

Damage Localization – a Precursor to Failure

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Abstract: This paper reviews the concept of damage localization and its application to time-independent (quasi-static) and time-dependent processes in heterogeneous materials.

For quasi-static process in a heterogeneous material with Weibull distribution, damage can be expressed by a function of state variable. Accordingly, the criterion for damage localization depends on Weibull modulus m . In particular, damage localization can appear even though stress gradient remains fixed and it is closely related to strain localization.

For time-dependent process, damage evolution law can be expressed by a function of stress and damage $f(\sigma, D)$. Accordingly, the criterion for damage localization can be expressed by $f_D > f/D$. Provided damage evolution law can be expressed by kinetics of microdamage, it is found that intrinsic Deborah number D^* plays a key role in damage localization. The criterion for damage localization has been applied to two extremes of time-dependent processes: spallation under wave loading and creep.

Keywords: damage localization, heterogeneity, intrinsic Deborah number

1. Introduction

“There is the other sort of problems, i.e. strength and plasticity theory, for which even essential physical formulation is still not available for engineering applications”, Tsien wrote in his well-known book “Physical Mechanics” about 40 years ago^[1]. This still remains a challenge till now. “Although much has been learned, it appears that damage mechanics is a formidable problem whose difficulty may be of the same dimension as turbulence”, see Bazant and Chen^[2]. They identified micromechanical basis of damage, etc. necessary and potentially profitable topics for the immediate future.

What are the main causes for the long lasting

challenge? From engineering points of view, as noted by Becker et al.^[3], this problem might be a 6 layer hierarchy. For instance, for a vehicle, these are platform, system, subsystem, component, element and material. “Though mission demands are made at the top level, failure is initiated at the lowest level”^[3]. In fact, the initial damage, like microcrack or microvoid, may come from the lower microstructural level in materials. But the eventual rupture may result from the evolution of microdamage. This is particularly true for heterogeneous materials. Perhaps, the tragedy of Columbia may result from such a similar process.

This paper intends to review the concept of damage localization and its application to the understanding of failure diagnostics in time-independent (quasi-static) and time-dependent processes of heterogeneous materials in engineering.

2. Damage Localization

It is well known that with progressive deformation an initially uniform damage field may become localized. And finally, along the localized damage, an approximately two-dimensional highly-damaged region may form and lead to eventual failure surface. It is assumed that damage localization occurs once the rate of relative damage gradient $|\Delta D/\Delta x|/D$ (D denotes damage and Δx is the increment of spatial coordinate) starts to become positive^[4-6], namely,

$$\frac{\Delta}{\Delta t} \left(\frac{|\Delta D|}{\Delta x} / D \right) \geq 0, \quad (1)$$

where Δt is the increment of time. Accordingly, damage localization can also be formulated as follows,

$$\frac{\Delta}{\Delta t} \left(\frac{|\Delta D|}{\Delta x} \right) / \left(\frac{|\Delta D|}{\Delta x} \right) \geq \frac{\Delta D}{\Delta t} / D. \quad (2)$$

That is to say, provided damage increases, only when the relative increment (defined by $(\Delta z/\Delta t)/z$ for argument z) of damage gradient overtakes the relative increment of damage itself, damage localization can occur. Obviously, this is true in geometrical and kinetic sense. By means of this simple definition of damage localization, one can identify the occurrence of damage localization. In next two sections, this criterion is applied to two typical processes, i.e. quasi-static and time-dependent processes to explore what special features will govern the damage localization in the two processes.

3. Damage Localization in Quasi-static Process

3.1 Theoretical analysis

In order to study damage localization of quasi-static process in heterogeneous materials, a theoretical model is proposed. Suppose that a heterogeneous sample be driven by boundary displacement quasi-statically. Moreover, the sample consists of linear elastic but brittle units, namely all units have the same elastic modulus E_0 but different breaking stress threshold σ_c (later in Section 3, we will take normalized stress and strain, i.e. $\sigma = \text{stress}/\eta$ and $\varepsilon = E_0 \text{strain}/\eta$ where η is the position factor in distribution function $h(\sigma_c)$). To depict heterogeneity, it is assumed that σ_c follows a probability distribution function $h(\sigma_c)$, like Weibull distribution [7],

$$h(\sigma_c) = m\sigma_c^{m-1}e^{-\sigma_c^m}. \quad (3)$$

where m is the shape factor (Weibull modulus). The smaller the Weibull modulus m is, the more diverse the threshold σ_c is, that is to say, the more heterogeneous the sample is.

Under uniaxial monotonic loading, mean field approximation gives the relations between nominal stress σ , nominal strain ε and damage D as follows,

$$\sigma = F(\varepsilon) = \varepsilon e^{-\varepsilon^m}, \quad (4)$$

$$\sigma = G(D) = (1-D)[-\ln(1-D)]^{\frac{1}{m}}, \quad (5)$$

$$\varepsilon = [-\ln(1-D)]^{\frac{1}{m}}. \quad (6)$$

In this quasi-static process, instead of time t the ‘‘temporal’’ variable should be a governing variable, like the boundary displacement U . Substituting the relation $G(D)$ into criterion Eq.(2) and assuming a fixed stress gradient, i.e.

$$\frac{\Delta}{\Delta U} \left(\frac{\Delta \sigma}{\Delta x} \right) = 0, \quad (7)$$

one can derive the following critical condition for damage localization,

$$-G'' \geq G'/D. \quad (8)$$

Conditions (7) and (8) imply that even though stress gradient remains fixed damage localization can still appear. Figure 1 gives a schematic illustration of the criterion (8) for Weibull distribution. Notice that initially $G(D)$ increases with increasing damage D , i.e. $G' > 0$ and the function is convex, hence $G'' < 0$. Moreover, prior to damage localization $-G'' < G'/D$, whereas as soon as $D > D_{DL}$, damage localization appears. For Weibull distribution, criterion (8) gives the following to calculate the critical damage value D_{DL} for damage localization,

$$m \ln(1-D) [m(1-D) \ln(1-D) + 1 - 2D] + (m-1)D = 0. \quad (9)$$

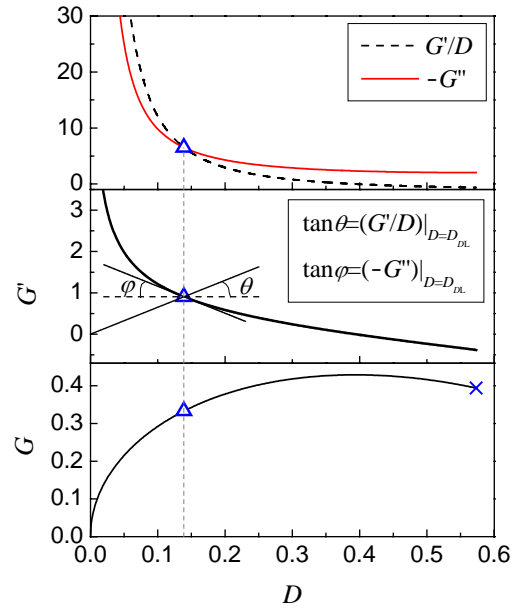


Figure 1 Curves of $-G''$, G'/D , G' and G versus damage D . Δ represents the critical damage to localization in accord with $-G''=G'/D$. \times represent catastrophic failure. (Weibull modulus $m=2$)

Similarly, the critical condition for strain localization can also be derived,

$$-F'' \geq F'/\varepsilon, \quad (10)$$

and

$$[1 + m \ln(1-D)]^2 + m^2 \ln(1-D) = 0. \quad (11)$$

We denote the solutions to Eqs.(9) and (11) by D_{DL} and D_{dL} respectively, i.e. the critical damages for the occurrence of damage localization and strain localization.

Later in the Section, we simply call the critical values as corresponding localization points. According to Eqs.(9) and (11), both D_{DL} and $D_{\varepsilon L}$ depends on Weibull modulus m , the characteristics of heterogeneity.

The nominal stress-strain curve and the nominal strain and damage at the two kinds of localization for $m=2, 4$ and 10 are shown in Fig.2. It can be seen that the stresses corresponding to D_{DL} and $D_{\varepsilon L}$ decrease with decreasing Weibull modulus m . That is to say, the more heterogeneous (less Weibull modulus m) the medium is, the more likely to localization it is. Moreover, both damage and strain localization happen prior to catastrophic failure. This indicates that localization might be a precursor to failure. In addition, strain localization (\square in Fig.2) always appears ahead of damage localization (\triangle in Fig.2) under mean field approximation.

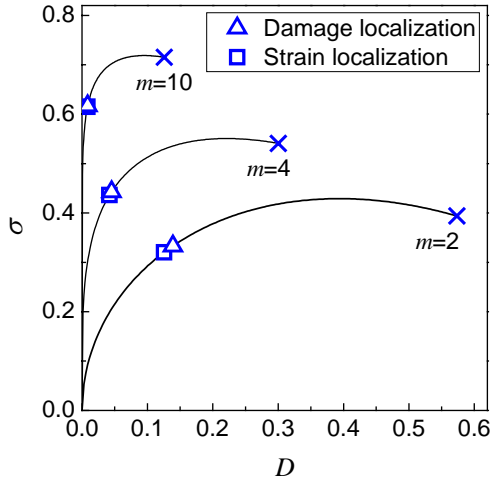


Figure 2 Nominal stress versus damage fraction curve (solid line). \square , \triangle and \times represent strain localization, damage localization and catastrophic failure.

3.2 Comparison of observed and predicted localization

As an example, typical raw and processed experimental nominal stress-strain curves $\sigma(\varepsilon)$ of a gabbro specimen with dimension of $5 \times 5 \times 13 \text{mm}^3$ under uniaxial compression are shown in Figs.3 and 4. Figure 5 shows seven patterns (corresponding to the states marked in Fig.3) of surface strain ε_{22}^* (subscript 2 denotes the loading axis) of the gabbro sample.

At the initial loading stage (between A and B in Figs.3 and 5), the surface strain field remains nearly homogeneous. This indicates that the mean field approximation should be valid before B. However, a small but high-strain-gradient domain appears at B, signifying the emergence of macroscopic strain inhomogeneity. This

is identified as strain localization point experimentally. Hereafter (from C to J in Figs.3 and 5), strain localization domain extends gradually and runs through the whole sample in the end. More importantly, it can be seen that the strain localization point is prior to the catastrophic point (J in Figs.3 and 5).

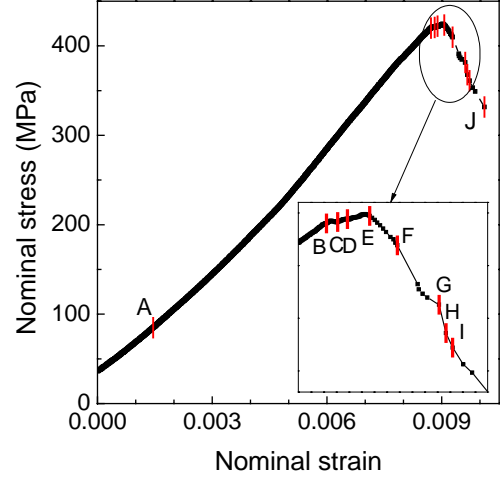


Figure 3 Raw experimental nominal stress-strain curve of a gabbro sample under uniaxial compression. Points A-J on the curve represent ten stages during the loading process, including the initial stage (A), strain localization point (B) and catastrophic point (J).

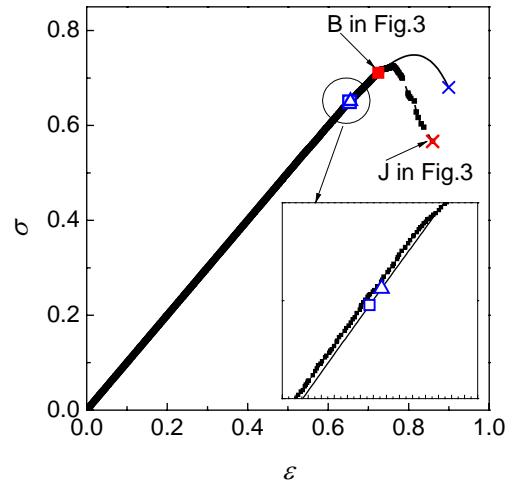


Figure 4 Processed dimensionless nominal stress-strain curve $\sigma(\varepsilon)$ (bulk solid line) and theoretical one (solid line). \blacksquare represents strain localization point determined by experiment, while \square and \triangle represent strain and damage localization points predicted by theory.

From the experimental measurement, the strain localization point corresponds to the following nominal stress, strain and damage (\blacksquare in Fig.4),

$$\varepsilon_{\varepsilon L}^{\text{Exp}} = 0.725, \sigma_{\varepsilon L}^{\text{Exp}} = 0.711 \text{ and } D_{\varepsilon L}^{\text{Exp}} = 0.0194. \quad (12)$$

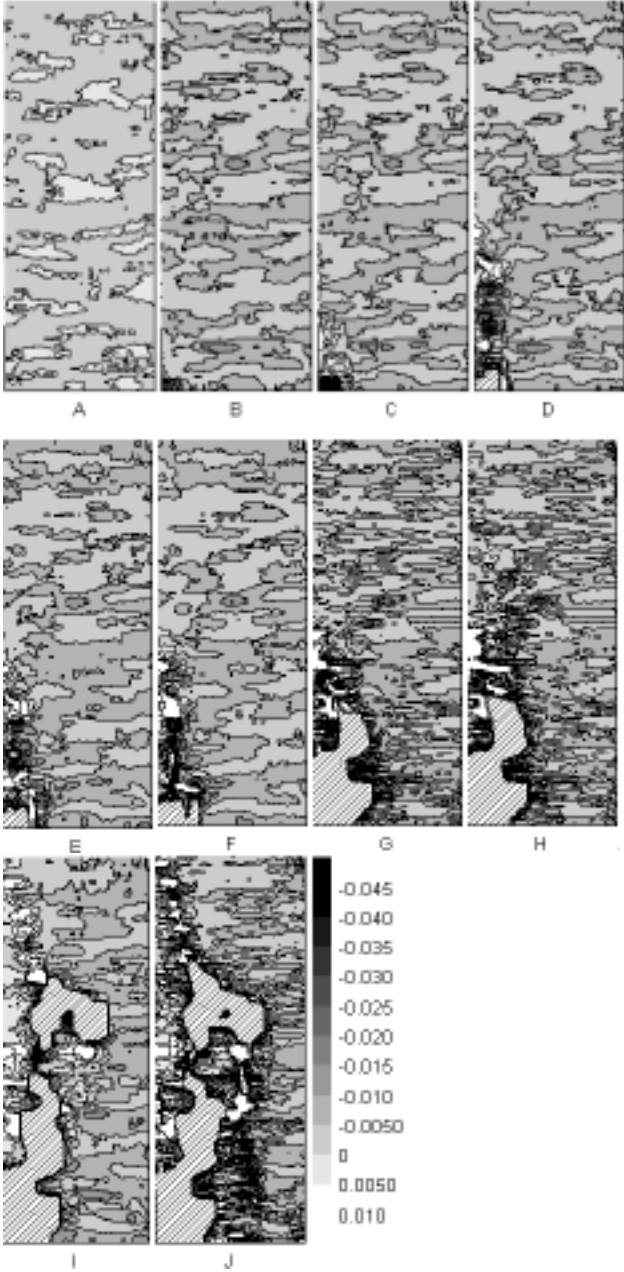



Figure 5 Evolution patterns of surface strain ε_{22}^* . The ten patterns, A-J, corresponding to the points marked by the same letters in Fig. 3.  indicates that the area where Digital Speckle Correlation Method ^[8-9] fails since the deformation is too big or the surface speckles fall off.

The raw experimental nominal stress-strain curve can be processed and the parameters included in the theoretical model introduced in Section 3.1 can be obtained from the processed nominal stress-strain curve as ^[9],

$$m=12.0, \eta=585.7 \text{ MPa and } E_0=53.0\text{GPa} . \quad (13)$$

Figure 4 shows the processed experimental nominal stress-strain curve (bulk solid line) and theoretical predicted curve (solid line) calculated by Eq. (4) with the above parameters. In addition, the theoretical predicted

nominal stress, strain and damage at damage and strain localization are (Δ and \square in Fig.4),

$$\begin{cases} \varepsilon_{\varepsilon L}^{\text{Theo}} = 0.653, \sigma_{\varepsilon L}^{\text{Theo}} = 0.650 \text{ and } D_{\varepsilon L}^{\text{Theo}} = 0.00592; \\ \varepsilon_{DL}^{\text{Theo}} = 0.655, \sigma_{DL}^{\text{Theo}} = 0.651 \text{ and } D_{DL}^{\text{Theo}} = 0.00613. \end{cases} \quad (14)$$

In this case study, the predicted strain localization appears prior to the experimental one. Also, the agreement between the theoretically predicted and experimental nominal stress-strain curves looks satisfactory prior to maximum nominal stress. And both strain and damage localizations occur prior to catastrophic failure.

4. Damage Localization in Time-dependent Process

4.1 Criterion for damage localization

According to damage mechanics, there should be a damage evolution law in time-dependent process. Generally speaking, the evolution law can be expressed by a function of stress σ and damage,

$$\dot{D} = f(\sigma, D). \quad (15)$$

The combination of the evolution law (15), the criterion for damage localization (2), and some approximations similar to the quasi-static case, gives the following form of the criterion for damage localization,

$$f_D > f/D, \quad (16)$$

where f_D denotes the partial differentiation of function f with respect to damage D . Now, the criterion for damage localization is no longer a geometrical description, like Eq.(2), but a physical assessment. This means that as soon as the tangent of the evolution law with respect to damage becomes greater than its secant on the section of current stress, damage localization is about to appear, see Fig.(6).

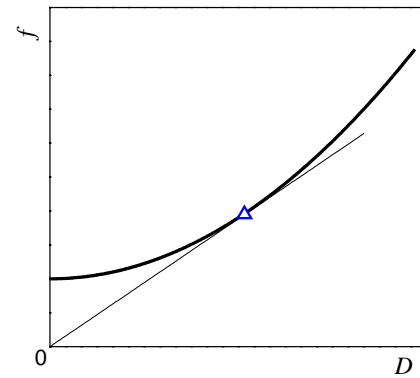


Figure 6 A schematic of criterion Eq.(16). The f - D plane is a section of constant stress. Bulk solid curve is the intersection of the evolution law $f(\sigma, D)$ and the section. Solid straight line is the secant of the curve equal to its tangent. Hence, Δ denotes damage localization.

4.2 Damage evolution law based on meso-kinetics

More interestingly, when turning to mesoscopic essence of damage evolution, we notice that the evolution of microdamage depends on two fundamental mesoscopic kinetics: the laws of nucleation rate n_N and growth rate V of microdamage. Generally, they can be expressed by,

$$n_N = n_N(c_0, \sigma) \text{ and } V = V(c, c_0, \sigma). \quad (17)$$

Based on the evolution equation of number density of microdamage, the other formulation of damage evolution law f in terms of these two mesoscopic kinetics is obtained approximately^[10-12],

$$f = \int_0^\infty n_N(c; \sigma) \tau(c) dc \cdot \left\{ 1 + \frac{\int_0^{c_f} n_N(c_0; \sigma) \int_{c_0}^{c_f} \tau'(c) dc dc_0}{\int_0^\infty n_N(c; \sigma) \tau(c) dc} \right\}, \quad (18)$$

where τ denotes the microdamage volume proportional to c^3 , and c_f is the time-dependent front of microdamage size,

$$t = \int_{c_0}^{c_f} \frac{dc}{V(c, c_0; \sigma)}. \quad (19)$$

4.3 Intrinsic Deborah number D^*

Notice that the expression of function f includes two meso-scopic time scales: the characteristic microdamage nucleation time $t_N = (n_N^* c^{*4})^{-1}$ and growth time $t_V = c^* / V^*$, where all letters with * denote the corresponding mesoscopic characteristic parameters. Clearly, the dimensionless number $D^* = t_V / t_N$ characterizes the ratio of the two intrinsic mesoscopic time scales: growth over nucleation. Since both time scales are relevant to intrinsic relaxation, D^* is called ‘‘intrinsic Deborah number’’. Furthermore, by means of the criterion for damage localization, (16), the magnitude of the critical damage to damage localization can be estimated by,

$$D_L \approx \frac{n_N^* c^{*5} \int_0^\infty \bar{\tau}(\bar{c}_f) \bar{n}_N(\bar{c}_0) d\bar{c}_0 \cdot \int_0^\infty \frac{\bar{\tau}(\bar{c}_f) \bar{n}_N(\bar{c}_0)}{\bar{V}(\bar{c}_0, \bar{c}_f)} d\bar{c}_0}{\int_0^\infty \bar{\tau}(\bar{c}_f) \bar{n}_N(\bar{c}_0) d\bar{c}_0} = D^* \cdot O(1), \quad (20)$$

where all variable with bar above are non-dimensional and normalized in order of $O(1)$, for instance $\bar{c}_0 = c_0 / c^* = O(1)$. Hence it is very clear that the intrinsic Deborah number D^* is a proper indicator of critical damage to damage localization.

A specific example of damage evolution under time-dependent loading, i.e. spallation in metals under stress wave loading, is shown in Fig.7. It is found that for the tested metals under such intensive but short stress wave loading, the intrinsic Deborah number D^* is very small, roughly speaking $D^* \sim (10^{-2} \sim 10^{-3})$. From the relationship of the intrinsic Deborah number D^* and the

critical damage D_L for damage localization, Eq.(20), damage localization should occur at the similar small damage of $(10^{-2} \sim 10^{-3})$. In experiments, according to the localization condition, we obtained the critical damage to localization $D_L \sim 4 \times 10^{-3}$ ^[12]. Clearly, the intrinsic Deborah number D^* does characterize the magnitude of the critical damage D_L , (also see Fig.8). The other important aspect is the time scale. The wave loading time in spallation is about $1 \mu s$, which is in the same range of the characteristic microdamage growth time scale t_V . This indicates that the competition between the imposed loading time and microdamage growth governs the spallation. Some further simulations demonstrate that the intrinsic Deborah number does give a proper indication of damage localization.

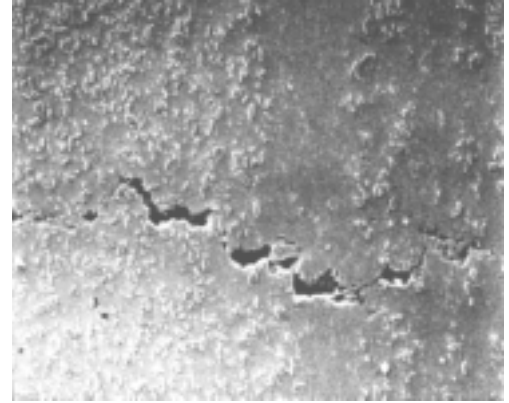


Figure 7 Microcracks on a section of an impacted aluminium alloy specimen.

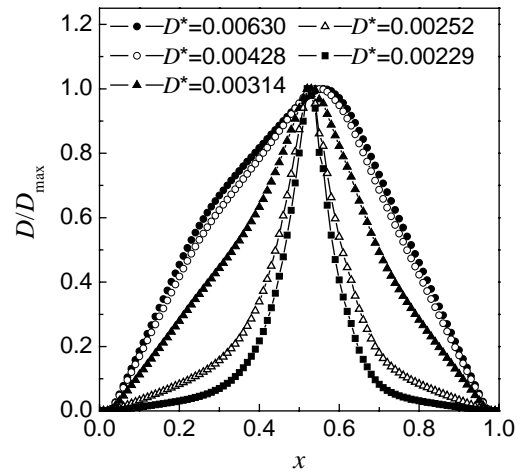


Figure 8 Effect of intrinsic Deborah number D^* on damage localization in spallation. All curves are calculated with fixed Mach number $M=0.0305$, damage number $S=0.153$, trans-scale Deborah number $De=65.9$ but different intrinsic Deborah number D^* (as shown in the Figure). The less the intrinsic Deborah number is, the more localized the damage becomes^[12].

On the other extreme of time-dependent processes, namely creep, the concept of damage localization seems to be effective too. We applied the criterion for damage localization to Hayhurst's experimental results of creep^[13]. For three metals: Cu, Al and 316 steel, the predicted and observed (in square bracket) ratios of damage localization time over whole life (namely damage $D=1$) are 0.66[0.83], 0.65[0.96] and 0.70[0.71]^[10]. The simple estimation seems to be reasonable.

5. Conclusion

In heterogeneous materials, damage localization may occur in either quasi-static or time-dependent process. As soon as damage localization appears, materials are subject to severe weakening, hence damage localization can be assumed as an early precursor to failure in engineering.

This paper reviews the concept of damage localization and its application to time-independent (quasi-static) and time-dependent processes in heterogeneous materials.

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