Influence of Aspect Ratio on the Onset of Thermocapillary Oscillatory Convection in a Floating Half Zone of Large Prandtl Number Fluid

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The onset of oscillatory thermocapillary convection in a floating half zone of 10 cst silicon oil (Prandtl number 105.6) is studied by the three-dimensional and unsteady numerical simulation in microgravity environment ($g = 10^{-4}g_{\rm earth}$). The results show that the steady and axi-symmetric convection, for a fixed liquid bridge volume ratio $V_\ell/V_0 = 1$, transits directly to the oscillatory convection if geometrical aspect ratio A is larger than the critical value $A_c = 1.25$, but transits to the oscillatory convection via the steady and non-axisymmetric flow if A is smaller than the critical value A_c. The result means that there are two bifurcation transitions in a liquid bridge of the large Prandtl number fluid with a smaller aspect ratio A.

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The onset of oscillatory thermocapillary convection in the floating half zone has been studied extensively since the preliminary studies of Chun and Wuest[1] and Schwabe and Scharmann[2] Usually, the transition from the steady and axis-symmetric thermocapillary convection to the oscillatory convection has been suggested for a liquid bridge of large Prandtl number fluid, which means that there is one bifurcation transition of thermocapillary convection. The hydrothermal wave instability, characterized by a travelling wave, was suggested to explain the onset of oscillatory thermocapillary convection.[3] The theoretical analyses have also been performed by the linear instability analyses,[4–8] the energy analysis method[9] and the unsteady and three-dimensional numerical simulation.[10–12]

Recently, two bifurcation transitions of thermocapillary convection in a floating half zone were obtained for smaller Prandtl number fluid $P_r = 0.01$ by the numerical simulation.[14] The result implies that the bifurcation mechanism is hydrodynamic instability but not the hydrothermal wave instability in the case of small Prandtl number fluid. Two bifurcation transitions in a fat silicon oil liquid bridge of larger Prandtl number ($P_r = 105.6$) were obtained by the numerical simulation in the microgravity environment for liquid bridges of different volume ratios.[11] The experimental and numerical studies also give the two bifurcation processes in Earth's gravity condition.[12] The linear instability analyses give the transition from the steady and axisymmetric convection to the steady and axi-asymmetric convection.[13] In this Letter, the influence of geometrical aspect ratio on the onset of oscillation is studied for larger Prandtl number fluid.

A liquid bridge floating between two co-axial copper rods of diameter $d_0 = 15$ mm, with the volume ratio $V_i = V_\ell/V_0 = 1$, is studied in the microgravity environment ($g = 10^{-4}g_{\rm earth}$) as shown in Fig. 1, where V_ℓ is the volume of liquid bridge and V_0 is the volume of cylinder $V_0 = \pi d_0^2\ell/4$. The aspect ratio A changes from 0.5 to 1.6. The lower rod keeps a constant temperature T_0, and the temperature at the upper rod is $T_0 + \Delta T$, where the applied temperature difference ΔT is positive and increases gradually at a certain heating rate. A cylindrical coordinate system (r, θ, z) is adopted.

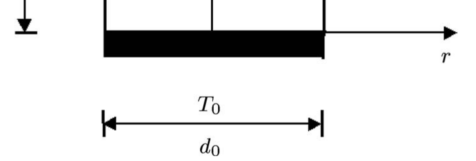

Fig. 1. Schematic diagram of a floating half zone in space.

The basic equations of the thermocapillary convection in the liquid bridge are the mass conservation, the momentum conservation and the energy conservation. The non-dimensional vector stream function and the vector vorticity are introduced respectively as $\boldsymbol{\psi} = (\psi_\xi, \psi_\eta, \psi_\zeta)$ and $\boldsymbol{\omega} = (\omega_\xi, \omega_\eta, \omega_\zeta)$, which are defined as follows.

$$\nabla \times \boldsymbol{\psi} = \boldsymbol{V}, \quad \nabla \times \boldsymbol{V} = \boldsymbol{\omega}, \tag{1}$$

$$\nabla \times \nabla \times \boldsymbol{\psi} = \boldsymbol{\omega}, \tag{2}$$

where $\boldsymbol{V}$ is the velocity vector. Then, the non-dimensional equations are written as[11]

$$\frac{\partial \boldsymbol{\omega}}{\partial \tau} + V \cdot \nabla \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \nabla V = \frac{1}{R_s}\Big(\nabla^2 \boldsymbol{\omega} + \frac{\ell^2}{\rho \nu v^*} \nabla \times F\Big), \tag{3}$$

$$\frac{\partial \boldsymbol{\Theta}}{\partial \tau} + \boldsymbol{V} \cdot \nabla \Theta = \frac{1}{\boldsymbol{M}_\alpha} \nabla^2 \Theta. \tag{4}$$

The boundary conditions are as follows: for $\zeta = 0$ and 1,

$$\psi_\xi = \psi_\eta = 0, \quad \frac{\partial \psi_\zeta}{\partial \zeta} = 0, \tag{5}$$

$$\omega_\xi = -\frac{\partial v}{\partial \zeta}, \quad \omega_\eta = \frac{\partial u}{\partial \zeta}, \quad \omega_\zeta = 0, \tag{6}$$

$$\Theta(\xi, \eta, 0, \tau) = 0, \quad \Theta(\xi, \eta, 1, \tau) = f(\tau); \tag{7}$$

for $\xi = 1/(2A)$:

$$\psi_s = \psi_\eta = 0, \quad \nabla \cdot \boldsymbol{\psi} = 0, \tag{8}$$

$$\begin{aligned} \omega_\eta = & -\frac{(1+f'^2)\partial T}{(1-f'^2)\partial S} - \frac{2f'}{(1-f'^2)}\Big[\frac{\partial}{\partial \xi}\Big(\frac{1}{\xi}\frac{\partial \psi_\zeta}{\partial \eta} - \frac{\partial \psi_\eta}{\partial \zeta}\Big) \\ & - \frac{\partial}{\partial \zeta}\Big(\frac{1}{\xi}\frac{\partial \xi \psi_\eta}{\partial \xi} - \frac{1}{\xi}\frac{\partial \psi_\xi}{\partial \eta}\Big)\Big] \\ & - 2\frac{\partial}{\partial \zeta}\Big(\frac{1}{\xi}\frac{\partial \psi_\zeta}{\partial \eta} - \frac{\partial \psi_\eta}{\partial \zeta}\Big) \end{aligned} \tag{9}$$

$$\begin{aligned} \omega_\xi = & \frac{(1+f'^2)^{1/2}}{\xi}\frac{\partial T}{\partial \eta} + 2\frac{\partial}{\partial \xi}\Big(\frac{\partial \psi_\xi}{\partial \zeta} - \frac{\partial \psi_\zeta}{\partial \xi}\Big) \\ & - f'\Big[\omega_\xi + 2\frac{\partial}{\partial \zeta}\Big(\frac{\partial \psi_\xi}{\partial \zeta} - \frac{\partial \psi_\zeta}{\partial \xi}\Big)\Big], \end{aligned} \tag{10}$$

$$\frac{\partial \Theta}{\partial n} = 0, \tag{11}$$

where the free surface is described as $\xi = 1/(2A) =$ constant, $\boldsymbol{n}$ and $\boldsymbol{s}$ are respectively the unit vector in the normal direction and in the direction perpendicular to both $\boldsymbol{n}$ and azimuthal direction of the free surface. The function $f(\tau)$ is related to the heating rate.[11] Non-dimensional quantities and parameters are introduced as follows,

$$\begin{aligned} & \xi = \frac{r}{\ell}, \quad \eta = \frac{r\theta}{\ell}, \quad \zeta = \frac{z}{\ell}, \\ & \boldsymbol{\tau} = \frac{tv^*}{\ell}, \quad U = \frac{u}{v^*}, \quad V = \frac{v}{v^*}, \quad W = \frac{w}{v^*}, \\ & P = \frac{p}{\rho \nu^{*2}}, \quad \Theta = \frac{T}{\Delta T^*}, \end{aligned} \tag{12}$$

where ℓ is the height of the liquid bridge. The typical velocity v^* is defined as $v^* = (|\mathrm{d}\sigma/\mathrm{d}T|\Delta T^*)/\rho\nu$, the applied temperature difference $\Delta T^* = T^* - T_0$ is a constant and T^* is a reference constant temperature defined as the highest temperature at the upper rod during the heating process. Here, ρ, p and T are respectively the density, pressure, temperature of the liquid, ν and κ are respectively the kinematics viscosity and thermal diffusion coefficients. The non-dimensional parameters are related by $M_a^* = R_s^* P_r$, and

$$R_s^* = \frac{v^* \ell}{\nu}, \quad M_a^* = \frac{\nu^* \ell}{\kappa}, \quad P_r = \frac{\nu}{\kappa}. \tag{13}$$

The finite element method (FEM) is applied to solve the problem, and the mesh numbers in the r, θ, and z directions are 12, 16, and 12, respectively. The floating half zone is divided into 10758 tetrahedron elements related to 2064 nodes. The non-linear convective terms in the vorticity equation and energy equation are calculated by the characteristic line method, and the diffusion terms are calculated by the FEM as the same as in Refs. [11,12]. The initial condition of the problem is related to the static state with zero temperature difference $\Delta T = 0$, then the applied temperature difference increases at the heating rate.

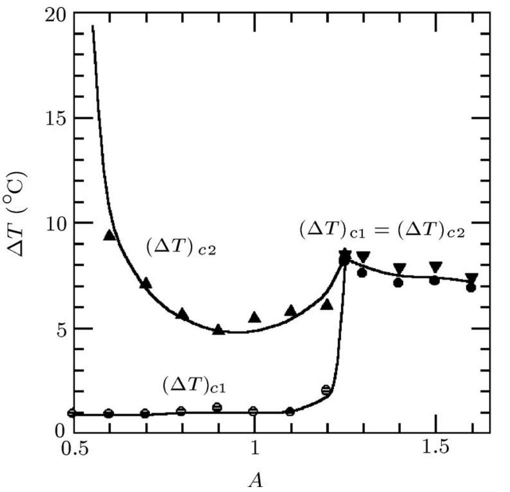

Fig. 2. Relation between the critical temperature difference and the aspect ratio.

Here we adopt the heating rate 0.05°C/s and the reference temperature $T^* = 25$°C. The first and second critical temperature differences, $(\Delta T)_{c1}$ and $(\Delta T)_{c2}$, are related respectively to the first and second bifurcations. Both $(\Delta T)_{c1}$ and $(\Delta T)_{c2}$ are smaller than the applied temperature difference ΔT^*, that is, $(\Delta T)_{c1} \le (\Delta T)_{c2} \le \Delta T^*$.

The onset of oscillatory thermocapillary convection of a liquid bridge of larger Prandtl number fluid in space environment was calculated. The critical applied temperature difference depending on the aspect ratio is shown in Fig. 2. The numerical results indicate that there is a critical aspect ratio $A_c = 1.25$. Two bifurcation transitions are obtained if the geometrical aspect ratio of a liquid bridge is smaller than the critical value A_c. This means that the steady and axis-symmetric thermocapillary convection transits to the oscillatory convection via the steady and asymmetric convection. Otherwise, the steady, axis-symmetric thermocapillary convection transits to oscillatory convection directly if $A > A_c$.

The coupling of the hydrodynamic and the thermal effects is strong in the case of the larger Prandtl number fluid. The conclusion of the present paper does not agree with that of the hydrothermal instability, which is related to a travelling wave but not a steady state. The studies show that the geometrical parameter A is not only a sensitive critical parameter for onset of oscillatory thermocapillary convection, but also important in the studies of oscillatory mechanism, which induces the different sort of bifurcation.

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