

## Effects of microstructural heterogeneity on the spallation behavior of materials

Haiying Wang<sup>1</sup>, Yong Liu<sup>1,2</sup>, Mengfen Xia<sup>1,3</sup>, Fujiu Ke<sup>1,4</sup> and Yilong Bai<sup>1</sup>

<sup>1</sup> LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, PR China

<sup>2</sup> Graduate School of Chinese Academy of Sciences, Beijing 100039, PR China

<sup>3</sup> Department of Physics, Peking University, Beijing 100871, PR China

<sup>4</sup> Department of Applied Physics, Behang University, Beijing 100084, PR China

**Abstract.** It is of utmost importance to understand the spallation behaviour of heterogeneous materials. In this paper, a driven nonlinear threshold model with stress fluctuation is presented to study the effects of microstructural heterogeneity on continuum damage evolution. The spallation behavior of heterogeneity material is analyzed with this model. The heterogeneity of mesoscopic units is characterized in terms of Weibull modulus  $m$  of strength distribution and stress fluctuation parameter  $k$ . At high stress, the maximum damage increases with  $m$ ; while at low stress, the maximum damage decreases. In addition, for low stress, severe stress fluctuation causes higher damage; while for high stress, causes lower damage.

### 1. INTRODUCTION

Heterogeneous materials, like ceramics and rocks, are widely used in armor engineering. Spallation is a typical failure process of these materials under impact loading. Therefore, it is of utmost importance to understand the spallation behavior of heterogeneous materials.

Studies reveal that spallation is resulted from the nucleation, growth and coalescence of microvoids or micro-cracks in materials [1, 2]. Hence, spallation is a process involving coupled multiple time and space scales. The diversity and coupling of physics at different scales present two fundamental difficulties for spallation modeling and simulation [3-5]. More importantly, the difficulties can be greatly enhanced by the disordered heterogeneity on multiple scales.

Different quantitative/predictive models for spallation have been proposed based on experimental observations [3, 4, 6-8]. Some of them [3, 4, 8] tried to link the mesoscopic scale and macroscopic scale by introducing microdamage nucleation and growth laws into continuum equations. However, the effects of microstructural heterogeneity on spallation cannot be studied even with these multiscale models.

In this paper, a driven nonlinear threshold model for damage evolution in heterogeneous materials is presented. In addition, stress fluctuation caused by heterogeneous damage is considered. The damage evolution in spallation is analyzed with the model. The effects of microstructural heterogeneity and stress fluctuation on damage evolution in spallation process are investigated.

### 2. MODEL

We consider a macroscopic representative volume element (RVE) (at  $\mathbf{x}$ ) comprised of a great number of interacting, nonlinear, mesoscopic units—that is, a driven nonlinear threshold model [9]. The heterogeneity of the mesoscopic units can be characterized by their broken threshold. The mesoscopic units are assumed to be statistically identical, and their broken threshold  $\sigma_c$  follows a statistical distribution function  $\varphi(\sigma_c, t, \mathbf{x})$ .

The RVE is subjected to nominal driving force  $\sigma_0(t, \mathbf{x})$ , which is adopted as macroscopic variable. In the RVE, a mesoscopic unit will have probability to break as the real driving force (true stress)  $\sigma$  on it becomes higher than its threshold. When a unit breaks, it will be excluded from the distribution

function. Hence, we introduce a time-dependent distribution function of intact units  $\varphi(\sigma_c, t, \mathbf{x})$  with initial condition

$$\varphi(\sigma_c, t = 0, \mathbf{x}) = h(\sigma_c), \tag{1}$$

where  $h(\sigma_c)$  is normalized as

$$\int_0^\infty h(\sigma_c) d\sigma_c = 1. \tag{2}$$

In Eq.(2),  $\sigma_c$  is non-dimensionalized and normalized by a parameter  $\sigma^*$ , the characteristic value of  $\sigma_c$ .

With function  $\varphi(\sigma_c, t, \mathbf{x})$ , the continuum damage of the RVE at time  $t$  can be defined by

$$D(t, \mathbf{x}) = 1 - \int_0^\infty \varphi(\sigma_c, t, \mathbf{x}) d\sigma_c. \tag{3}$$

Due to the heterogeneity, the true stress applied on the intact units in RVE fluctuates. We assume that the true stress follows a statistical distribution function  $\xi(\sigma; t, \mathbf{x})$ . Hence, the probability that the real driving force (true stress) applied on intact units is  $\sigma$  can be denoted by  $\xi(\sigma; t, \mathbf{x})$ . Rough speaking, function  $\xi(\sigma; t, \mathbf{x})$  is determined by the nominal stress  $\sigma_0(t, \mathbf{x})$  and continuum damage  $D(t, \mathbf{x})$  that is,

$$\xi(\sigma; t, \mathbf{x}) = \xi(\sigma; \sigma_0(t, \mathbf{x}), D(t, \mathbf{x})). \tag{4}$$

In addition, function  $\xi(\sigma; t, \mathbf{x})$  should be normalized as

$$\int_0^\infty \xi(\sigma; t, \mathbf{x}) d\sigma = 1, \tag{5}$$

and the mean value of driving force on intact units follows

$$\int_0^\infty \sigma \xi(\sigma; t, \mathbf{x}) d\sigma = \frac{\sigma_0(t, \mathbf{x})}{1 - D(t, \mathbf{x})}. \tag{6}$$

By assuming statistical independency between broken threshold  $\sigma_c$  and true stress  $\sigma$ , the evolution of distribution function  $\varphi(\sigma_c; t, \mathbf{x})$  is suggested to follow an equation based on relaxation time model:

$$\frac{\partial \varphi(\sigma_c; t, \mathbf{x})}{\partial t} = - \int_0^\infty \frac{\varphi(\sigma_c; t, \mathbf{x})}{\tau(\sigma, \sigma_c)} \xi(\sigma; t, \mathbf{x}) d\sigma, \tag{7}$$

where  $\tau$  is the characteristic relaxation time of damage. In general,  $\tau$  is determined by the true driving force and the threshold of mesoscopic units,  $\tau = \tau(\sigma, \sigma_c)$ .

Integrating Eq.(7) and substituting the definition of continuum damage (Eq.(3)) to the obtained equation, we obtain the evolution equation of continuum damage:

$$\frac{dD(t, \mathbf{x})}{dt} = f = - \int_0^\infty \frac{\partial \varphi(\sigma_c, t, \mathbf{x})}{\partial t} d\sigma_c = \int_0^\infty \int_0^\infty \frac{\varphi(\sigma_c, t, \mathbf{x})}{\tau(\sigma, \sigma_c)} \xi(\sigma; t, \mathbf{x}) d\sigma d\sigma_c \tag{8}$$

Where  $f$  is the dynamic function of damage (DFD), the agent linking mesoscopic microdamage relaxation and macroscopic damage evolution.

Similar to [4], in order to establish a closed, complete formulation, Eq.(8) should be associated with traditional, macroscopic equations of continuum, momentum, and energy, constitutive relationship and Eq.(4). This is a formulation with intrinsic trans-scale closure. Eq.(4) and Eq.(8) relate the macroscopic and mesoscopic scales.

With the abovementioned formulation, we numerically investigated the process of spallation and analyzed the effects of microstructural heterogeneity in terms of the distribution of broken threshold and the fluctuation of true stress on the propagation of damage.

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4. RESULTS

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**3. NUMERICAL ANALYSIS OF SPALLATION**

Consider a problem of damage evolution owing to the normal impact of a flying plate of thickness  $L$  with velocity  $v_f$  striking on a target plate, i.e. spallation. For simplicity, we assume that the impactor-plate system deforms in uniaxial strain. For the time-dependent damage process, an associated equations of continuum, momentum and damage evolution should be formed [10].

Similar to Weibull's statistical strength theory [11], we suppose that the initial distribution of threshold  $h(\sigma_c)$  follows Weibull distribution:

$$h(\sigma_c) = m \frac{(\sigma_c - \sigma^*)^{m-1}}{(\sigma^*)^m} \exp \left[ - \left( \frac{\sigma_c - \sigma^*}{\sigma^*} \right)^m \right] \quad (9)$$

where  $m$  is the Weibull modulus and  $\sigma^*$  the characteristic value of  $\sigma_c$ . The smaller Weibull modulus ( $m$ ), the broader the distribution becomes, and the material is more heterogeneous. On the other hand, larger  $m$  value represents a homogeneous material in which the stress threshold is almost constant.

There are various ways to determine the characteristic relaxation time of damage  $\tau$ [10]. For simplicity, we may assume that if  $\sigma \geq \sigma_c$ , the damage relaxation time is a fixed value  $\tau_D$ .

The fluctuation of true stress exerted on intact mesoscopic units can be dealt with different approaches. We assume the statistical distribution function of true stress  $\xi(\sigma, t, \mathbf{x})$  as follows:

$$\xi(\sigma, T, X) = \begin{cases} \frac{2F_0}{\sigma_2 - \sigma_1} (\sigma - \sigma_1), & \text{as } \sigma_1 < \sigma \leq \frac{\sigma_1 + \sigma_2}{2} \\ \frac{2F_0}{\sigma_2 - \sigma_1} (\sigma_2 - \sigma), & \text{as } \frac{\sigma_1 + \sigma_2}{2} < \sigma < \sigma_2 \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

where  $\sigma_1 = \frac{1-\sqrt{6kD}}{1-D}\sigma_0$ ,  $\sigma_2 = \frac{1+\sqrt{6kD}}{1-D}\sigma_0$ ,  $F_0 = \frac{1-D}{\sigma_0\sqrt{6kD}}$ , and  $k = \frac{(\sigma - \bar{\sigma})^2}{(D(\frac{\sigma_0}{1-D})^2)}$  is the stress fluctuation parameter.

Due to the trans-scale nature of spallation, it is helpful to non-dimensionalize the variables in the associated equations. The non-dimensionalization shows that some dimensionless numbers govern the damage evolution process in the target plate [10]. In this paper, we will focus on the effects of microstructural heterogeneity in terms of Weibull modulus  $m$  and stress fluctuation parameter  $k$ .

**4. RESULTS AND DISCUSSIONS**

**4.1 Effects of Weibull modulus  $m$  on damage evolution**

Fixing all other parameters, and set  $k = 0$ , we studied the effects of Weibull modulus  $m$  on the damage evolution. Fig.1(a) and (b) show the effects of  $m$  on the maximum damage in target plate at two different stresses. For low stress cases (Fig.1(a)), the increase of  $m$  leads to smaller damage. While for high stress cases (Fig.1(b)), the maximum damage increases with  $m$ .

What causes the opposite effects of  $m$  on damage evolution? Actually, if  $D \ll 1$  and loading time is long enough, the damage will reach a saturated value at constant loading. The saturated damage  $P(\sigma_0)$  can be expressed in terms of nominal stress:

$$P(\sigma_0) = 1 - \exp \left[ - \left( \frac{\sigma_0}{\sigma^*} - 1 \right)^m \right] \quad (11)$$

Obviously, for  $\sigma_0/\sigma^* < 2$  ( $\sigma^* = 1$ ), the saturated damage decreases with  $m$ ; while for  $\sigma_0/\sigma^* > 2$ , the saturated damage increases with  $m$ . This trend is consistent with what we obtained in numerical study.

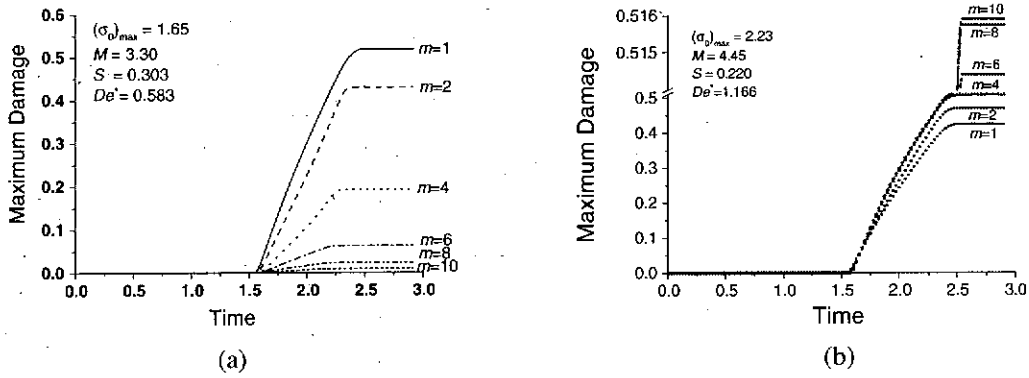


Figure 1. Effects of  $m$  on the maximum damage. (a): low stress cases,  $(\sigma_0)_{max} = 1.65$  (b): high stress cases,  $(\sigma_0)_{max} = 2.23$ .

4.2 Effects of stress fluctuation parameter  $k$  on damage evolution

Fixing all other parameters, and set  $m = 5$ , we studied the effects of stress fluctuation parameter  $k$  on the damage evolution. Fig.2(a) and (b) show the effects of  $k$  on the maximum damage in target plate at two different stresses. For low stress cases (Fig.2(a)), the maximum damage increases with  $k$ . While for high stress cases (Fig.2(b)), the maximum damage decreases with  $k$ .

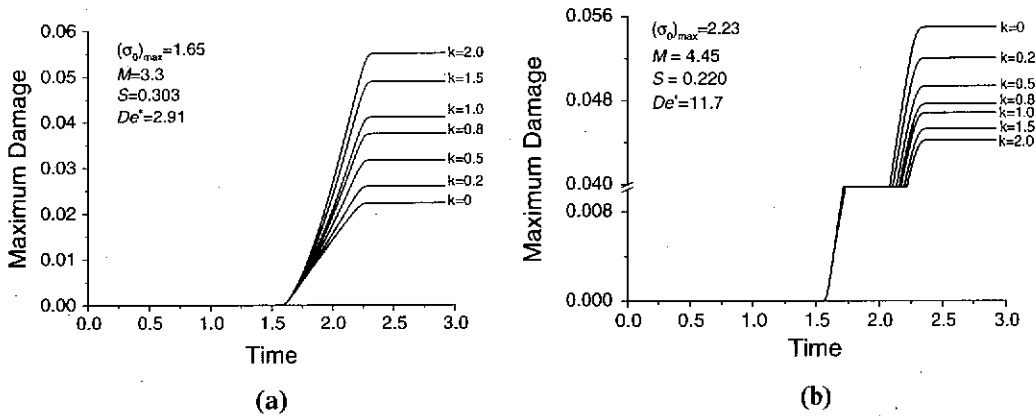


Figure 2. Effects of  $k$  on the maximum damage. (a): low stress cases,  $(\sigma_0)_{max} = 1.65$  (b): high stress cases,  $(\sigma_0)_{max} = 2.23$ .

Why does  $k$  have opposite effects on maximum damage in these two cases? The saturated damage curve (Fig.3) may give us a clue. The figure shows that the curve is concave up for  $\sigma_0 = 1.65$ . Hence, for  $\sigma_0 = 1.65$ , larger fluctuation leads to a higher mean value of saturated damage. On the other hand, the curve is concave down for  $\sigma_0 = 2.23$ ; and therefore, larger fluctuation results in a lower mean value of saturated damage. Therefore, for low stress, severe stress fluctuation causes higher damage, while for high stress, severe stress fluctuation causes lower damage.

5. SUMMARY

In this paper, a driven nonlinear threshold model with stress fluctuation is presented to study the effects of microstructural heterogeneity on continuum damage evolution. The spallation behavior

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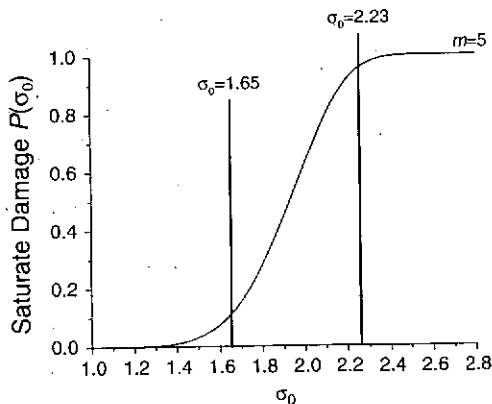


Figure 3. Curve of saturated damage vs. nominal stress ( $m = 5$ ).

of heterogeneity material is analyzed with this model. The heterogeneity of mesoscopic units is characterized in terms of Weibull modulus  $m$  of strength distribution and stress fluctuation parameter  $k$ . At high stress, the maximum damage increases with  $m$ ; while at low stress, the maximum damage decreases. In addition, for low stress, severe stress fluctuation causes higher damage; while for high stress, causes lower damage.

### Acknowledgments

The authors would like to acknowledge the support from NSFC(10302029, 10472118, 12372012, 10232040).

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