

## **A semi-empirical equation of penetration depth on concrete target impacted by ogive-nose projectiles**

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**Abstract.** In this paper, the penetration process of ogive-nose projectiles into the semi-infinite concrete target is investigated by the dimensional analysis method and FEM simulation. With the dimensional analysis, main non-dimensional parameters which control the penetration depth are obtained with some reasonable hypothesis. Then, a new semi-empirical equation is present based on the original work of Forrestal *et al.*, has only two non-dimensional combined variables with definite physical meanings. To verify this equation, prediction results are compared with experiments in a wide variation region of velocity. Then, a commercial FEM code, LS-DYNA, is used to simulate the complex penetration process, that also show the novel semi-empirical equation is reasonable for determining the penetration depth in a concrete target.

### **1. INTRODUCTION**

Currently, the investigation on the penetration in rock and concrete targets remains an active research field because of the development of military and civil guard engineering. Over the last decades, some important survey references have been published, and it is shown that three kinds of methodology are used to obtain the penetration mechanism. Firstly, real size or reduction scale model tests are conducted to obtain some key parameters and their relationship, such as penetration depth, acceleration of a projectile, *etc.* Secondly, various theoretical models are built to describe the penetration process, such as r cavity-expansion theory. Lastly, some algorithm and code are developed to simulate the details of penetration in a target, including of FEM, FDM, SPH *etc.*

However, because of the complexity of penetration process, the analytical solution on this problem is based on a lot of hypothesis, and with a complex form [1-4]. Concerning the numerical simulation method, the lack of reasonable and efficient algorithm about the failure, and the proper constitutive model and relative material constants, hinder of the extensive application of numerical method in this field [5].

Some empirical equations are founded in this field, based on abundant testing. Specially, equations developed by Sandia National Laboratory and Waterway Test Station are applied in most abroad way [6-8]. However, there are some deficiencies. For instance, numerous constants appear in these equations which need to be fitted through testing results, and many physical parameters are adopted directly without any combination. It is inconvenient for understanding the penetration mechanism.

In this paper, based on the semi-empirical equation presented by Forrestal *et al* [8-9], a novel non-dimensional equation is developed by using dimensional analysis method and similarity theory. In this equation, the penetration depth is only depended on two non-dimensional parameters and a material constant. This equation is verified by comparing with the experimental results and the FEM simulation.

## 2. EXISTING EMPIRICAL EQUATIONS

About the penetration in the concrete target, many equations have been built, as described in section 1. The equation [6] presented by Waterway Testing Station is,

$$Z = \frac{W}{A} \cdot \frac{N_{rc}}{\rho} \cdot \left[ \frac{43.715V}{3} \cdot \sqrt{\frac{\rho}{\sigma_{rc}}} - \frac{175.562}{9} \cdot \log \left( 1 + \frac{2.989}{4} \cdot V \cdot \sqrt{\frac{\rho}{\sigma_{rc}}} \right) \right] \quad (1)$$

For a ogive-nose penetrator,

$$N_{rc} = 0.863 \left( \frac{4\psi^2}{4\psi - 1} \right)^{0.25}, \quad \sigma_{rc} = \sigma_c \left( \frac{RQD}{100} \right)^{0.2} \quad (2)$$

For a conical one,

$$N_{rc} = 0.805 [\sin(\eta_c)]^{-0.5} \quad (3)$$

Where,  $\psi$  denotes the ratio between radius of curvature of head and diameter of penetrator,  $\eta_c$  is one half of the cone angle,  $W, A, V$  denote the weight, cross section area and the impacting velocity of the penetrator.  $\rho, \sigma_c$  denote the density and unidirection compression strength of projectile materials. RQD is a characteristic parameter of rock properties.

Forrestal *et al.* [8-9] presented a semi-empirical penetration equation. In this model, the penetration in concrete is divided into two phases, cratering and tunneling. For the cratering region, the resistance pressure on the projectile nose is linear dependence on the penetration depth. So,

$$F = cz, \quad 0 < z < 4a \quad (4)$$

Where,  $F, c, z, a$  are the resisting force, coefficient, penetration depth and radius of penetrator. For the tunneling region, the resisting force is deduced by using the cavity expansion method,

$$F = \pi a^2 (\tau_0 A + NB\rho V^2) \quad (5)$$

And, it also can be written as,

$$F = \pi a^2 (Sf'_c + N\rho V^2), \quad 4a < z < P \quad (6)$$

Where,  $N = \frac{8\psi-1}{24\psi^2}$ .  $f'_c$  is unidirection compression strength of projectile materials,  $S$  is described with Equation 9. In some recent references,  $R$  is used to replace  $Sf'_c$ . The relationship between the penetration depth and impacting velocity can be gotten with the following equation,

$$P = \frac{m}{2\pi a^2 \rho N} \ln \left( 1 + \frac{N\rho V_1^2}{Sf'_c} \right) + 4a, \quad P > 4a \quad (7)$$

where,

$$V_1^2 = \frac{mV_s^2 - 4\pi a^3 Sf'_c}{m + 4\pi a^3 N\rho} \quad (8)$$

$$S = \frac{N\rho V_s^2}{f'_c} \frac{1}{\left( 1 + \frac{4\pi a^3 N\rho}{m} \right) \exp \left( \frac{2\pi a^2 (P - 4a) N\rho}{m} \right) - 1} \quad (9)$$

Equations 7-9 are the groundwork of our new model, which will be given in next section.

## 3. A NOVEL NON-DEMENSIONAL SEMI-EMPIRICAL PENETRATION EQUATION

In last century, some similar criterions of penetration in metal target were presented. Based on these methods and results [10], the main non-dimensional controlling parameters can be determined for the

penetration process in concrete target by using dimensional analysis, when the impacting velocity is lower than 1000m/s,

$$\frac{P}{d} = f\left(\frac{\rho_p}{\rho}, \frac{\rho_p V_s^2}{Sf'_c}, N, \frac{L}{d}\right) \quad (10)$$

In the right side of this equation, every nondimensional parameter has definite physical meaning, represents the density ratio of projectile and target materials, verified Johnson's Number, ratio of length and diameter of projectile, respectively.

It must be noted that equation 10 only can be obtained with following assumptions. The deformation and failure of a penetrator are neglected, the compressibility of target material is ignored, and the target is considered as a semi-infinite region

For an ogive-nose penetrator, a parameter, called mass- equivalent length,  $L_0$ , should be introduced:

$$L_0 = \frac{4m}{\rho_p \pi d^2}, \quad d = 2a \quad (11)$$

Then, other two combined nondimensional parameters are presented,

$$\bar{J} = \frac{L_0 \rho_p V_s^2}{d Sf'_c}, \quad \bar{M} = \frac{\rho_p L_0}{\rho} \frac{1}{d N} \quad (12)$$

The first parameter is the product of Johnson's Number and the ratio of length and diameter of penetrator, and describes the relation between the inertia force and target strength. Wherein, the second combined variable is related to the projectile mass and properties of target materials.

To substitute equations 11 and 12 into equations 7-9, we have,

$$\frac{P}{d} = \frac{1}{2} \bar{M} \ln\left(1 + \frac{\bar{J} - 2}{\bar{M} + 2}\right) + 2 \quad (13)$$

Generally,  $M \gg 2$ . So, equation 13 can be rewritten as,

$$\frac{P}{d} = \frac{1}{2} \bar{M} \ln(1 + \bar{R}) + 2 \quad (14)$$

where,  $\bar{R} = N \frac{\rho V_s^2}{Sf'_c}$ .

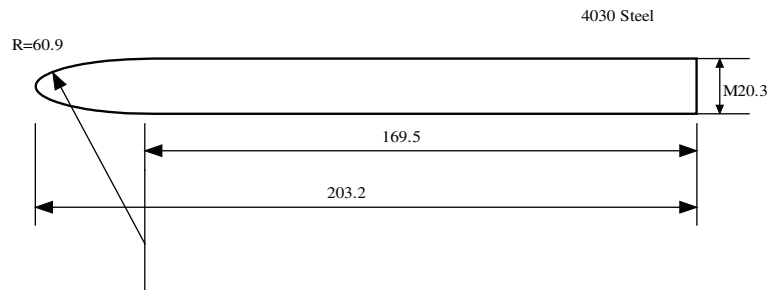
It is shown in equation 14 that, if shape and velocity of projectile are fixed, and target materials is the same, the penetration depth is linear related to the ratio of length and diameter of projectile, and density ratio of bullet and target materials.

In Equation 14, the nondimensional penetration depth is only dependent on two parameters,  $\bar{M}$  and  $\bar{R}$ . This result can be used to decrease the test times and cost, and to make the test easier. For instance, the penetration under higher impacting velocity can be simulated with changing the nose shape or length/diameter ratio of projectile.

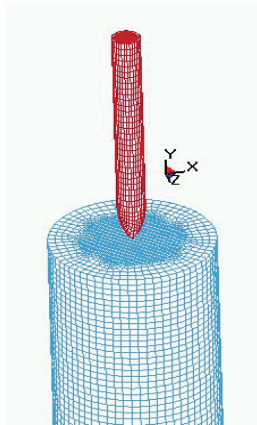
#### 4. FEM NUMERICAL SIMULATION

In this section, a commercial FEM code, LS-DYNA, is adopted to investigate the penetration process for a semi-infinite concrete target. The shape and dimension of a bullet are shown in Fig.1.

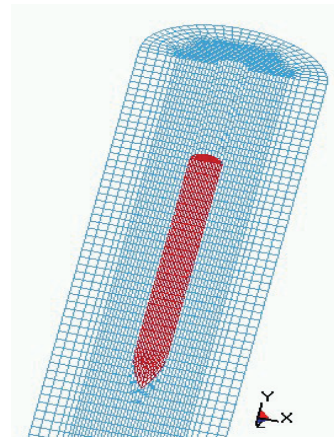
In numerical simulation, the FEM mesh is divided with 12018 elements and 17544 nodes, as plotted in Fig.2. The outside surface is considered as a non-reflection boundary to simulate the semi-infinite effect. The various materials constants can be obtained from references [9-11]. Where, the density of the concrete target is 2320kg/m<sup>3</sup>, the unidirection compression strength is 58.4MPa. The projectile is made of 4030 steel, with the weight of 0.478 kg.



**Figure 1.** The sketch map of a projectile.



**Figure 2.** FEM model and mesh.



**Figure 3.** Mesh deformation in penetration process.

In the numerical analysis, the impacting velocity is set to 442, 815, 1165 m/s respectively, and the penetrator is considered as a rigid body. This assumption is applied to speed up the simulation process, and is acceptable and widely used in similar impact conditions.

Fig.3 shows the deformation of target with the impact velocity of 442m/s, when the penetrator arrives at the maximum depth.

Fig.4 represents the velocity decaying of penetrators with different initial velocities. To differentiate these curves, the acceleration and resisting loading of a penetrator can be obtained expediently. The penetrator moving in targets with different initial velocity is depicted in Fig.5.

## 5. CONCLUSIONS AND DISCUSSIONS

In this paper, a novel semi-empirical equation about penetration in concrete targets is presented to describe the effect of various parameters on the penetration depth, with a simple form, based on Forrestal *et al* work and dimension analysis method.

In Fig.6, the prediction results of this new equation are compared with those acquired experiments [10-11] and FEM simulation. Where, lines denote the prediction values, when stars and other symbol represent the numerical and experimental ones, respectively. From Fig.6, we find that these results obtained through different methods fit well with each other. It shows that the new semi-empirical is reasonable.

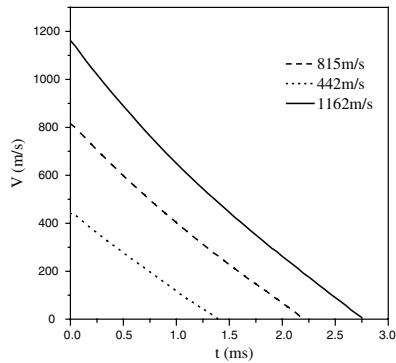


Figure 4. Deceleration of penetrator.

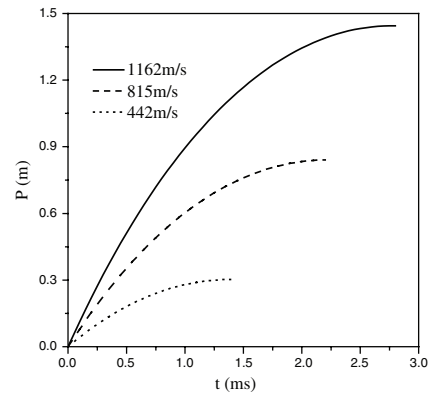


Figure 5. Penetration depth of projectile.

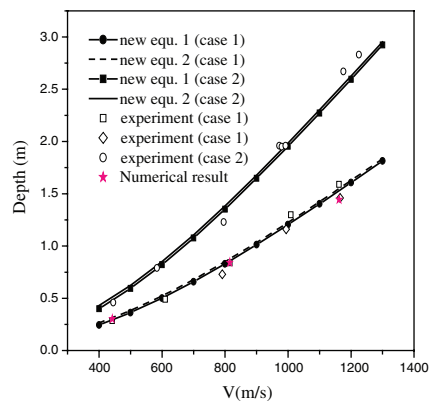


Figure 6. Verifying of new equation.

The new equation only includes two non-dimensional parameters, and a material constant which can be determined through material properties testing. The simple form has obvious advantages in understanding the penetration mechanism and simplifying the testing schedule.

To the knowledge of authors, we suggest that following research is important,

1. To consider the effect of a more complex structure of penetrator
2. To verify the new equation with more abundant testing data
3. To conduct numerical simulation with much more input conditions, to check the applicability of this equation under different combined parameter values.

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