Instability of Two-Layer Rayleigh–Bénard Convection with Interfacial Thermocapillary Effect *

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The linear instability analysis of the Rayleigh–Marangoni–Bénard convection in a two-layer system of silicon oil 10 cS and fluorinert FC70 liquids are performed in a larger range of two-layer depth ratios H_r from 0.2 to 5.0 for different total depth $H \leq 12$ mm. Our results are different from the previous study on the Rayleigh–Bénard instability and show strong effects of thermocapillary force at the interface on the time-dependent oscillations arising from the onset of instability convection.

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Both surface tension gradients and buoyancy may drive convective motion in a liquid layer with a free surface when heated by the bottom. The thermocapillary forces at a free upper boundary play a major role when the depth of the liquid layer is small or the liquid layer is in a gravity reduced field. The Bénard–Marangoni convection in single layers has been a typical phenomenon since the Bénard experiments [1] around 1990. The convective instabilities and mechanisms in two or more superposed layers of liquid–liquid systems become more complex than single layer systems, due to the competition between instabilities in the separate layers and the various interfacial surface tension driven modes. Many scientists have extensively studied two- or multiple-layer convection due to several interfacial phenomena in nature (layered earth's mantle convection [2,3]) and in numerous engineering applications (liquid encapsulated crystal growth techniques,[4,5] film processing,[6,7] multilayer coating,[8] transportation of oil [9]). In the basic research the study of two-layer convection constitutes an important new direction for the field of pattern formation and bifurcation phenomenon in non-equilibrium systems. For the flow in a two- or multilayer system, one of the more interesting problems is the possibility of finding time-dependent states at the onset to convection. The oscillatory convection in the two-layer Rayleigh–Bénard system where thermocapillary is negligible has been investigated theoretically [10–13] and experimentally.[14–18] The onset of thermocapillary oscillatory convection in a floating half zone of large Prandtl number fluids are studied numerically.[19] Both instability analyses and experimental observation found two possible convective states: thermal coupling, or mechanical coupling in two-layer Rayleigh–Bénard convection for different combinations of two liquids. In the first coupling case, the superposed convection rolls are co-rotating, and in the second case the superposed rolls are counter-rotating. In a narrow transition region between the two different states the time-dependent convection (Hopf modes) may appear.[20] Colinet and Legros [13] revisited the problem theoretically by assuming a non-deformable interface and by selecting the non-identical fluids properties of the two-layer system, and gave a typical stability diagram for one range of layer depth ratios, as shown in Fig. 1, in which the oscillatory modes arise in between the two different stationary convective states. Recently, the experiments on the two-layer Rayleigh–Bénard system with two different pairs of fluids were performed by Degen *et al.*[18] They found time-dependent patterns at or near convective onset, but some evident differences such as the periods of the time-dependent flow and the time-dependent region of layer depth ratios have also been shown in comparison with the theoretical predictions.[20] In fact, for the two-liquid system of silicon oil 10 cS and fluorinert FC70 used in Degen's experiments, the oscillatory convection region for the total layer depth $H = 12$ mm is too small to be practically accessible for experiments or possible even nonexistent at the onset of convection. What will the instability behaviour of convection be in the above-discussed two-layer system while simultaneously considering the thermocapillary effects at the interface, for example in the cases when the two-layer depth is thinner than 12 mm? It is the main objective of the present work to investigate theoretically the thermocapillary effects on two-layer Rayleigh–Bénard convective instabilities, and much attention is paid to the oscillatory instability at the onset of convection. The instability analysis results presented here are the first part of study on the two-layer Rayleigh–Marangoni–Bénard convection.

The theoretical model of two-layer Rayleigh–Marangoni–Bénard system is assumed to be infinite in the horizontal direction as shown schematically in

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Fig. 2. By using the total depth of two layers H as the non-dimensional scale for length, the layers have non-dimensional depths $h_1 = H_1/H$ and $h_2 = H_2/H$, where the subscripts 1 and 2 refer to the top and bottom fluid layers, respectively. The depth ratio is defined as $H_r = H_1/H_2$. A temperature difference $\Delta T = T_2 - T_1$ is imposed parallel to the acceleration of gravity g between the top and bottom isothermal and rigid plates. When $\Delta T > 0$, the bottom boundary is hotter than the top boundary ($T_2 > T_1$). The dimensionless ratio of the fluid properties are $\kappa^* = \kappa_1/\kappa_2$ (thermal diffusivity), $\beta^* = \beta_1/\beta_2$ (volumetric expansion coefficient), $\chi^* = \chi_1/ = chi_2$ (thermal conductivity), $\mu^* = \mu_1/\mu_2$ (dynamic viscosity), $\rho^* = \rho_1/\rho_2$ (density) and $\nu^* = \nu_1/\nu_2$ (kinematical viscosity), respectively. The interface between the immiscible liquids is assumed to be flat. The interfacial tension at the interface is considered to be a linear function of temperature: $\sigma = \sigma_0 + (\partial\sigma/\partial T)(T - T_0)$, where T_0 is the reference temperature of interface, and $\partial\sigma/\partial T$ is usually negative. The governing equations for each fluid layer are the heat transport equation and the Navier–Stokes equations with the Boussinesq approximation, i.e. only the densities ρ_i are dependent on the temperature, $\rho_i = \rho_{0i}[1 - \beta_i(T_i - T_0)]$. In a two-layer Rayleigh–Marangoni–Bénard system, the convection arises due to buoyancy and temperature dependence of the interfacial tension, and their contributions are estimated by two important non-dimensional parameters: the Rayleigh number $Ra = g\beta_2\Delta T H^3/(\nu_2\kappa_2)$, and the Marangoni number $Ma = (-\partial\sigma/\partial T)\Delta T H/\mu_2\kappa_2)$. At the onset of convection, these parameters correspond to the critical values Ra_c, Ma_c with the critical temperature difference ΔT_c. The linear stability analysis is performed on the base state of the system with a flat interface at $z = 0$, a zero velocity field and a temperature field which varies linearly with z in each fluid. We introduce spatial normal perturbations proportional to $\exp[\lambda t + i(k_x + k_y)]$ into the linearized full governing equations referred to in chapter 2 of Ref. [21]. By using ν_2/H, H^2/ν_2, H and ΔT as the scaling factors for velocity, time, length and temperature, respectively, the dimensionless linear governing equations of the two-layer system are formulated in the form of the amplitudes of perturbation quantities w_i, the velocity component in the vertical direction z and θ_i, the temperature in each layer:

$$\nu^*(D^2 - k^2)^2 w_1 - \frac{Ra}{Pr}\beta^* k^2\theta_1 = \lambda(D^2 - k^2)w_1, \quad (1)$$

$$\kappa^*(D^2 - k^2)\theta_1 - \frac{\partial T_1}{\partial z}Pr w_1 = \lambda Pr\theta_1, \quad (2)$$

$$(D^2 - k^2)^2 w_2 - \frac{Ra}{Pr}k^2\theta_2 = \lambda(D^2 - k^2)w_2, \quad (3)$$

$$(D^2 - k^2)\theta_2 - \frac{\partial T_2}{\partial z}Pr w_2 = \lambda Pr\theta_2. \quad (4)$$

together with the boundary conditions

$$w_1 = Dw_1 = \theta_1 = 0 \quad \text{at} \quad z = -h_1, \quad (5)$$

$$w_1 = w_2 = 0, \quad Dw_1 = Dw_2,$$
$$\theta_1 = \theta_2, \quad \chi^* D\theta_1 = D\theta_2,$$
$$D^2 w_2 - \mu^* D^2 w_1 = -\frac{Ma}{Pr}k^2\theta_2 \quad \text{at } z = 0, \quad (6)$$

$$w_2 = Dw_2 = \theta_2 = 0 \quad \text{at } z = h_2, \quad (7)$$

where D is the dimensionless differential operator d/dz, Ra is the Rayleigh number, Ma is the Marangoni number, $Pr = \nu_2/\kappa_2$ is the Prandtl number of fluid-2, λ is the time growth rate, k is the dimensionless wavenumber, and $\partial T_i/\partial z$ is the temperature gradient of liquid-i at the steady state.

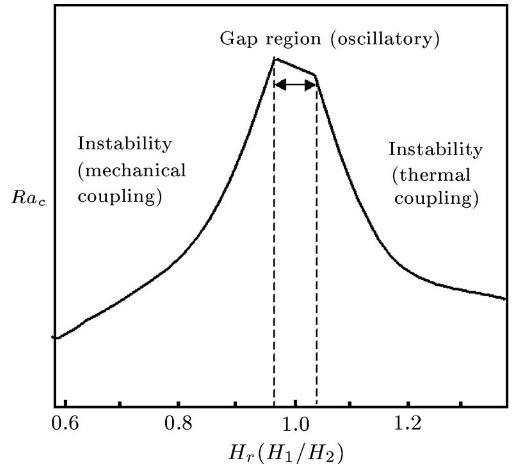

Fig. 1. Critical bifurcation diagram for the onset of Rayleigh–Bénard convection in a two-layer fluid system.

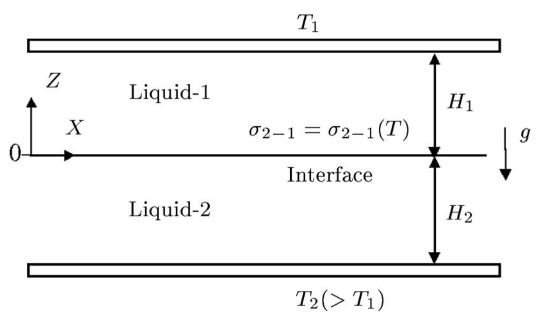

Fig. 2. Schematic diagram of two-layer liquids.

A spectral numerical method (Tau–Chebychev) was used to resolve the eigenvalue problem in λ for the above linear equations together with its boundary conditions of the two-layer Rayleigh–Marangoni–Bénard system. The complex growth rates λ were computed in complex double precision. A liquid system of silicon oil (10 cSt) (in the top layer) and fluorinert (FC70) (in the bottom layer) is selected here since this fluid pair has been more recently investigated theoretically[21–23] and experimentally.[18] The oscillatory convection regions at the onset state are given in Table 1 for different depths H of two-layer fluids. The four different cases for $H = 12$, 6, 4 and

Table 1. Oscillatory convection at the onset in the system of silicon oil (10 cSt) and fluorinert (FC70) for different total depths H of two layers ($Pr = 406$, $\kappa^* = 2.762$, $\beta^* = 1.1$, $\chi^* = 1.917$, $\mu^* = 0.344$, $\rho^* = 0.482$, $\nu^* = \nu_1/\nu_2 = 0.714$, and $\partial\sigma/\partial T = -4.46 \times 10^{-5}$ N/mK).

Two-layer depth $H(H_1+H_2)$ (mm)	Oscillatory region for H_r	Ra_c in oscillatory region	k_c in oscillatory region	Γ (Ra/Ma)
12	1.461–1.564	26840–26321	5.13–5.08	61.38
6	1.5–2.1	25010–21520	5.17–4.87	15.35
4	1.55–2.95	22400–18349	5.24–4.65	6.82
3	1.6–3.5	19715–18418	5.35–4.62	3.84

3 mm were investigated numerically here when considering both thermogravitational and thermocapillary effects which may be represented by the relation between the Ra and Ma numbers, $\Gamma = Ra/Ma = -g\beta_2\rho_2 H^2/(\partial\sigma/\partial T)$.

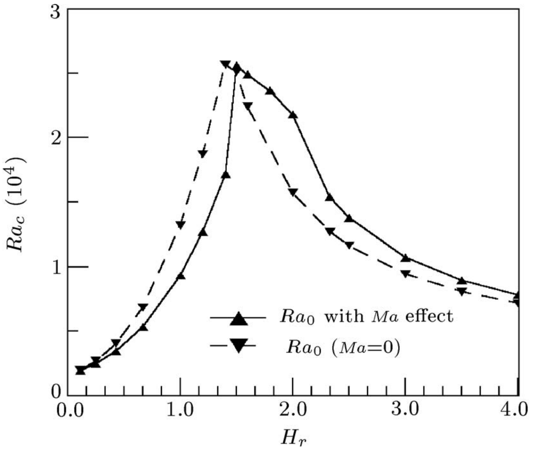

Fig. 3. Variation of the critical parameters Ra_c, at the onset of the Rayleigh–Marangoni–Bénard convection in the system $H = 6$ mm and $g = 9.8\,\mathrm{ms}^{-2}$ for different depth ratios H_r, in comparison with the case without the Marangoni effect ($Ma = 0$).

A narrow gap $1.461 \leq H_r \leq 1.564$ in which the oscillatory onsets of the Rayleigh–Marangoni–Bénard convection could occur in this system was found in the neutral stability curve of the $Ra_c - -H_r$ plane for $H = 12$ mm. The corresponding critical Rayleigh number of the system varies from 26840 to 26321, and the critical wavenumber k_c from 5.13 to 5.08. When the total depth H is reduced from 6 to 3 mm the oscillatory instability at onset occurs in the larger and larger gap regions of two-layer depth ratio H_r from 1.5–2.1 to 1.6–3.5. This variation of the gap regions is due to the augment of thermocapillary effect at the interface which corresponds to the decrease of the ratio $\Gamma = Ra/Ma$ when we increase H from 12 to 3 mm given in Table 1. The contribution of the thermocapillary effect on the instability of the system is shown in Fig. 3 where the neutral stability curve of the system displaces to the right while we consider the Marangoni effect at the interface in comparison with the Rayleigh–Bénard instability of the system without the Marangoni effect ($Ma = 0$) corresponding to the case considered in Colinet and Legros' works.[13] It is notable that the more larger oscillatory regime for $1.5 \leq H_r \leq 2.1$ found in the Rayleigh–Marangoni–Bénard convective instability of the system replaces the very narrow oscillatory onset gap in the Rayleigh–Bénard instability of the system when neglecting the thermocapillary effect ($Ma = 0$).

In summary, we have presented a new feature of oscillatory instability of the Rayleigh–Marangoni–Bénard convection in a thin two-layer system when considering the real thermocapillary effect at the interface. The oscillatory regime in the oscillatory gap region given in Table 1 is more complex in the time-dependant pattern formation than that in the Rayleigh–Bénard convection in the case of $Ma = 0$. This new phenomenon resulting from the competition between the thermocapillary forces and buoyancy forces has been confirmed recently by our preliminary numerical investigation of nonlinear convective instability in the same system [23] and will be discussed in details elsewhere.

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