtains the following governing equation

$$\begin{aligned}\frac{\mu_2}{\kappa_2+1}\{2\overline{I^+(t)} + 2t\big[(\overline{I'^+(t)} - I'^-(t)\big] + I^+(t) - I^-(t) + (3\Lambda_2 - \Lambda_1)I(-t) - \\ 12\Lambda_2 tI'(-t) + 4t^2\Lambda_2 I''(-t)\big\} = -\sigma \qquad 0 < t < a\end{aligned} \tag{16}$$

2.4 Stress Intensity Factor

The stress distribution ahead of the crack tip B can be expressed as follows

$$\sigma_x + \mathrm{i}\tau_{xy} = (2 + 3\Lambda_1 - \Lambda_2)I(t) + 2(\Lambda_1 - \Lambda_2)t\overline{I'(t)} \tag{17}$$

On the right side of the Eq.(17), the function $I(t)$ can be expanded as

$$I(t) = \frac{1}{\sqrt{\xi^2-1}} \sum_{m=1}^{\infty} \alpha_m \big[\xi - \sqrt{\xi^2-1}\big]^m - \frac{\beta_0}{\sin\pi\lambda_0}\Big[\Big(\frac{\xi-1}{\xi+1}\Big)^{\lambda_0} - 1\Big] \tag{18}$$

If $\lambda_0 > 0.5$, the most strong stress singularity is produced by the special term, which is related to the parameter β_0. The stresses near the crack tip B can be represented as

$$\sigma_{ij} = \frac{Q_{\mathrm{I}}}{(2\pi r)^{\lambda_0}} f_{ij}(\theta) \tag{19}$$

where Q_{I} is the generalised stress intensity factor. From Eqs.(17) and (18), one obtains

$$Q_{\mathrm{I}}(B) = \lim_{r\to 0}(2\pi r)^{\lambda_0}\sigma_x = -\frac{\beta_0}{\sin\pi\lambda_0}(2\pi a_0)^{\lambda_0}2^{\lambda_0}[(2 + 3\lambda_0 - \Lambda_2) - 2\lambda_0(\Lambda_1 - \Lambda_2)] \tag{20}$$

The normal stress ahead of the crack tip B can be rewritten as

$$\sigma_x = \frac{Q_{\mathrm{I}}}{(2\pi r)^{\lambda_0}} f_x(\theta) + \sigma_x^*$$

For stress σ_x^*, we have

$$\lim_{r\to 0}\sqrt{2\pi r}\sigma_x^* = -\frac{2\mu_2}{(\kappa_2+1)}(1+\Lambda_1)\sqrt{\pi a_0}\sum_{m=1}^{\infty}(-1)^m\alpha_m$$

Of course such a singularity should be excluded. It follows that

$$\sum_{m=1}^{\infty}(-1)^m\alpha_m = 0 \tag{21}$$

This is the second constraint condition for our problem. Similarly we can find the standard stress intensity factor at crack tip A

$$K_{\mathrm{I}}(A) = -\frac{2\mu_2}{(\kappa_2+1)}\sqrt{\pi a_0}\sum_{m=1}^{\infty}\alpha_m \tag{22}$$

The governing equation (16) is a function equation, which contains a set of unknown coefficients α_m and β_0. The infinite series in Eq.(8) can be approximated with a sufficient degree of accuracy by the corresponding truncated series. The crack surface is discretized into $M+1$ elements. The nodal points are given by the following expression

$$t_k = c + a_0(1+\cos\theta_k) \qquad \theta_k = \frac{k\pi}{(M+1)} \qquad k = 1, 2, \cdots, M \tag{23}$$

According to the boundary collocation method, the governing Eq.(16) should be satisfied on these nodal points. Then the governing equation is transformed into a set of linear algebraic equations. From these equations and the two constraint conditions (9) and (21), one can get a set of unknown coefficients $\alpha_m (m = 0, 1, 2, \cdots, M)$ and β_0.

2.5 T Stress

The stress fields ahead of the crack tip can be expressed as an asymptotic series

$$\sigma_{ij} = \frac{Q_I}{(2\pi r)^{\lambda_0}} f_{ij}(\theta) + T_{ij} \tag{24}$$

The parameters T_{ij} define the T stress, which characterises the second term of the eigenfunction expansion and plays an important role in the analysis of fracture process. From above equations and Eq.(17), we obtain

$$(T_x^*)_{\mathrm{I}} = \frac{\mu_2}{\kappa_2+1}(2+3\Lambda_1-\Lambda_2)\sum_{m=1}^{\infty}(-1)^m m\alpha_m \tag{25}$$

where T_x^* is the x- component of T stress contributed by the second solution of the symmetric problem. On the other hand we have

$$\sigma_x + \sigma_y = 4\mathrm{Re}\{\Phi(z)\} = \frac{4\mu_2}{\kappa_2+1}\mathrm{Re}\{(1+\Lambda_1)I(t)\}$$

Similar analysis leads to

$$(T_x^* + T_y^*)_{\mathrm{I}} = \frac{4\mu_2}{\kappa_2+1}(1+\Lambda_1)\sum_{m=1}^{\infty}(-1)^m m\alpha_m$$

From above equation and Eq.(25), it follows

$$(T_y^*)_{\mathrm{I}} = \frac{\mu_2}{\kappa_2+1}(2+\Lambda_1+\Lambda_2)\sum_{m=1}^{\infty}(-1)^m m\alpha_m \tag{26}$$

Now we obtain

$$\frac{(T_x^*)_{\mathrm{I}}}{(T_y^*)_{\mathrm{I}}} = \frac{(2+3\Lambda_1-\Lambda_2)}{(2+\Lambda_1+\Lambda_2)} = 1 + 2\beta \tag{27}$$

From the stress jump Eq.(10), it follows

$$(T_x^*)_{\mathrm{I}} = \frac{(1+\alpha)}{(1-\alpha)}(T_x^*)_{\mathrm{II}} + \frac{2T_y^*}{(1-\alpha)}(2\beta-\alpha) \tag{28}$$

where $T_y^* = (T_y^*)_{\mathrm{I}} = (T_y^*)_{\mathrm{II}}$. Using Eqs.(27) and (28), one obtains

$$\left.\begin{aligned} T_y^* &= -\frac{(1+\alpha)\sigma}{[(1-\alpha)(1+2\beta)-2(2\beta-\alpha)]} = -\frac{1}{1-2\beta}\sigma \\ (T_x^*)_{\mathrm{I}} &= -\frac{1+2\beta}{1-2\beta}\sigma \end{aligned}\right\} \tag{29}$$

The total T_y stress around the crack tip B is described as

$$\left.\begin{aligned} T_y &= \sigma_y^\infty + T_y^* = q_T(\sigma_x^\infty)_{\mathrm{II}} \\ q_T &= \frac{\sigma_y^\infty}{(\sigma_x^\infty)_{\mathrm{II}}} - \frac{1}{1-2\beta} \end{aligned}\right\} \tag{30a}$$

Parameter q_T characterises the effect of the remote stresses $(\sigma_x^\infty)_{\mathrm{II}}$ and σ_y^∞ on the T_y stress. The total T_x stress around the crack tip B in material I can be expressed as

$$\left.\begin{aligned} (T_x)_{\mathrm{I}} &= \frac{1+\alpha}{1-\alpha}\sigma + \frac{2(2\beta-\alpha)}{1-\alpha}\sigma_y^\infty + (T_x^*)_{\mathrm{I}} = p_T(\sigma_x^\infty)_{\mathrm{II}} \\ p_T &= \frac{1+\alpha}{1-\alpha} + \frac{2(2\beta-\alpha)}{1-\alpha}\frac{\sigma_y^\infty}{(\sigma_x^\infty)_{\mathrm{II}}} - \frac{1+2\beta}{1-2\beta} \end{aligned}\right\} \tag{30b}$$

The parameter p_T characterises the effect of the remote stresses $(\sigma_x^\infty)_{\mathrm{II}}$ and σ_y^∞ on the $(T_x)_{\mathrm{I}}$ stress.

3 CALCULATION RESULTS

Calculations were carried out for different material pairs. The convergence of the series in Eq.(8) was very quick. A typical example for material pairs Aluminum-Epoxy in the case of plane strain is tested. $M = 150, 180$ and 210 gave same results of stress intensity factors at the crack tips A and B with four digits of accuracy and the coefficients α_m approaches to zero rapidly, as m increases. For example, $\alpha_1 = 0.2487, \alpha_{10} = -0.1421 \times 10^{-3}, \alpha_{100} = 0.4015 \times 10^{-6}$, $\alpha_{180} = -0.2342 \times 10^{-8}$. All of results given in this paper were calculated with $M = 180$.

3.1 Stress Intensity Factor

The calculated stress intensity factors for different material pairs in the case of plane stress are shown in Table 1 and Table 2. The present results agree very well with the results by Meguid et al.[8] and Chen[9].

Table 1 Nondimensional stress intensity factor $\bar{Q}_{\mathrm{I}}(B) = \dfrac{\sqrt{2}Q_{\mathrm{I}}(B)}{\sigma(2\pi a_0)^{\lambda_0}}$

μ_1/μ_2	ν_1	ν_2	present result	Meguid et al.[8]	Chen[9]	Lin & Mar[6]	Cook & Erdogn[2]
0.00722	0.35	0.3	0.0192	0.018	0.0192	0.196	0.079
0.0433	0.35	0.3	0.0955	0.094	0.095	0.095	0.074
23.08	0.3	0.35	4.232	4.240	4.231	4.241	4.176
138.46	0.3	0.35	5.002	5.004	5.001	4.978	4.922

Table 2 Nondimensional stress intensity factor $\bar{K}_I(A) = \dfrac{K_I(A)}{\sigma(2\pi a_0)^{1/2}}$

μ_1/μ_2	ν_1	ν_2	present result	Chen[9]	Lin & Mar[6]	Cook & Erdogn[2]
0.007 22	0.35	0.3	1.474	1.474	1.529	1.509
0.043 3	0.35	0.3	1.340	1.340	1.371	1.353
23.08	0.3	0.35	0.879	0.879	0.855	0.879
138.46	0.3	0.35	0.870	0.870	0.833	0.871

3.2 Stress Distribution Ahead of the Crack Tip B

The stress distribution ahead of the crack tip B is most interesting in this study.

Figure 2 and Fig.3 show the stress distributions ahead of the crack tip B for the material pairs Epoxy-Boron in the case of plane stress. The material constants are $\mu_1/\mu_2 = 0.007\,22, \nu_1 = 0.35, \nu_2 = 0.3$. It is clear that the normal stress σ_x is dominated by the Q field in the region of $0 < r/a_0 < 1$, meanwhile the normal stress σ_y is characterised by the Q field in the region of $0 < r/a_0 < 0.05$.

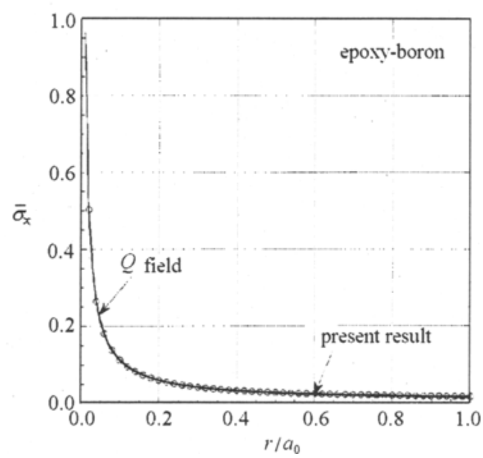

Fig.2 Normal stress σ_x distribution ahead of the crack tip B for material pairs Epoxy-Boron in the case of plane stress. Here $x = 0$, r is the distance from the crack tip B. $\bar{\sigma}_x = \sigma_x/\sigma$

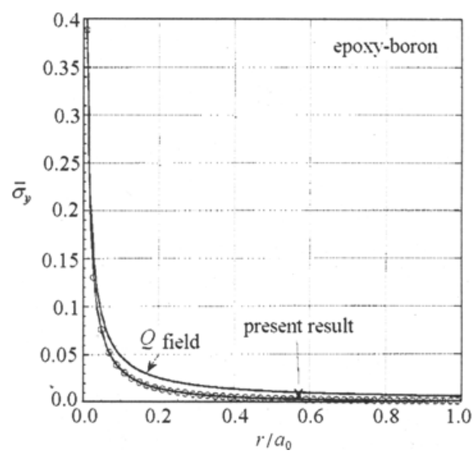

Fig.3 Normal stress σ_y distribution ahead of the crack tip B for material pairs Epoxy-Boron in the case of plane stress. $\bar{\sigma}_y = \sigma_y/\sigma$

The tendency becomes different as the crack is within the weaker material as shown in Fig.4 and Fig.5 for the case of plane stress. Figure 4 shows the comparison between the Q field, Q field plus T_x stress and the normal stress σ_x for material pairs Boron-Epoxy. The material constants are $\mu_1/\mu_2 = 138.46$, $\nu_1 = 0.3$, $\nu_2 = 0.35$.

The Q field plus T_x stress gives very good prediction for the normal stress σ_x in the region of $0 < r/a_0 < 1$. But the Q field is remarkably deviated from the normal stress σ_x. It clearly shows that the T_x stress has a tremendous contribution to the normal stress. The Q field plus the T_y stress also agrees very well with the normal stress σ_y in the region $0 < r/a_0 < 0.4$ as shown in Fig.5. The stress distributions along the interface are plotted on Fig.6 and Fig.7. The normal stress σ_x has a very large jump across the interface.

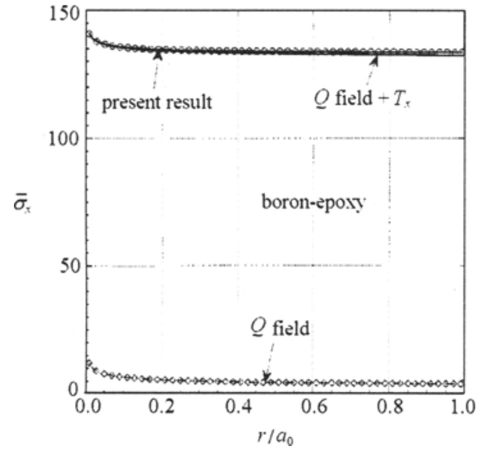

Fig.4 Normal stress σ_x distribution ahead of the crack tip B for material pairs Boron-Epoxy in the case of plane stress

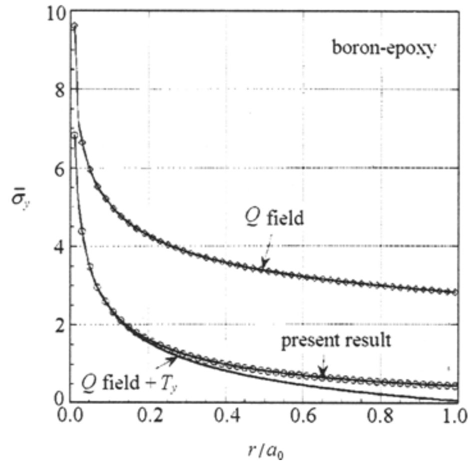

Fig.5 Comparison of the normal stress σ_y with the Q field and Q field plus T_y stress for material pairs Boron-Epoxy in the case of plane stress

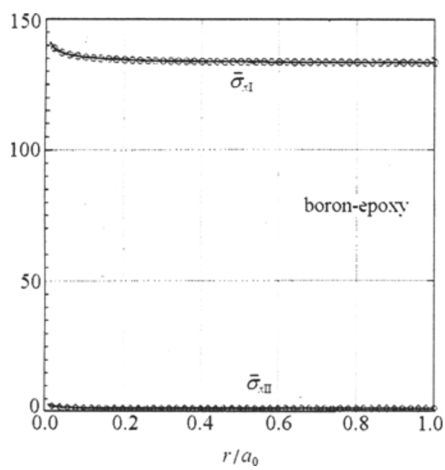

Fig.6 Normal stress distribution σ_x along the interface for material pairs Boron-Epoxy in the case of plane stress

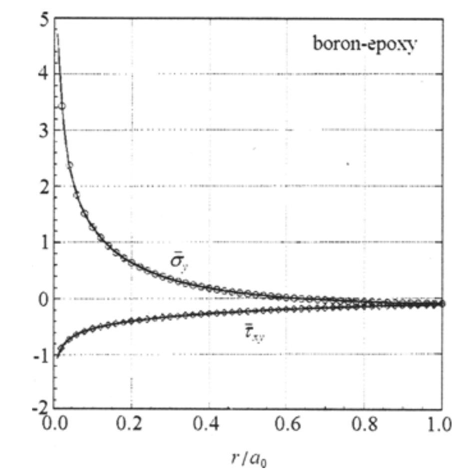

Fig.7 Normal stress σ_y and shear stress τ_{xy} distributions along the interface for material pairs Boron-Epoxy in the case of plane stress. $\bar{\sigma}_y = \sigma_y/\sigma$, $\bar{\tau}_{xy} = \tau_{xy}/\sigma$

4 CONCLUSIONS AND DISCUSSION

From this study, we can draw the following conclusions:

(1) The normal stress σ_x ahead of the crack tip B, is characterised by the Q field in the region of $0 < r/a_0 < 1$ and the normal stress σ_y, which is parallel to the crack surface, is dominated by the Q field in the region of $0 < r/a_0 < c$, if the crack is within a stiff material and the parameters $|p_T|$ and $|q_T|$ are very small. The parameter c is 0.05 for material pairs Epoxy-Boron and 0.4 for material pair's Epoxy-Aluminum.

(2) The normal stress σ_x ahead of the crack tip B is characterised by the Q field plus T_x stress in the region of $0 < r/a_0 < 1$ and the normal stress σ_y is dominated also by

the Q field plus T_y stress in the region of $0 < r/a_0 < 0.4$, if the crack is within a weaker material and the parameter $|p_T|$ or $|q_T|$ is very large. In this case, the Q fields deviate remarkably from the normal stresses σ_x and σ_y and the T stress components have tremendous contributions to the normal stresses.

(3) The stress σ_x has a jump across the interface due to the elastic constant mismatch. The larger is the elastic constant mismatch, the larger the jump will be. For materials pairs Aluminum-Epoxy, the stress jump can be as large as about 25 times of the remote stress $(\sigma_x^\infty)_{\text{II}}$.

Above conclusions are based on the analysis for an infinite bimaterial, which is bonded by two half planes containing a finite crack perpendicular to the interface and subject to uniform remote load at infinite. For finite bimaterial and complex loading, the situation will be complicated. In order to gain a better understanding for fracture characteristics of a crack perpendicular to the interface further investigations are needed.

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