Marangoni–Bénard Instability with the Exchange of Evaporation at Liquid–Vapour Interface *

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A new two-sided model rather than the one-sided model in previous works is put forward. The linear instability analysis is performed on the Marangoni–Bénard convection in the two-layer system with an evaporation interface. We define a new evaporation Biot number which is different from that in the one-sided model, and obtain the curves of critical Marangoni number versus wavenumber. The influence of evaporation velocity and Biot number on the system is discussed and a new phenomenon uninterpreted before is now explained from our numerical results.

Evaporative convection and instability give rise to both scientific and technological interests. Practically, many industrial applications such as thin-film evaporators, boiling equipments and heat pipes are concerned about the evaporation process, i.e. the heat and mass transfer through the gas–liquid interface. From a physical viewpoint, one of the interesting questions is the mechanism of convection instability in thin liquid layers induced by the coupling of the evaporation phenomenon and the Marangoni effect at the mass exchanged interface. The phenomenon of cellular convection was discovered by Bénard[1] in 1900 when a horizontal layer is heated below. As surface tension gradients at a free surface act as tangential stresses on the adjacent fluids, a dependence of surface tension on temperature will provide a coupling of temperature and velocity fluctuations at the free surface.[2,3] The convection driven by surface tension gradients is called the Marangoni–Bénard convection. The Marangoni–Bénard convection in a single layer has been such a typical phenomenon since Bénard's experiments[1] that many scientists have studied this phenomenon.[4,5] In previous works, most studies on Marangoni–Bénard instability were carried out for single liquid phase systems, and the gas phase adjacent to the liquid layer was considered to be passive. In this case, the dynamics of the thermal and mechanical perturbations in the gas was neglected. The heat transfer across the interface can be described by Newton's cooling law.[3] In the dimensionless governing equations of the system, the Biot number is introduced into the boundary conditions of thermal equilibrium, i.e. $dT/dz + BiT = 0$. Here $Bi = qh/\lambda$, where q is the Newton cooling coefficient, h is the depth of the liquid layer, λ is the thermo-conductivity coefficient. In this case, the system is considered as a one-sided model. Pearson[3] obtained the critical Marangoni number which is defined as $\sigma_T \Delta T h/(\mu\kappa)$. In Pearson's theory, convective cells cannot appear in the liquid layer until the temperature difference of the liquid layer exceeds the critical value. This theory is correct when the liquid layer is heated from the bottom. Nevertheless, more regular cellular flow patterns were observed when a hydrocarbon liquid is cooled from the bottom in Block's experiment.[2] This phenomenon has confused many investigators and most theoretical analysis on evaporating convection available till now has been presented with the assumption that the liquid layer is heated from the bottom or cooled from the top so as to be consistent with Pearson's result. Many investigators have focused on the study of instabilities of evaporating liquid surface by considering a one-sided model with an evaporation surface similar to Pearson's model.[6−9] Recently, Chai and others[10,11] have studied experimentally the effects of evaporating on Marangoni–Bénard convection in thin liquid layers evaporating at room temperature and proposed a modified Marangoni number to gauge the convection stability status in evaporating liquid layer.

Here we propose a theoretical model of the two-layer Marangoni–Bénard system with an evaporating interface assumed to be infinite in the horizontal direction, as shown schematically in Fig. 1. A similar model has been presented at recent international conferences by ourselves and other scientists.[12−14] Both the top wall and the bottom wall are considered as rigid perfectly conducting boundaries. The top wall is assumed to be a porous medium to vapour, through which the gas phase can pass at a certain velocity, in order that the evaporation flux and vapour pressure in the system can be controlled. In the basic state, the liquid is evaporating at a certain steady evaporating rate, and it is assumed that there is no convection between the vapour layer and the evaporating liquid layer. By using the depth of liquid layer H_2 as the non-dimensional scale for length, the layers have non-dimensional depths $h_1 = H_1/H_2$ and

$h_2 = 1$, where the subscripts 1 and 2 refer to the vapour and liquid layers, respectively. A temperature difference $\Delta T = T_2 - T_1$ is imposed perpendicular to the evaporating interface. When $\Delta T > 0$, the bottom boundary is hotter than the top boundary ($T_2 > T_1$). The dimensionless ratio of the fluid properties are $\kappa^\star = \kappa_1/\kappa_2$ (thermal diffusivity), $\beta^\star = \beta_1/\beta_2$ (volumetric expansion coefficient), $\chi^\star = \chi_1/\chi_2$ (thermal conductivity), $\mu^\star = \mu_1/\mu_2$ (dynamic viscosity), $\rho^\star = \rho_1/\rho_2$ (density) and $\nu^\star = \nu_1/\nu_2$ (kinematical viscosity), respectively. The evaporating interface is assumed to be flat. The interfacial tension at the interface is considered to be a linear function of temperature: $\sigma = \sigma_0 - \sigma_T(T - T_0)$, where T_0 is the reference temperature of interface. The governing equations for each fluid layer are the continuity equation, the energy equation and the Navier-Stokes equations with the Boussinesq approximation,[15,16] i.e. only the densities ρ_i ($i = 1, 2$) are dependent on the temperature. At the evaporating interface, the mass flux J is assumed to obey the Hertz Knudsen equation:[17]

$$J = \alpha\sqrt{\frac{M}{2\pi RT}}(p_s(T) - p_0(T)), \qquad (1)$$

where α is the accommodation coefficient; M is the molecular weight of vapour; $p_s(T)$ is the saturation pressure at surface interface temperature T; $p_0(T)$ is the vapour pressure just beyond the interface; and R is the universal gas constant. To linearize the above equation, the constitutive equation is obtained:[9]

$$J = \alpha\rho_v L\sqrt{\frac{M}{2\pi RT_s^3}}(T - T_s), \qquad (2)$$

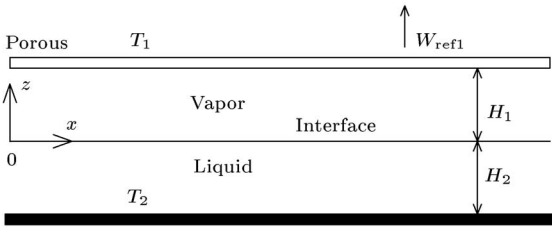where L is the latent heat, ρ_v is the density of the vapour.

Fig. 1. Schematic diagram of the definite-depth liquid–vapour layer system.

We introduce spatial normal perturbations proportional to $\exp[\lambda t + i(k_x x + k_y y)]$ into the linearized full governing equations (see Ref. [18]). Using ν_2/H_2, H_2^2/ν_2, H_2 and ΔT as the scaling factors for velocity, time, length and temperature, respectively. The dimensionless linear governing equations of the two-layer system are formulated in the form of the amplitudes of perturbation quantities W_i, the velocity component in the vertical direction z and Θ_i, the temperature in each layer:

$$\nu^\star(D^2 - k^2)^2 W_1 - W_{\text{ref1}} D(D^2 - k^2)W_1$$

$$-\beta^\star Gr k^2 \Theta_1 = \lambda(D^2 - k^2)W_1, \qquad (3)$$

$$\kappa^\star(D^2 - k^2)\Theta_1 - W_{\text{ref1}}Pr D\Theta_1 - Pr\frac{\partial T_{\text{ref1}}}{\partial Z}W_1$$
$$= \lambda Pr\,\Theta_1, \qquad (4)$$

$$(D^2 - k^2)^2 W_2 - Gr k^2 \Theta_1 = \lambda(D^2 - k^2)W_2, \qquad (5)$$

$$(D^2 - k^2)\Theta_2 - Pr\frac{\partial T_{ref2}}{\partial Z}W_2 = \lambda Pr\,\Theta_2, \qquad (6)$$

$$W_1 = 0,\; DW_1 = 0,\; \Theta_1 = 0,\; \text{at } z = h_1, \qquad (7)$$

$$W_2 = 0,\; DW_2 = 0,\; \Theta_2 = 0,\; \text{at } z = -1, \qquad (8)$$

$$DW_1 = DW_2,\; W_1 - W_2 = Ma \cdot E \cdot Bi_{ev} \cdot \Theta_2, \qquad (9)$$

$$W_2 = \rho^\star W_1,\; \Theta_2 = \Theta_1,$$
$$D\Theta_2 - \chi^\star D\Theta_1 + Bi_{ev} \cdot \Theta_2 = 0,$$
$$\mu^\star(D^2 + k^2)W_1 - (D^2 + k^2)W_2 = k^2\frac{Ma}{Pr}\Theta_2,$$

where D is the dimensionless differential operator d/dz, Ma is the Marangoni number defined as $\sigma_T\Delta TH_2/(\mu_2\kappa_2)$, where $\Delta T = \Delta T_1 + \Delta T_2$, ΔT_1 and ΔT_2 are the temperature differences in the vapour and liquid layers, respectively; Pr is the Prandtl number of fluid defined as ν_2/κ_2, λ is the time growth rate, k is the dimensionless wavenumber, and $dT_{\text{ref } i}/dz$ is the temperature gradient of fluid-i at the steady state. W_{ref1} is the dimensionless evaporation velocity of vapour leaving the interface in the basic state. The evaporating Biot number induced by liquid evaporation is defined as

$$Bi_{ev} = \alpha\rho_1 L^2 H_2 \sqrt{M/2\pi RT^3}/\chi_2, \qquad (10)$$

and the dimensionless numbers E is defined as

$$E = (1 - \rho^\star)(\kappa_2\chi_2)/(\rho^\star\sigma_T LH_2). \qquad (11)$$

The linear equations (3)–(6) together with its boundary conditions (7)–(9) are discretized by using the spectral numerical method (Tau–Chebyshev)[18] and then are resolved as the eigenvalue problem. The complex growth rates λ were computed in complex double precision.

The alcohol liquid with its own vapour is selected in the present study and the depth of the liquid layer is taken to be 1 mm. The ratios of physical properties (see Ref. [19]) and dimensionless numbers of the liquid–vapour system are $\nu^\star = 34.7$, $\rho^\star = 2.3 \times 10^{-4}$, $\chi^\star = 8.5 \times 10^{-2}$, $\kappa^\star = 6.1 \times 10^2$, $Pr = 14.9$, and $Gr = 0$ (here considered in microgravity condition or for a very thin liquid-layer), respectively. The neutral stability curves of the system with dimensionless evaporation velocity $W_{\text{ref1}} = 0.5$ are plotted in Fig. 2 for different evaporation Biot numbers. In our two-sided model, the evaporating Biot number presents the capability of heat exchange induced by evaporating at the liquid–vapour interface. In Fig. 2, the critical Marangoni number corresponding to the minimum of each curve increases with the evaporating Biot number; this means that the liquid layer becomes more stable with the increase of Bi_{ev}. When the Bi_{ev} number

reduces below a certain value, the critical Marangoni number Ma_c of the system is less than zero. This means that in the two-sided model, the liquid–vapour layer system become instable even while it is cooled from below. This is a new mechanisms of evaporating on the stabilities of the layer that we found in our two-sided model. In the case of $Bi_{ev} = 0$, the evaporating heat passing through the interface is zero and no perturbation of evaporation velocity, the results of instability analysis is the classic Marangoni–Bénard convection in a two-layer system described in Refs. [16, 20]. When $Bi_{ev} \to \infty$, the perturbation of the temperature at the interface is zero, and the system is absolutely stable.
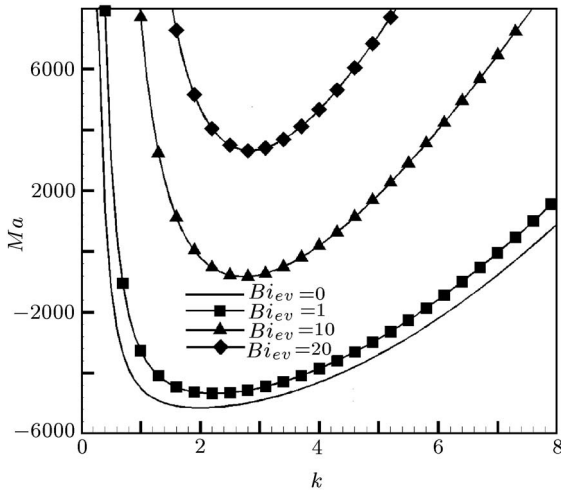

Fig. 2. Marangoni number versus dimensionless wavenumber for different evaporation Biot numbers in the definite-depth system ($Gr = 0, W_{\mathrm{ref1}} = 0.5$).
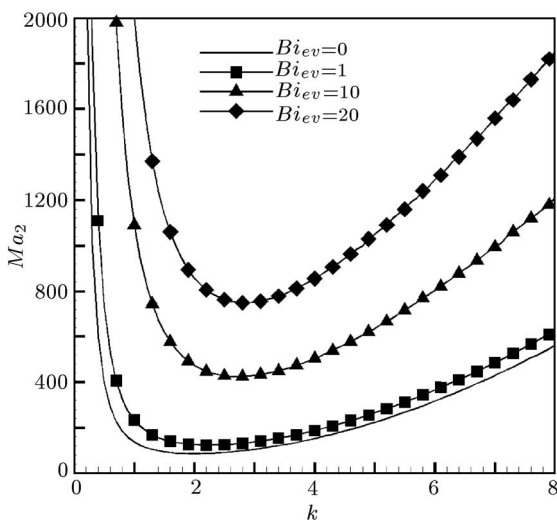

Fig. 3. Marangoni number of the liquid layer versus dimensionless wavenumber for different evaporation Biot numbers in the definite-depth system ($Gr = 0, W_{\mathrm{ref1}} = 0.5$).

Figure 3 gives a different Marangoni number corresponding to the liquid layer, Ma_2 versus wavenumber,

where Ma_2 is defined as $\sigma_T \Delta T_2 H_2/(\mu_2 \kappa_2)$. The reason why Ma_2 is introduced is that in the definition of Ma, the temperature difference of the liquid layer ΔT_2 is not able to be obtained directly. The relation between the temperature difference of two layers and that of the liquid layer in the definite-depth liquid–vapour system (see Ref. [12]) is

$$\Delta T_2 = (JLH_1 + \chi_1 \Delta T)H_2/(\chi_2 H_1 + \chi_1 H_2). \quad (12)$$

From Eq. (12), even the liquid–vapour system is cooled from below ($\Delta T < 0$ and $Ma < 0$ in Fig. 2), the temperature difference in the liquid layer can also be positive ($\Delta T_2 > 0$ and $Ma_2 > 0$ in Fig. 3) for the positive value of JLH_1. The corresponding critical Marangoni numbers of the liquid layer, Ma_2c for different Bi_{ev} are positive. This is the second mechanism of evaporations on the Marangoni instability in the definite-depth liquid–vapour system.

In summary, classical theories have only successfully explained the convection in a liquid layer heated from bottom without evaporation. However, these theories are unable to explain the convection in an evaporating thin layer, especially with the liquid layer cooled from bottom. In our two-sided definite depth model, instability occurs even when the evaporating liquid layer is cooled from bottom. This new phenomenon we found in the two-sided model of liquid–vapour system is impossible to exist in the classical one-sided model. In our further studies, influence of the interface deflection and vapour-recoil effect on the instability of the system and the evaporating Biot number will be discussed.

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